PARAMETRIC STUDIES OF SLOPE STABILITY ANALYSES USING THREE-DIMENSIONAL FINITE ELEMENT TECHNIQUE: GEOMETRIC EFFECT

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ABSTRACT

Many existing factors are found to affect the factor of safety (FS) for slope stability analysis due to the different analysis methods employed. Even when the same analysis method is used, such as finite element (FE) method; if the element type, node number, boundary conditions, and model dimensions change, different FS numbers will be present. The remarkable improvement on computational tools makes three-dimensional (3-D) FE approaches easier to access. Therefore, the purpose of this paper is to present the advantages of 3-D FE techniques used in slope stability analysis in terms of geometry and boundary conditions. Furthermore, this paper presents the parametric studies on several cases using 2-D limit equilibrium methods (LE), as well as the 2-D and 3-D FE analyses. The elastic-plastic Mohr-Coulomb failure criterion is employed in the analyses. The comparisons are made between these numerical methods for homogeneous and nonhomogeneous slopes. The strength reduction factor (SFR) obtained from the 2-D analyses is considered to be relatively conservative compared to 3-D FE analysis. However, the discrepancy on the SFR between 2-D and 3-D FE analyses is very limited unless the boundary conditions change on z planes. The boundary conditions assumed in the z planes are important to 3-D finite element analysis, while the effect of length in the z direction can be ignored if the roller-roller type of conditions is applied. The progressive failure can also be observed in the increments of analysis. Moreover, the limitations of LE and 2-D FE analyses are also identified when the complex failure mechanism of a slope is concerned.

Key words: Factor of safety, 3-D finite element analysis, limit equilibrium method, Mohr-Coulomb, strength reduction method, progressive failure.

1. INTRODUCTION

Slope stability has been an important issue in geotechnical problems. Many procedures based on 2-D and 3-D LE or FE methods (Griffiths and Lane 1999; Stolle and Guo 2008; Nian *et al.* 2012) have been proposed. The limitations for LE methods are mainly the slip surface determination and the side forces assumptions between slices or columns (Griffiths and Lane 1999; Lam and Fredlund 2003; Cheng and Yip 2007; Nian *et al.* 2012; Zhang *et al.* 2013). The finite element (FE) method was first introduced into geotechnical engineering by Clough and Woodward (1967). The FE method is a great tool to solve geotechnical problems due to its ability to model nonlinear stress- strain behavior of materials. The majority slope stability analysis was conducted using the 2-D FE method. Recently, the 3-D FE method became more attractive due to the progress of the computational tools including the computer and computer programs. Additionally, the 3-D numerical model created can make the analysis closer to the reality. The FE methods used in slope stability have been well documented as well. Therefore, to improve the confidence and present the advantages using the FE method, particularly for 3-D FE method are necessary. The background information for the LE method is also briefly reviewed in this paper to emphasize the advantages of FE applied in slope stability analysis. Meanwhile, the comparisons will be made between the 2-D, 3-D FE methods and the conventional LE method. The 3-D FE method has been used and is considered to be more rigorous in the slope stability. In many cases, particularly for slope stability analyses, the 2-D and 3-D slope stability analyses will not give a significant difference in the results of analysis (Griffiths and Marquez 2007). Hence, the factors to result in this insignificant need to be investigated. Meanwhile, the advantages of using 3-D FE analysis can also be identified.

2. BACKGROUND OF SLOPE STABILITY ANALYSES

2.1 Limit Equilibrium Analyses

Limit equilibrium (LE) methods have been the primary method used in estimating the stability of slope for decades. The procedures are based on finding a factor of safety (FS) for the slope. The FS represents the factor by which the shear strength must be reduced, so that the reduced strength just reaches equilibrium with the shear stress. In other words, when the FS is 1.0, the slope is in a state of limiting equilibrium. The definition of limit equilibrium can be expressed in the form of Eq. 1.

$$\tau = \frac{s}{\text{FS}} \tag{1}$$

where

s: Shear strength of the soil;

τ: Shear stress in the soil mass.

Slopes are usually classified as infinite slopes or finite slopes. An infinite slope mainly indicates a slope with translational failure along a single plane. The ratio of depth to failure surface to length of failure zone is relative small (< 10%). The soil type in this failure is usually granular.

The method of slices is a common method to solve slope stability problem using LE methods. This methodology divides a sliding mass into several slices, and moment and force equilibrium are summed for the entire sliding mass (Abramson *et al.* 1995). Numerous methods have been developed and are summarized in the following.

1. Ordinary Method of Slices: This is one of the simplest methods, which neglects all interslice forces and fails to satisfy for both entire soil mass and individual mass (Abramson *et al.* 2002). This method only satisfies the moment equilibrium. The method is convenient for hand calculations but less accurate than other procedures of slices (Duncan and Wright 2005).

2. Bishop's Simplified Method: The interslice shear forces are assumed to be zero by Bishop (1955), leaving the solution over-determined as horizontal force equilibrium will not satisfy for one slice. This method satisfies the vertical force equilibrium for each slice and the overall moment about the center of the circular trial surface.

3. Janbu's Simplified Method: Similar to Bishop (1955), Janbu (1973) also assumes zero interslice shear forces. This method leads the solution to satisfy the vertical force equilibrium and the overall horizontal force equilibrium for the entire slice mass. However, the method will not satisfy the moment equilibrium conditions. Janbu (1973) proposed a correction factor, f_0, to account for this incompleteness. An assessment based on 3-D simplified Janbu and Hovland methods was also reported by Ahmed *et al.* (2012).

4. Spencer's Method: Spencer (1967) assumes that the resultant of the side forces on each side is at the mid-height of each slice. However, no assumption is made as to the inclination of resultants. Therefore, inclination becomes one unknown which is a part of the solution. This method is considered to be more accurate than Bishop's method. Jiang and Yamagami (2004) proposed an extended Spencer's method to apply on 3-D slope stability analysis.

5. Morgenstern-Price Method: Morgenstern and Price (1965) present a method similar to Spencer's method. However, no assumptions are made on inclination or applied point of resultants and are parts of unknowns. This method requires a computer for solving the basic equation.

The Morgenstern-Price method is considered to provide the most rigorous limit equilibrium solution; however, the application of the method is quite cumbersome due to its complexity. The simplicity of Bishop's method and the ease of computation of the Bishop, Janbu, and Spencer methods make them the most practical LE solutions (Duncan and Wright 2005). However, some of the shortcomings of LE methods are concluded as follows: (1) stress-deformation relationship is not considered, (2) the stress distribution along the slip surface is the same, (3) the

assumptions of forces between "slices" or "columns" are improper, and (4) the progressive failure is not taken into consideration.

2.2 Background of Strength Reduction Method

In recent years, the strength reduction method (SRM) has increasingly been used associated with the finite element method (and finite difference methods) for slope stability analyses. The successful use of this method is also well documented (Zienkiewicz 1975; Matsui and San 1992; Griffiths and Lane 1999, Chang and Huang 2004, Stianson *et al.* 2011; Nian *et al.* 2012; Zhang *et al.* 2013). In the conventional LE method for slope stability analysis, the critical slip surface has to be determined. The average shear strengths along the slip surface are treated the same. The factor of safety (FS) used in the LE method is the ratio of the average shear strength to the driving shear stress along the potential slip surface. However, in the FE analysis, no potential slip surface has to be determined in advance; and the stress-strain relationship in the slope can also be considered. The analysis is based on the FE analysis and is unable to directly output a global FS. In order to quantify an equivalent FS from the LE methods, the SRM (Zienkiewicz *et al.* 1975; Ugai 1989; Griffiths and Marquez 2007; Nian *et al.* 2012; Zhang *et al.* 2013) is employed. The strength reduction factor (SRF) is applied to reduce the strength of soil to the point of failure. The SRF can be regarded as the factor equivalent to the FS in the LE analyses. If the strength parameters of the soil are c' and ϕ', the factors c_f' and ϕ_f' will bring the slope to failure, which can be defined as:

$$c_f' = \frac{c'}{\text{SRF}} \tag{2}$$

$$\phi_f' = \arctan\left(\frac{\tan \phi'}{\text{SRF}}\right) \tag{3}$$

In the application of the SRM in FE analyses, successive applications of increasing SRFs are applied to reduce the soil strength in the model until the solution cannot converge. In the ABAQUS FE model, a SRF field variable was created. The unconverged solution takes place when the largest strength reduction factor is reached. This largest SRF is regarded as the factor equivalent to FS given in LEM.

2.3 Three-Dimensional Finite Element Analysis

The 3-D FE model has been used in slope stability analysis for more than thirty years. According to Duncan (1996), the FS for 3-D analysis is greater than the FS for 2-D analysis. Only a few studies indicated that the FS for 2-D is greater than the results from 3-D models with inaccurate analyses such as the studies by Hovland (1977), Chen and Chameau (1983), and Seed *et al.* (1990). Azzouz and Baligh (1978) indicate that the use of the Ordinary Method in 3-D analyses is inadequate by assuming zero normal stress applied on the vertical interfaces. Also, Seed *et al.* (1990) found that all conditions of equilibrium cannot be satisfied in 2-D and 3-D analyses. Hutchinson and Sharma (1985) also pointed out that 2-D and 3-D analyses should give the same FS on cohesionless soils because the slip surface is a shallow plane and is parallel to the surface of the slope. Azzouz *et al.* (1981) also found that if the 3-D effects are ignored in the anal-

yses to back-calculate shear strengths, the results from back calculation will be overestimated. Griffiths and Marquez (2007) compare the results of 2-D slope and 3-D slope analyses. It was found that a higher FS is resulted from the 3-D analysis. When the width in the third direction is increasing, the analysis tends to be the plane-strain solution, which is close to the result from 2-D analysis. Therefore, the boundary condition assumptions are significant in 3-D FE analyses due to side forces that are not readily justified.

The 3-D FE method has been used and is considered to be more rigorous in the slope stability analysis compared to the 2-D FE model. In terms of equivalent plastic strain contour shown in the FE model, the potential slip surface can also be identified in the FE model. However, in many cases for slope stability analyses, the 2-D and 3-D slope stability analyses will not produce a significant difference in the results (Griffiths and Marquez 2007).

3. PARAMETRIC STUDY

The cases of slope conditions are presented in this paper including 2-D and 3-D LE or FE methods. In order to validate the analysis results, the parameters used in the model for soil are consistent with some previous studies done by Griffiths and Lane (1999) and Rockscience (2004). The element used in the 2-D FE model in this paper including the 4-node, 6-node and 8-node of plane-strain elements. The boundary condition on the bottom is hinged ($u_x = u_y = 0$), which is restricted on vertical and horizontal displacements. The vertical boundaries are modeled using a roller to confine the movement on the vertical direction only. In the 3-D FE model, the model is meshed using 8-node 3-D stress elements (C3D8) in ABAQUS. The boundary conditions are restricted on the bottom surface in vertical and horizontal directions. In addition, three types of boundary conditions could be assumed in the z planes: Roller-roller, fixed-roller and fixed-fixed. According to the prototype of a slope shown in Fig. 1, the boundary conditions applied in z direction of a slope are illustrated in Table 1. Three typical types of examples for parametric studies are presented and discussed herein.

3.1 Case Study 1

The geometry of a homogeneous slope without foundation is shown in Fig. 2. This case follows the analyses performed by Griffiths and Lane (1999) and Rocscience (2004), which were used as benchmark cases to study the applicability of 3-D FE analyses to slope stability. The slope angle is 26.25° or 2H:1V (H: horizontal distance; V: vertical distance) for this case. According to Griffiths and Land (1999), the adopted parameters are based on $c'/\gamma H = 0.05$ (c': cohesion of soil; γ: unit weight of soil). The height of the slope is assumed to be 40 m, thus the corresponding parameters are summarized in Table 2. In the 2-D FE analysis, three steps are created in the analysis: Initial, load, and reduced. In the load step, a soil strength corresponding to SRF = 0.5 is applied leaving the slope in a stable condition initially. After the load step is completed, the SRF starts to increase to reduce the soil strength until the iteration achieved cannot make the solution converge. Similar to 2-D analysis, the 3-D FE analysis using ABAQUS, the analysis steps remain the same. However, the length in the third dimension is defaulted to be 10m.
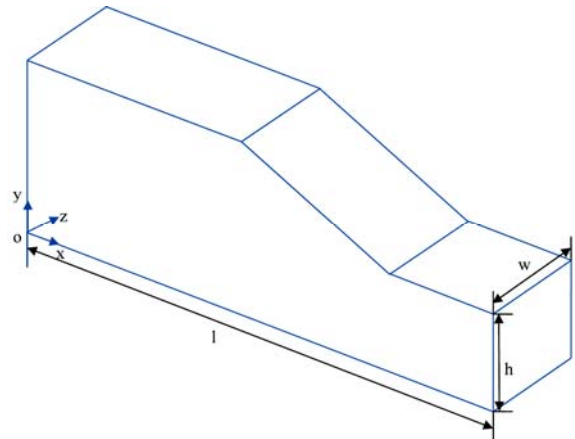
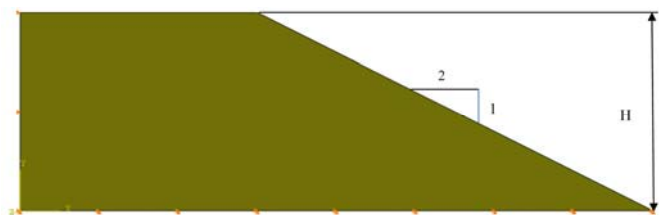

Fig. 1   Dimensions of 3-D FE slope model

Fig. 2   Homogeneous slope without foundation

Table 1   Boundary conditions in *z* planes in 3-D FE models

Plane	Fixed-fixed	Roller-fixed	Roller-roller
$x = 0$	$u_x = 0$	$u_x = 0$	$u_x = 0$
$x = 1$	$u_x = 0$	$u_x = 0$	$u_x = 0$
$y = 0$	$u_x = u_y = u_z = 0$	$u_x = u_y = u_z = 0$	$u_x = u_y = u_z = 0$
$z = 0$	$u_x = u_y = u_z = 0$	$u_z = 0$	$u_z = 0$
$z = w$	$u_x = u_y = u_z = 0$	$u_x = u_y = u_z = 0$	$u_z = 0$

*u_x, u_y and u_z = displacement in x, y and z directions, respectively.

Table 2   Slope dimension and material properties

c' (kPa)	ϕ' (°)	γ (kN/m³)	H (m)	E (MPa)	ν
40	20	20	40	100	0.3

The boundary conditions on both sides are assigned to be roller-roller which restricts the lateral movement on z direction ($u_z = 0$). However, the movement in x-y plane is permitted. It was observed that the analysis is independent on the length in z dimension if the roller-roller boundary conditions are applied. The SRF applied is equivalent to the FS output in LE analyses.

Analyses and Results

The slope stability analyses using the computer program, SLOPE/W, is shown in Fig. 3. The FS based on the Bishop method is shown as 1.386. The overall results are summarized in Table 3. Figure 4 shows the initial step when the equivalent plastic strain (PEMAG) starts to form at the 6th increment. In Fig. 5, the contour indicates the location of the potential slip surface formed in the failing slope at the last increment of the second step. The different contours shown in the figure can indicate the
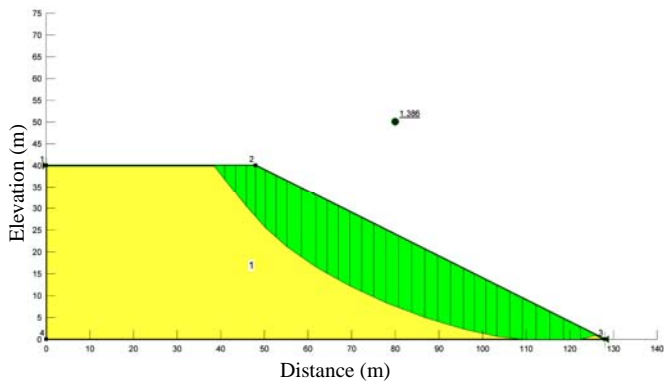
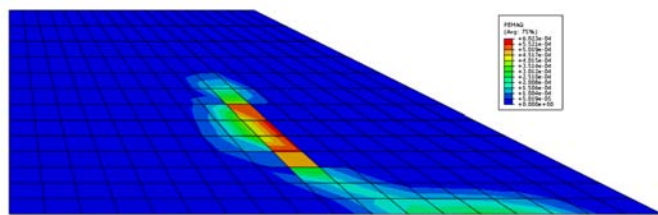**Fig. 3   Analysis using SLOPE/W (Bishop method) – slope failure, FS = 1.386**

Fig. 4   PEMAG contour at *t* = 0.456 (6th increment of second step)

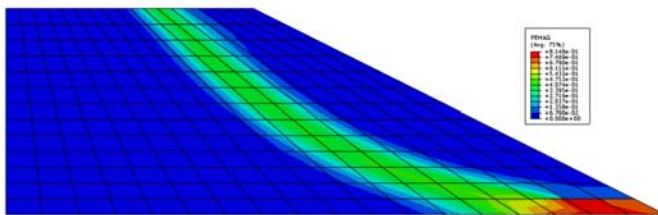**Fig. 5   PEMAG contour at *t* = 0.6077 (19th increment of second step)**

Table 3   Results of numerical analysis in slope stability, homogeneous slope *w/o* foundation

Method	Janbu	Bishop	Spencer	GLE	Ordinary	Morgenstern-Price
*Toe	1.298	1.383	1.382	1.385	1.318	1.385
**Slope	1.302	1.386	1.376	1.373	1.317	1.373

* Slip surface tangent to the boundary at toe

** Slip surface exits on slope

location where the progressive failure will begin. This is also one of the advantages of using FE method. Figure 6 shows the deformed meshed and the PEMAG contour which presents the slip surface in 3-D FE analysis. Both computational results (see Fig. 7) obtained from 2-D and 3-D FE analyses do not give a significant difference in terms of the SRF. The comparisons of the SRF using different analysis methods (LE and FE) are presented in Table 4. The FS for the case using 2-D FE analysis with the Strength Reduction Method (SRM) have been conducted by Griffiths and Lane (1999) and Rocscience (2004). It can be seen
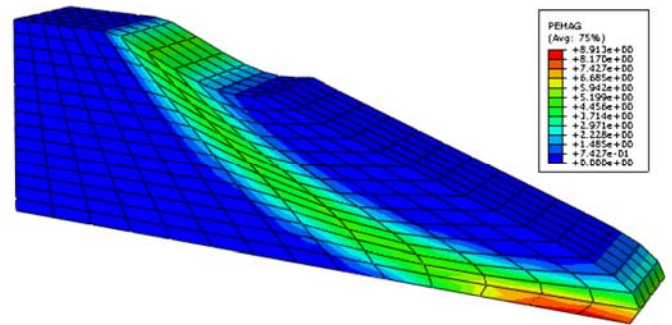

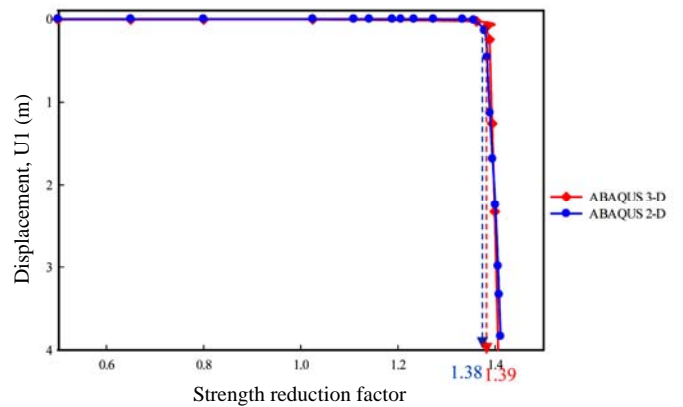**Fig. 6   PEMAG contour at *t* = 0.6435 (57th increment of second step)**

**Fig. 7   Displacement (U1) versus strength reduction factor for slope stability analyses**

Table 4   Comparisons of FS using LE and FE methods for homogeneous slope *w/o* foundation

| Method	Bishop	Griffiths	Rocscience inc.	ABAQUS 2D	ABAQUS 3D
FS	1.38	1.4	1.42	1.38	1.39

that the values of 1.4 and 1.42 are slightly higher than the FS of 1.38 using Bishop LE analysis. The results are comparable with other studies, and the SRF given from the 3-D FE analysis in this paper is slightly higher than the one from the 2-D FE analysis. However, the difference is not very significant.   The implication of a conservative 2-D slope stability analysis is that the unconservative estimation will be obtained based on the back-analysis of a failed slope (Arellano and Stark 2000; Griffiths and Marquez 2007).

3.2 Case Study 2

In this case, a 2H:1V slope is underlain by a 0.5H thickness for the foundation (see Fig. 8). The slope stability analyses are conducted using SLOPE/W (see Fig. 9), ABAQUS 2-D (see Fig. 10), and ABAUQS 3-D FE analyses (see Fig. 11). The parameters used in these analyses are the same as the parameter shown in Table 2. The slope is also assumed to be homogeneous. In the 2-D FE analysis, the left and right vertical boundary conditions are restricted in horizontal movement and the bottom is restricted
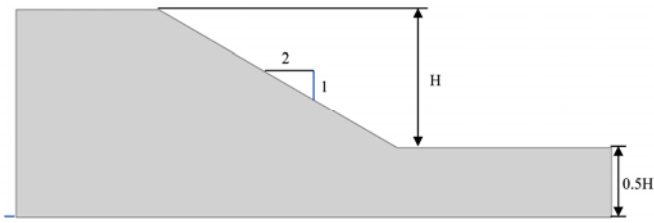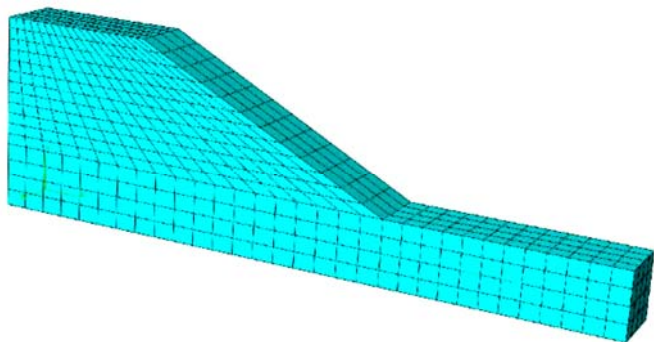
Fig. 8   Homogeneous slope with a 0.5H high of foundation underneath
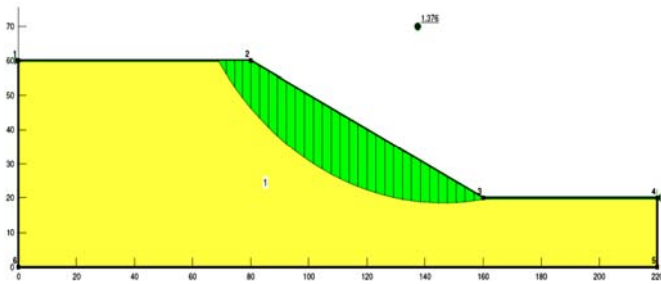
Fig. 9   **Analysis using SLOPE/W (Bishop method), FS = 1.376**
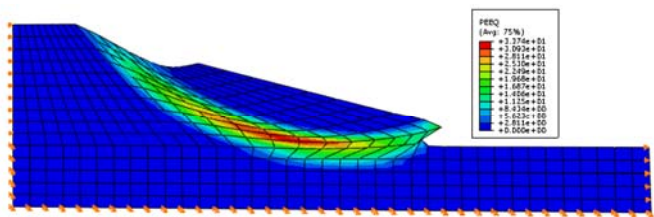
Fig. 10   Deformed mesh and PEEQ contour at the last increment

Fig. 11   Meshed 3-D FE model for a homogeneous slope with a foundation

in both horizontal and vertical directions. In the 3-D FE analysis, the 10m in *z* direction is adopted since the SRFs are independent on the length used in the third direction if the roller-roller boundary conditions are applied. In order to reduce the computational time, the length is defaulted to be 10m.

Analyses and Results

Similar analyses of these cases were also presented in literature, such as the papers by Griffiths and Lane (1999) and

Rocscience (2004). The results of slope stability analyses using the LE and FE methods are summarized in Table 5. Two types of slip surfaces, circular and log-spiral, are assumed in SLOPE/W which is based on LE methods. The results shown in the table are about 1.37 for circular failure slip surface assumed and 1.35 ~ 1.36 for log spiral slip surface, except in the Janbu and Ordinary slice methods. For further verification of the method used in this paper, a series of 2-D and 3-D FE analyses are also conducted. The 2-D FE analysis conducted by Griffiths and Land (1999) shows the SRF = 1.37. The SRF = 1.37 and 1.38 for the results from ABAQUS 2-D and 3-D FE analyses, respectively. Accordingly, the assumptions for the 2-D analysis result in more conservative solutions as compared to the 3-D FE analysis. Even the solution of the 3-D FE analysis gives a little higher FS, but the result is not significantly different between 2-D and 3-D FE models. The results are similar to the conclusions made previously (*i.e.* Duncan 1996; Hutchinson and Sarma 1985; Hungr *et al.* 1989; Griffiths and Marquez 2007; Nian *et al.* 2012).   Figure 12 presents the undeformed shape with the displacement contour in the 3-D FE model for slope stability analysis. The maximum soil movement will take place on the slope and goes through the foundation portion at the toe. The plastic strain contour comes with the deformed meshes as shown in Fig. 13 which can also indicate the location of the potential slip surface. The soil mass can be seen to fail along the log-spiral failure surface in the model. In the FE analyses, the maximum SRFs can be determined from the displacement and SRF relationship (see Fig. 14). The 3-D analysis shows a higher SRF than the 2-D analysis does. The advantages of the 3-D FE technique have been well- documented due to the complex geometries, boundary conditions, and property variations which can be modeled for the third dimension (Griffiths and Marquez 2007; Nian *et al.* 2012). In addition, the accuracy of the solutions is also dependent on the element type, node number, and the mesh method for the FE model. The appropriate partition technique is required to avoid the "mesh incomplete" and "computation" errors. The different element types, T6 and Q8, are also used to mesh the model in ABAQUS 2-D FE analyses for this homogeneous slope. The results are summarized in Table 6. The SRFs from ABAQUS FE models are 1.38 for 2-D (CPE8) analysis and 1.39 for 3-D (C3D8) analysis. If the T6 element is used in the ABAQUS 2-D analysis, the SRF = 1.41, and the SRF = 1.38 when the Q8 element is important. It is also observed that the number of nodes per element and the shape of elements can also result in the different outcomes.

Discussion

The SRF from the 2-D FE analysis by Griffiths (1999) is 1.37 which shows that the slip surface passes through the toe rather tangent to the foundation. The FS = 1.7 is found when the slip surface is assumed to be tangent to the base in LE method. The FE analysis can overcome this limitation to avoid assuming the location of the slip surface. Meanwhile, the stress distribution and the deformation along the slip surface are not the same. However, the stress distribution along the slip surface is assumed to be the same in LE methods. Hence, the progressive failure can only be observed using the FE methods. The failure begins at the plastic strain initiated in the early stage of the analysis. Based on the 2-D and 3-D FE analyses for this case, the SRF is similar to

**Table 5 Factor of safety for stability analyses using LE and FE methods**

Slope stability analysis method				
Limit equilibrium method			Finite element method	
Method	Type of failure surface/FS		Method	Factor of safety
	Circular	Log-spiral		
Janbu	1.290	1.280	ABAQUS 2-D (Q8)	1.38
Bishop	1.376	1.361	ABAQUS 2-D (T6)	1.41
Spencer	1.373	1.363	ABAQUS 3-D	1.39
GLE	1.378	1.348	Griffiths & Lane (1999)	1.37
Ordinary	1.309	1.287		
M-P	1.373	1.348		

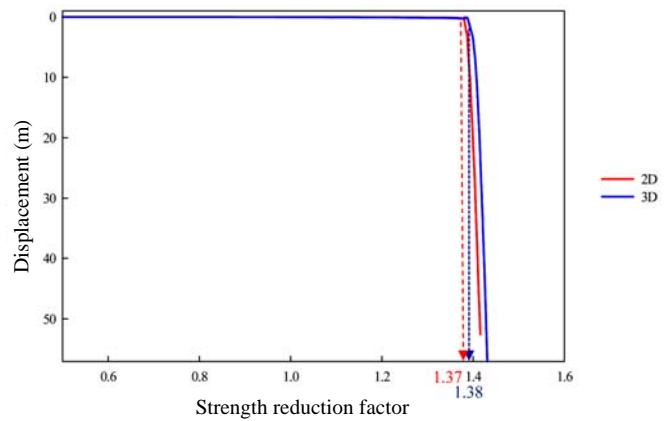**Table 6 Comparisons of other FE results and ABAQUS in slope stability, homogeneous slope**

Program	Griffiths and lane	Rocscience	ABAQUS 2-D
Element T6	NA	1.018	1.41
Element Q8	1.37	0.997	1.38
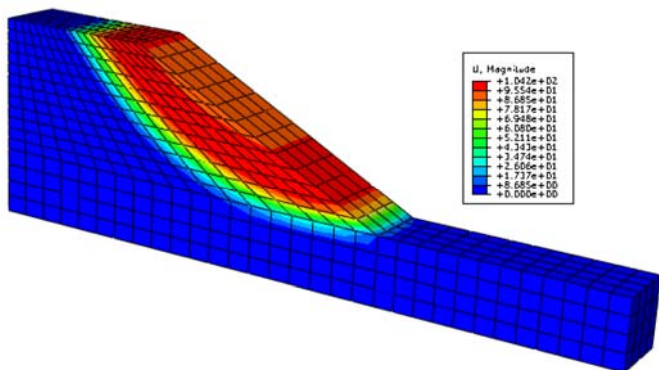
Fig. 12   Undeformed mesh and displacement contour at the last increment
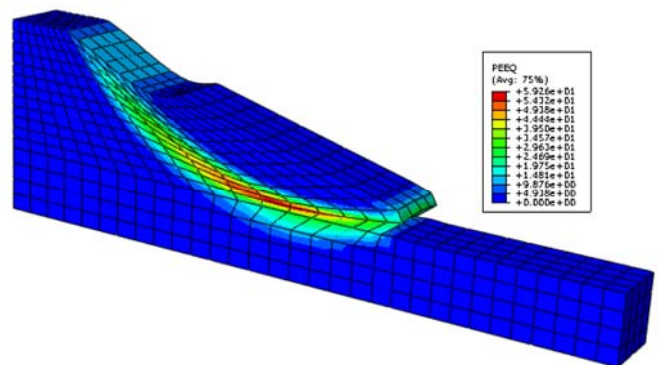
Fig. 13   Deformed mesh and PEEQ contour at the last increment for 3-D FE model

Fig. 14 Results of 2-D and 3-D FE with strength reduction method for slope stability analysis using ABAQUS

the results from LE methods by assuming the slip surface is either log-spiral shape or circular, except when using the Janbu and Ordinary methods. As for the element type and the node number adopted in the analyses, the triangular element with 6-node (T6) is found to give a less conservative FS. The 3-D FE model with roller-roller boundary conditions in z planes still does not give an obviously less conservative SRF compared to 2-D FE analysis.

3.3 Case Study 3

A nonhomogeneous example with a foundation underneath is shown in Fig. 15. The thickness of the foundation is equal to the height of the slope (H) which means $D = 2.0$ in the figure. The properties of the soil are summarized in Table 7. The undrained shear strength of the slope soil (C_{u1}) and the foundation soil (C_{u2}) are assigned to be the ratios from 0.5 to 2.0. To make the proper comparisons with the noted literatures (Griffiths and Lane 1999; Rockscience 2004), the ratio, $C_{u1} / \gamma H$, equal to 0.25 was also adopted. The 2-D FE and LE methods are both performed to validate the accuracy of the results of slope stability analysis. In the analysis, the undrained shear strength C_{u1} remains 200kPa, the C_{u2} changes in different stratum types in the analyses accordingly. The boundary conditions applied in 3-D FE model are very important, particularly in z planes in 3-D model. The boundary type will be addressed in each stratum type. The ratios of C_{u2} / C_{u1} equal to 0.5, 1.0, 1.5, and 2.0 are used in the analysis and discussed herein. Moreover, the special case of $C_{u2} / C_{u1} = $ 1.0 was also performed to compare with the results from Case Study 2. It can be seen the effect of the size of the underneath foundation assumed in slope stability analysis. In 2-D FE slope stability analysis, two types of mesh (triangular and quadrilateral) are also employed as shown in Figs. 16(a) and 16(b). The results of FE analysis and SLOPE/W based on $C_{u2} / C_{u1} = $ 0.5, 1.0, 1.5, and 2.0 are investigated and presented accordingly. Moreover, the 3-D FE analyses are also conducted to compare with the 2-D FE results. The four stratum types of the slopes are discussed as follows.

Stratum Type 1: $C_{u2} / C_{u1} = 0.5$

The resulting factors of safety using SLOPE/W and ABAQUS 2-D analyses with $C_{u2} / C_{u1} = 0.5$ are summarized in Table 8. The 2-D FE method using T6 and Q8 elements give the
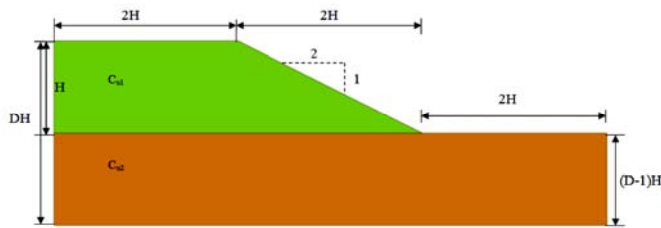
Table 7   Slope dimension and material properties

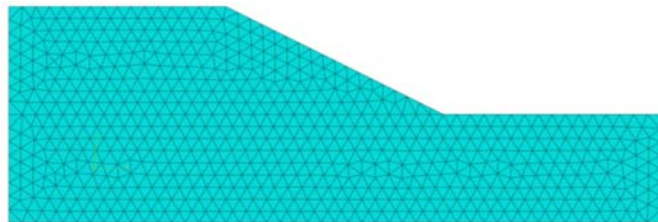| E (kN/m^2)	ϕ (°)	γ (kN/m^3)	ν	C_{u1} (kN/m^2)	H (m)
100000	0.01	20	0.3	200	40

Table 8   Results of numerical analysis in slope stability, $C_{u2}/C_{u1} = 0.5$, 2-D

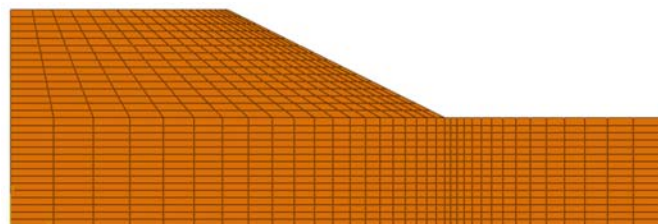| Method	Janbu	Bishop	Spencer	GLE	Ordinary	M-P[*]	ABAQUS (T6)	ABAQUS (Q8)
Circular	0.893	0.876	0.876	0.876	0.876	0.876	0.88	0.88
Wedge	0.822	0.801	0.760	0.267	0.969	0.737	NA	NA

[*] Morgenstern and Price

Fig. 15   Geometry of nonhomogeneous slope with foundation


**Fig. 16   (a) Triangular and (b) quadrilateral meshed 2-D FE model in ABAQUS**

same results of SRF for slope stability analyses; both SRFs are 0.88. Compared to the results of using the LE methods, the values using the ABAQUS 2-D analyses are comparable to the results using these noted methods. The SRF is also given approximately 0.88, which is reported by Griffiths and Lane (1999). In this case, the boundary condition applied in z-planes is also roller-roller type. The length in z direction is 10m. The equivalent plastic strain contours (PEMAG) from the ABAQUS 2-D and 3-D analyses are shown in Figs. 17(a) and 17(b). The potential slip surface goes through the base, and the shape is circular which is similar to the assumption made in SLOPE/W. Figure
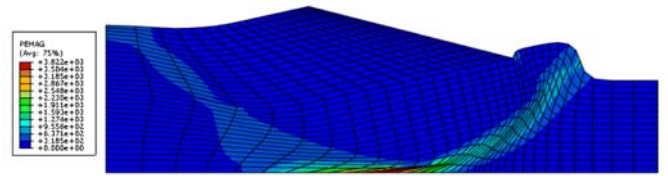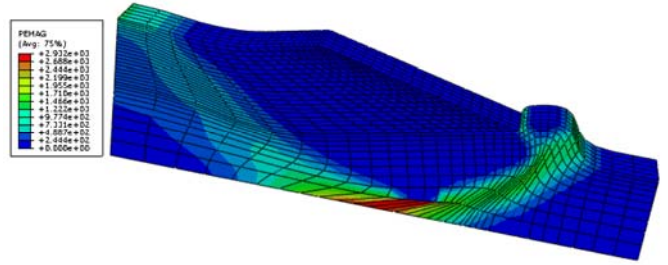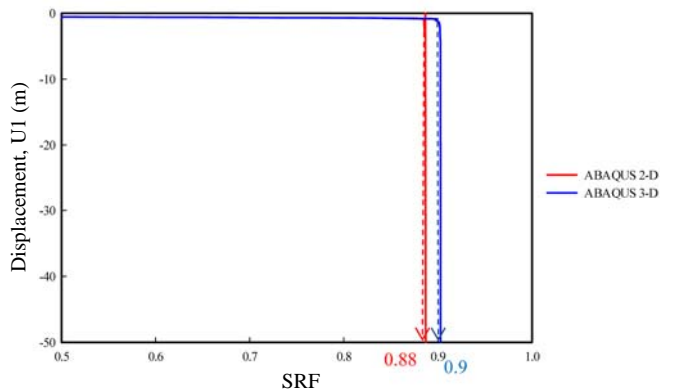
Fig. 17   (a) 2-D FE model, (b) 3-D FE model, (c) computation results using SRM ($C_{u2}/C_{u1} = 0.5$)

17(c) compares the computational results in ABAQUS with SRM. It can be seen that the SRF = 0.88 in the ABAQUS 2-D analysis, and the SRF = 0.9 in the ABAQUS 3-D analysis. The ABAQUS 3-D model gives a slightly higher SRF, However, the difference is still insignificant. Based on the slope stability analysis, the limit equilibrium and finite element methods give similar factors of safety and failure mechanisms in this case.

Stratum Type 2: $C_{u2} / C_{u1} = 1.0$

The given FS' from analyses for this case, $C_{u2} / C_{u1} = 1.0$ are summarized in Table 9. This case is actually a homogeneous slope condition and similar to Case 2 in this paper. The difference between these two cases are the thickness of the underneath foundations. For the case here, the foundation is twice as thick as that analyzed in Case Study 2. The SRF for the slope using FE method is similar to most of the LE methods except for Janbu's solution. The reasons for Janbu's lower factor of safety are not readily apparent. The results for the slope stability analysis using the ABAQUS 2-D analyses are slightly higher than these for LE methods: 1.50 with T6 element and 1.49 with Q8 element. The equivalent plastic strain contour is shown in Fig. 18(a) for the 2-D FE analysis and Fig. 18(b) for 3-D FE analysis. The boundary condition adopted in z planes in this stratum is also roller-roller type and the dimension in z direction remains 10m. Both of

Table 9   Results of numerical analysis in slope stability,
C_{u2}/C_{u1} = 1.0, 2-D

Method	Janbu	Bishop	Spencer	GLE	Ordinary	M-P	ABAQUS (T6)	ABAQUS (Q8)
Circular	1.408	1.477	1.477	1.477	1.477	1.477	1.50	1.49
Wedge	1.390	1.484	1.365	0.425	1.621	1.369	NA	NA

(a)

(b)

(c)
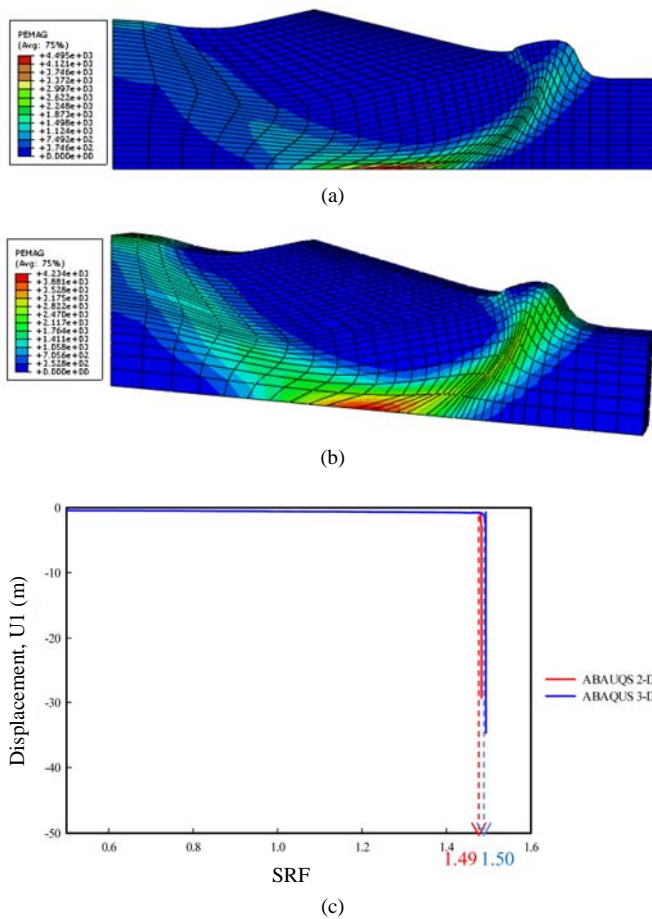
Fig. 18 (a) 2-D FE model, (b) 3-D FE model and (c) Computation results using SRM C_{u2}/C_{u1} = 1.0

them present the same location and similar type of the potential slip surface based on the location with the maximum plastic shear strain. The failure mechanism is similar to the case of C_{u2} / C_{u1} = 0.5, but different from the failure mechanism observed in Case Study 2. However, the SRF for the example given in Case Study 2 is 1.38 for the 2-D analysis and 1.39 for the 3-D analysis. Both of these SRFs in Case Study 2 are lower than the SRFs shown in this case. Hence, the size of the foundation presented or adopted will also influence the results of the analysis and the failure type. From Fig. 18(c), the SRFs given from the 2-D and 3-D FE analyses for this stratum type are computed to be 1.49 and 1.50, respectively. The SRF is governed by the dominated failure surface in the FE analysis. The greater circle tangent to the firm base will yield a higher SRF.

Stratum Type 3: C_{u2} / C_{u1} = 1.5

Continuing to increase the undrained shear strength of the soil in the foundation (C_{u2}), the computed FS for the case of C_{u2} / C_{u1} = 1.5 using LE methods by SLOPE/W and FE methods by ABAQUS are all summarized in Table 10. The majority of the LE methods provide an FS for this case is 2.078, except for the Janbu's method. The reason for the discrepancy between Janbu's result and the other limit equilibrium methods is unclear. The Wedge form of slip surface assumed in SLOPE/W gives the FS ranging from 0.653 to 1.967, and the results are found to be unreasonable. The analyses using T6 and Q8 elements provide similar SRF in 2-D FE analyses, which are 2.09 and 2.08, respectively. The equivalent plastic strain contour in the 2-D FE analysis using ABAQUS for this case is shown in Figs. 19(a) and 19(b). If a comparison is made between Figs. 19(a) and 19(b), the initial stage can show the failure starts close to the bottom of the underneath foundation. Then, another slip surface is formed on the boundary between the slope and the underneath foundation until the slope fails. For the 3-D FE analysis, the boundary conditions in z planes are still applied to be roller-roller type. The length in z direction is also assumed to be 10m. The failure mechanism shown in Fig. 19(c) is similar to what is observed in 2-D FE analysis. It was found by Griffiths and Lane (1999) that the SRF for this slope is 2.10 using the 2-D FE method. In Fig. 19(d), the SRFs for the 2-D and 3-D FE analyses are computed to be 2.08 and 2.09, respectively. These surfaces formed simultaneously in the analyses and which is a weaker surface cannot be ascertained in LEM analyses. However, in FE analyses, the weaker failure surface can be identified from the different time step. These results also make good agreements with the results presented by Griffiths and Lane (1999) for the same slope case.

Stratum Type 4: C_{u2} / C_{u1} = 2.0

A strength ratio, C_{u2} / C_{u1} = 2.0 for the slope is also performed. The results of the computation using both LEM and 2-D FE methods are summarized in Table 11. The analyses performed with T6 and Q8 elements have different but close results with a FS of 2.15 and 2.11, respectively. The equivalent plastic strain contour in the 2-D FE analysis using ABAQUS is shown in Figs. 20(a) and 20(b). The potential slip surface in terms of maximum plastic strain found in the FE analysis occurs at the toe on the slope and tangent to the foundation. If a comparison is made between Figs. 20(a) and 20(b), the initial stage will show the failure starts at the boundary between the slope and underneath foundation. For the 3-D FE analysis, the boundary conditions are also restricted the movement in the z direction for both z planes, and the length remains to be 10m. The 3-D failure type is presented in Fig. 20(c). In this case, the ABAQUS 2-D analysis performed is found to give a higher FS than the ABAQUS 3-D does. The computational results for 2-D and 3-D analyses are shown in Fig. 20(d) and the SRFs are 2.11 and 2.10, respectively. However, the discrepancy is still not very significant. The FE and the LE methods diverge on their results for this case due to the different assumptions for the type of slip surfaces. The results for FE analysis are closer to that for LE methods when the slip surface is assumed to form in the slope. Meanwhile, if the slip surface is assumed to pass through the foundation in LE methods, the analysis results are presented to be lower. The FE results make intuitive sense when the foundation soils are twice as strong as the

Table 10 Results of numerical analysis in slope stability, $C_{u2}/C_{u1} = 1.5$

Method	Janbu	Bishop	Spencer	GLE	Ordinary	M-P	ABAQUS (T6)	ABAQUS (Q8)
Circular	1.915	2.078	2.078	2.078	2.078	2.078	2.09	2.08
Wedge	1.776	1.916	1.939	0.653	1.802	1.967	NA	NA

Table 11 Results of numerical analysis in slope stability, $C_{u2}/C_{u1} = 2.0$
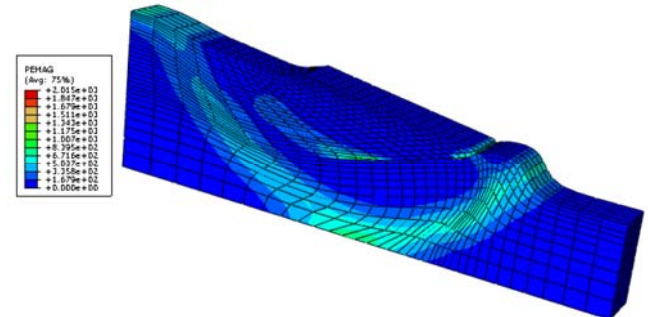
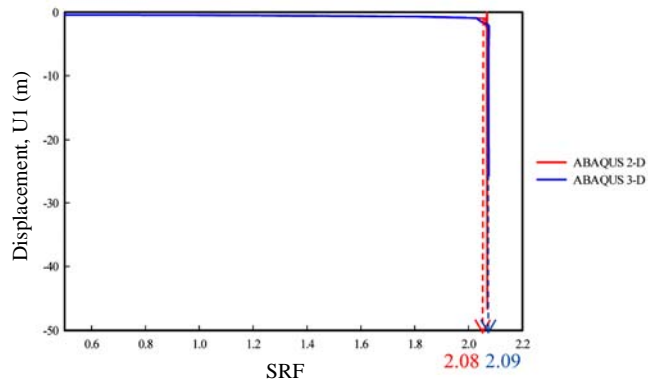| Method	Janbu	Bishop	Spencer	GLE	Ordinary	M-P	ABAQUS (T6)	ABAQUS (Q8)
Slope	2.083	2.125	2.125	2.125	2.125	2.125	2.15	2.11
Base	1.940	1.977	1.969	1.967	2.017	1.997	NA	NA
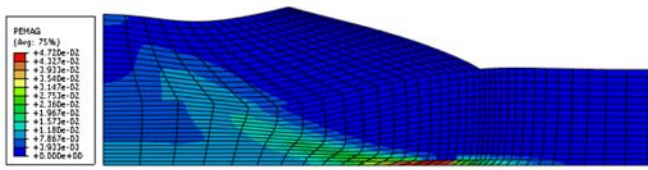
(a)
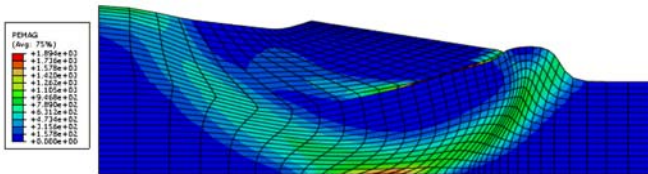
(b)

(c)

(d)

Fig. 19 (a) 5th increment of the second step, (b) the last increment of the second step in 2-D FE model, (c) PEMAG contour in 3-D FE model, (d) computation results using SRM ($C_{u2}/C_{u1} = 1.5$)
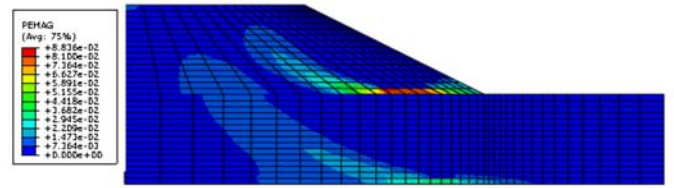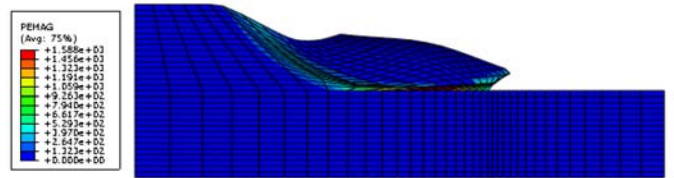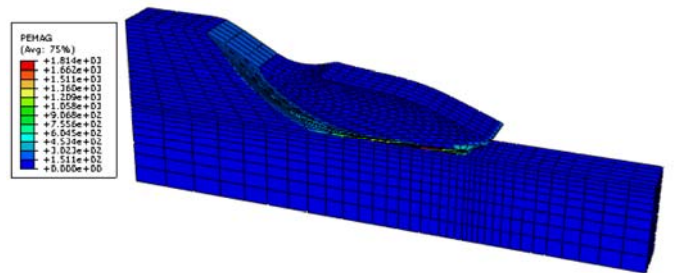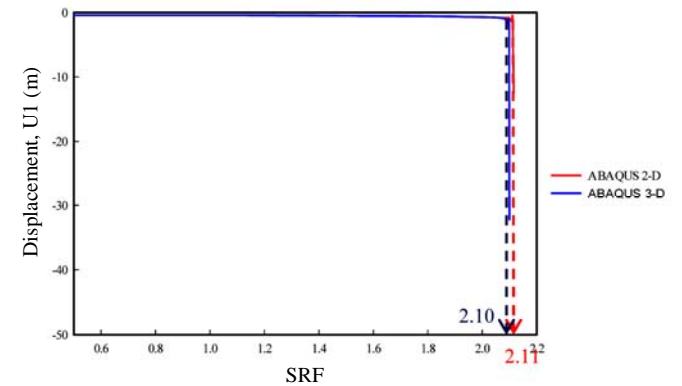

(a)

(b)

(c)

(d)

Fig. 20 (a) 5th increment of the second step, (b) the last increment of the second step in 2-D FE model, (c) PEMAG contour in 3-D FE model, (d) computation results using SRM ($C_{u2}/C_{u1} = 2.0$)

slope soils, there is not an expectation that a failure surface would necessarily move into the foundation. However, this non-homogeneous effect can easily be ignored in the LE methods using slices. These 2-D FE slope stability analysis results are similar to those presented by Griffiths and Lane (1999).

For the same case, if the length of the slope in the third dimension, w, changes (see Figs. 1 and 21(a)), the SRF is found to be independent of w when the boundary conditions on z planes are roller-roller type. The SRF remains a constant regardless of the length of w. However, if the boundary conditions on z planes change, the SRF computed will change. To make $w = 2h = 80$m, Fig. 21(b) shows the slope failure for the boundary conditions in z planes are free-fixed type, and Fig. 21(c) shows the failure mode for boundary conditions in z planes are applied to be the fixed-fixed type. Figure 21(d) presents the computational results

(a)

(b)

(c)

(d)
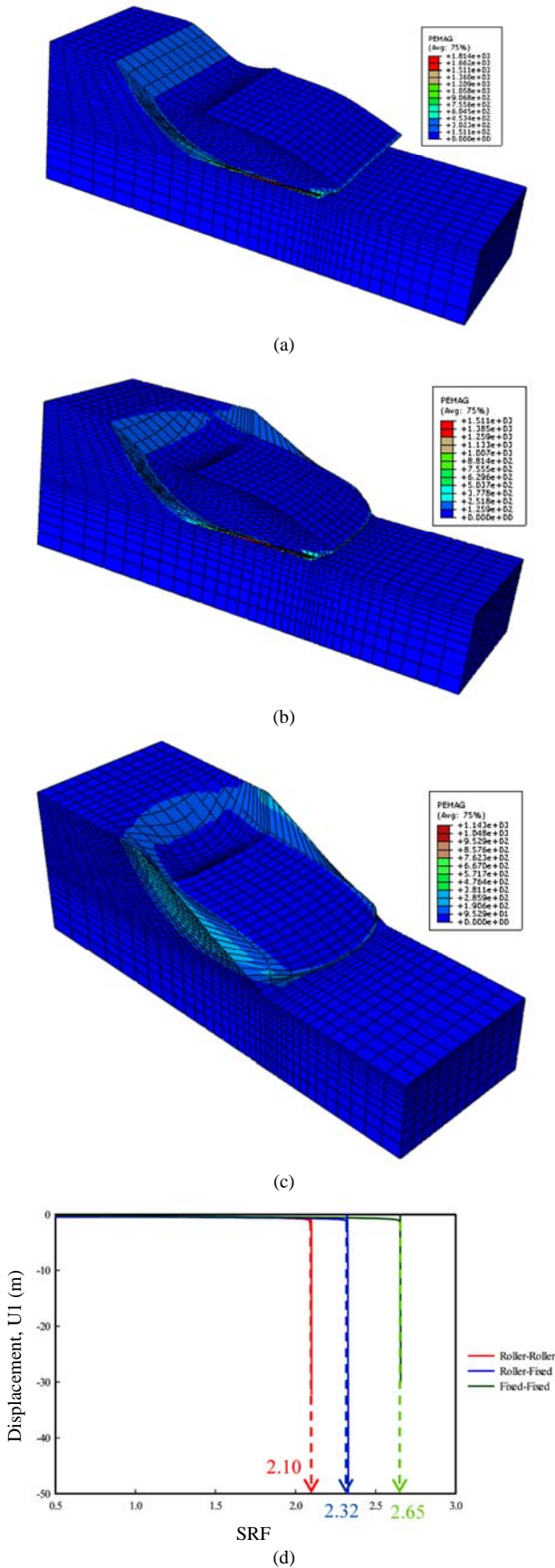
Fig. 21 (a) Roller-roller, (b) roller-fixed, (c) fixed-fixed boundary conditions on z planes, (d) computation results using SRM ($C_{u2}/C_{u1} = 2.0$; $L/H = 2.0$)

for the 3-D FE method by applying different types of boundary conditions in z planes. The SRF for fixed-fixed conditions on z planes is 2.65; SRF = 2.32 for free-fixed conditions; and FS = 2.10 for free-free conditions. The boundary conditions assumed on the z-planes are found to be more significantly than the dimension assumed along the z-axis.

4. DISCUSSION

Comparing the results for the case of $C_{u2} / C_{u1} = 1.0$ in FE analysis to the values of the homogeneous slope with foundation in Case Study 2, the potential slip surface is in a different location. In Case Study 2, the thickness of the foundation is only half of the height of the slope ($D = 1.5$, see Fig. 15). For the example in Case 3, the thickness of the foundation is the same to the height of the slope ($D = 2.0$). When $D = 1.5$, the potential slip surface occurs within the slope with a lower factor of safety; however, for $D = 2.0$, the potential slip surface forms circularly through the bottom of the foundation results in a higher SRF. Based on the similar failure surface assumed in LE methods, the results are similar to both the FE and the conventional LEM methods using ABAQUS and SLOPE/W.

If the ratio $C_{u2} / C_{u1} = 1.5$ is used in the FE analysis, slip surfaces will form in two locations simultaneously to the end of analysis; one is along the boundary of slope and foundation, and the other one is a great circle tangent to the bottom of the foundation. Both slip surfaces dominate the failure mechanism of the slope stability which is not considered in LE methods. Even there is not a significant difference on FS given for both FE and LE analyses, however, the failure mechanism becomes very important when a construction design on such a slope needs to be proposed. Either of the slip surfaces cannot be ignored.

As for the case of C_{u2} / C_{u1} equals to 2.0, the undrained shear strength of the foundation soil is much stronger than the soil in the slope. Thus, the slip surface will occur circularly along the boundary between the slope and the foundation only, not through the foundation soil. Only the soil in the slope governs the failure mechanism of the slope. Under this condition, the given SRF is the highest among these examples in Case Study 3. The case for $C_{u2} / C_{u1} = 2.0$ has a similar failure mechanism to Case study 1 which is a homogeneous slope without a foundation.

The 3-D FE analyses in this case do not show the attractive point compared to the 2-D FE analyses if the geometry is uniform in the third dimension. The SRF, however, is found to be independent on the third direction, H. Additionally, the thickness of the foundation assumed in the analyses and the type of boundary conditions of z planes applied will affect the SRF outputs in the 3-D FE analyses. The $SRF_{(Fixed-Fixed)} > SRF_{(Fixed-Roller)} > SRF_{(Roller-Roller)}$ is observed in the 3-D FE computations.

5. CONCLUSIONS

Three cases of slopes are analyzed using numerical methods presented in this paper, the range of boundary assumed is very important to a slope stability analysis. Based on these cases, the foundation and the thickness of the foundation attached to the slope in the FE analyses are found to affect the computational results. Even 3-D FE analysis has been considered less conservative by many previous researchers compared to 2-D FE analysis; however, the discrepancy of the results is insignificant if the geometry is uniform in the third direction unless the boundary conditions in z planes change. Based on these comparisons and observations in the results of computations, several conclusions can be drawn as follows.

1. The FE analysis used takes account of stress-strain relationship and the progressive failure mechanisms which cannot properly be considered in LE methods. Moreover, LE methods are also found to have limitations to conduct complex soil geometry.

2. The thickness of the foundation underneath a homogeneous slope affects the computational result which is associated with the failure mechanisms. The failure mechanism will determine the SRF. For a homogeneous slope with a thicker foundation, a deeper slip surface will be located and gives a higher SRF.

3. The progressive failure can be observed using the FE element method which is unable to be considered in LEM. If the remediation method has to be adopted to increase the slope stability. Understanding the location of the failure initiated in early stage using FE methods is important. It is helpful to design a proper remediation method to stabilize the slope in the early stage.

4. The 2-D and 3-D FE methods will not give significant difference on SRFs if the geometry relatively simple and the boundary conditions in z planes are less restricted. Even 3-D analysis has been widely regarded as less conservation compared to 2-D analysis. However, the advantages of the 3-D FE analysis will not be revealed unless the proper boundary conditions are employed.

5. The boundary conditions assumed for a slope are important to the 3-D FE analysis. The analysis results show the $FS_{(fixed-fixed)} > FS_{(fixed-roller)} > FS_{(roller-roller)}$. Hence, the selection of the length in z direction becomes more important if the boundary conditions other than the roller-roller type on both z- planes are assumed.

6. The advantages of using 3-D FE analysis are (a) able to monitor the progressive failure of a slope, (b) making the geometry closer to a real slope, and (c) to apply the proper boundary conditions in z planes to obtain a reasonable SRF.

7. In the future, the study for the effect of different boundary conditions applied in z planes to the length in z direction will need to be further investigated.

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