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Note:
A SIMPLE METHOD FOR CALCULATING THE SEEPAGE FROM EARTH DAMS WITH CLAY CORE

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ABSTRACT

There are various evaluation methods available for the discharge rate passing from the body of dam 1) Analytical methods, 2) Numerical methods and 3) Experimental methods. In the initial stages of dam design, it is necessary to approximately calculate the seepage and board of the project. In this research, almost 600 geometric models have been considered for embankment dam with clay core which have been solved numerically. These models have vertical and oblique cores with distinct thicknesses. additionally, the height of the water in the reservoir, the width of dam crest and the central angle of the core are among the variables subject of study. The seepage passing through the dam core has been studied for each model. Ultimately, the results of the analysis have been used to develop new approximate equations to calculate the discharge rate passage from the body of the dam. Results obtained from the approximate equations have been compared with the results from numerical simulations and the equations proposed by other researchers. The above-mentioned comparison shows that the proposed equations are capable of predicting seepage of embankment dam body with high precision. Also, an equation has been presented to calculate the seepage from the body of embankment dam with oblique core which, the passing discharge rate is a function of core angle.

Key words: Embankment dam, seepage analysis, core, oblique core, permeability.

1. INTRODUCTION

Studying the causes of destruction in 200 destroyed embankment dam around the world, we conclude that 25 percent of destructions have been due to wash out of the fine granules of the body or the dam foundation (Foster and Fell 1999). The analytical methods of solving the seepage equations in porous environments are based on solving the governing differential equations using simplifying hypotheses. Those hypotheses are acceptable in certain conditions and therefore, the scope of their applications is limited due to the problems with special geometry and boundary conditions. In addition other numerical methods are available to investigate the seepage such as finite elements, finite differences and finite volumes.

The analytical calculation of seepage in dams has received many efforts. Dupuit (1863) assumed that the hydraulic gradients has the same slope of free surfaces. Then he used the Darcy's Law to calculate the discharge rate passing for each vertical shape of the dam according to Eq. (1).

$$q = k \cdot \frac{h_1^2 - h_2^2}{2l} \tag{1}$$

In this equation, l, h_1, and h_2 are the length of flow path, height of the water on top and bottom of dam, as shown in Fig. 1.

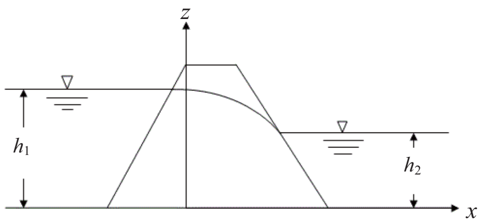

Fig. 1 Specifications used in Dupuit solution

Schaffernak (1917) presented Eqs. (2) and (3) to calculate seepage from the body of a homogenous embankment dam placed on an impervious foundation, as shown in Fig. 2.

$$q = k\ l\ \sin\beta \tan\beta \tag{2}$$

$$l = \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{\cos^2\beta} - \frac{H^2}{\sin^2\beta}} \tag{3}$$

In these equations, β is the angle of downstream slope as per degree, d is the length of drainage path in meter, and H is the height of upstream water per meter.

Casagrande (1937) evaluate the amount of discharge rate passing through body of embankment dam by assuming that the hydraulic slope (dz/dx) equals to (dz/ds), where s is the length measured along the phreatic surface in meter, (see Eqs. (4) ~ (6))

$$q = k \cdot a \cdot \sin^2\alpha \tag{4}$$

$$a = s - \sqrt{s^2 - \frac{h^2}{\sin^2\alpha}} \tag{5}$$

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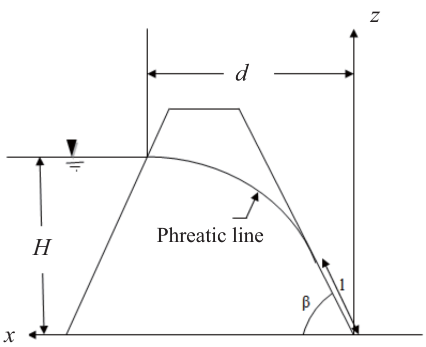

Fig. 2 Specifications used in Schaffernak solution

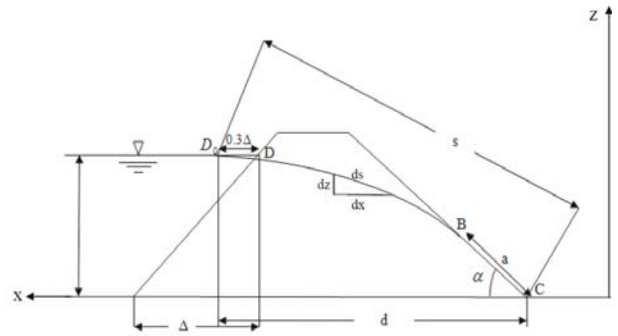

Fig. 3 Specifications used in Casagrande solutions (Das 1983)

$$s = \sqrt{d^2 + h^2} \;, \quad d = l - 0.7\Delta \tag{6}$$

where α is angle of downstream slope of the dam per degree, h is height of upstream water of the dam in meter, l is width of core floor, and Δ is shown in Fig. 3.

Stello (1987) presented a design chart using method of fragments to predict the phreatic line of the dam body and discharge rate that pass through the dams which are located on an impervious foundation. Their investigation revealed that the results of this method, compared to the software results, show an average 18 percent difference. Stello (1987) proposed Eq. (7) to calculate the seepage passing from the body of the dam.

$$q = k \cdot m \cdot h \tag{7}$$

In this equation, m is calculable for various angles of upstream slope and the d/h is a ratio in which h is the height of upstream water and d is calculable from Eq. (6).

In recent years, many researchers have calculated the seepage passing through the foundation and body of embankment dams using analytical and numerical methods.

Boger (1998) solved the 3D permeability equation with unstable flow assuming capillary forces trivially. In addition, the permeability equations were studied for steady state condition considering the size of capillary force. Akyuz and Merdun (2003) used the physical model and presented applied equations to calculate the amount of discharge rate passing through embankment dams located on an impervious base. Comparing the results of this research with the results of Dupuit, Schaffernak and casagrande shows that this method had the highest compatibility with the equation proposed by Dupuit.

Al-Damluji *et al*. (2004) used the boundary elements methods for solving the flow issue in steady state conditions and compared them with the results obtained from finite elements method. Rezk *et al*. (2010) presented the analytical solution to calculate seepage from body of dam with inner core placed on an impervious base and compared the results with expremental results. Also, Rezk *et al*. (2010) has studied the impact of core permeability and fall of phreatic surface per various discharge rates that pass through dam body. Comparisons showed that the phreatic surface obtained in analytical method is very close to the phreatic surface obtained in lab methods. They ultimately presented a design chart for calculation of discharge rate passing through dam body.

In this research, the Seep/W (Krahn 2007) software is used and around 600 numerical models have been studied based on finite elements method. Ultimately, based on the results of those analyses, an equation has been introduced to assess the discharge rate passing through homogenous embankment dams. In addition, the results obtained from proposed equations have been compared with the results of methods presented by other researchers.

2. MODELS USED IN THE RESEARCH

In this research, in order to achieve a simple equation for fast estimation of seepage from the body of dam, various models such as Figs. 4 and 5 have been used. The limit of variable parameters in those models is listed in Table 1. The numerical analyses have been done for each one of the models and the amount of seepage passing through the dam core is obtained.

3. RESULTS AND DISCUSSION

After more than 600 numerical experiments on dams with vertical core and the homogenous dams, it was concluded that the exiting seepage passing from body of the dam could be explained in Eq. (8) as follows.

$$q = f \cdot k \cdot h \tag{8}$$

In this equation, f is the seepage factor, a dimensional coefficient, which is calculated in this research.

Figure 6 indicates that by the increase in the size of width of dam crest (W), the seepage factor, decreases exponentially. In addition, by lowering the height of the water in the reservoir of dam (h) in accordance with Fig. 7 is the amount of seepage factor has been decreased exponentially, too. In Figs. 7 and 8, $c = b - 0.7\Delta$. For the dams with oblique core, as the angle of the central axis of the core increases with horizon, the discharge rate from the body of dam will decrease exponentially in accordance with Fig. 8. The studies show that for the constant upstream slope of the core, the angle of central axis of the core increases with the horizon, and the discharge rate passing through the core will be decreased exponentially in accordance with Fig. 9.

After performing numerical analyses and extracting the results for each model, it is tried to present a correlation between f parameters and other characteristics of the dam. Studies show that with an acceptable precision, it can be shown that for a dam with vertical core Eq. (9) correlates parameters f, h, tanα, W, H and the d/h relation. The difference in the results of this equation with Seep/W results in all cases is in average less than 20 percent.

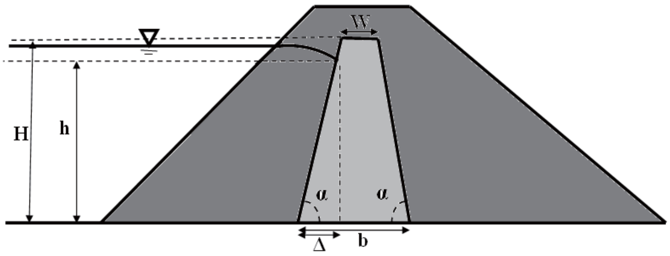

Fig. 4 The model used to analyze a dam with vertical core

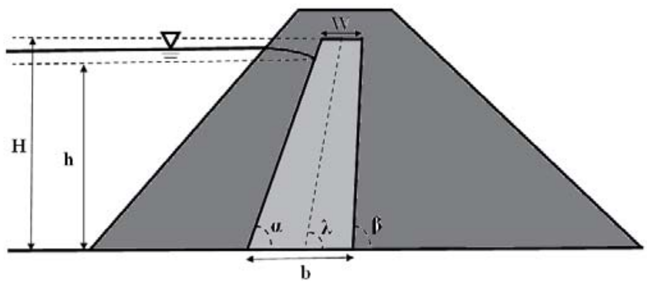

Fig. 5 The model used for analyzing a dam with oblique core

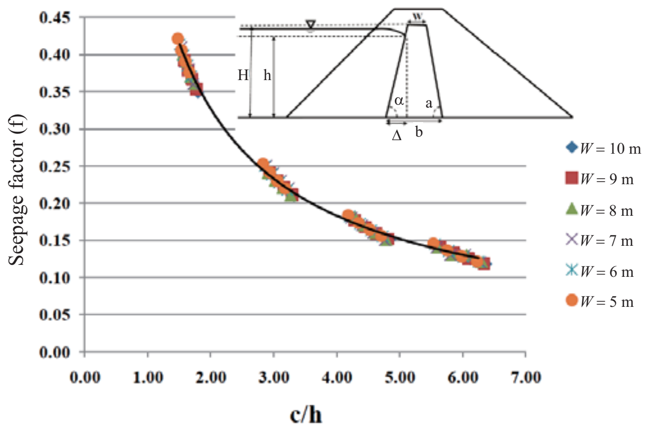

Fig. 6 Changes in seepage factor by *c*/*h* at different *W* for vertical core dams

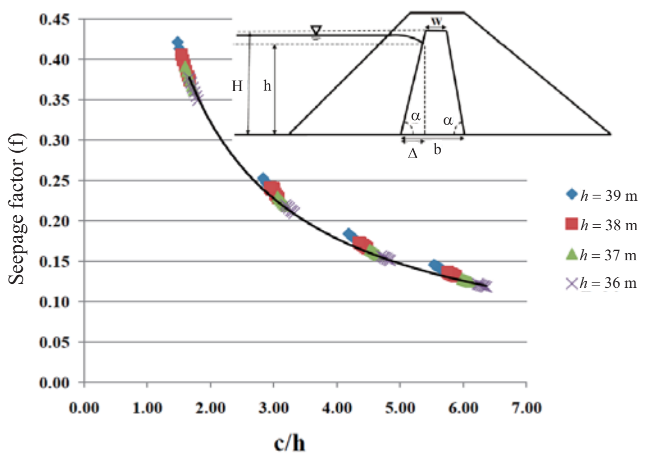

Fig. 7 Changes in seepage factor by *c*/*h* at different *h* for vertical core dams

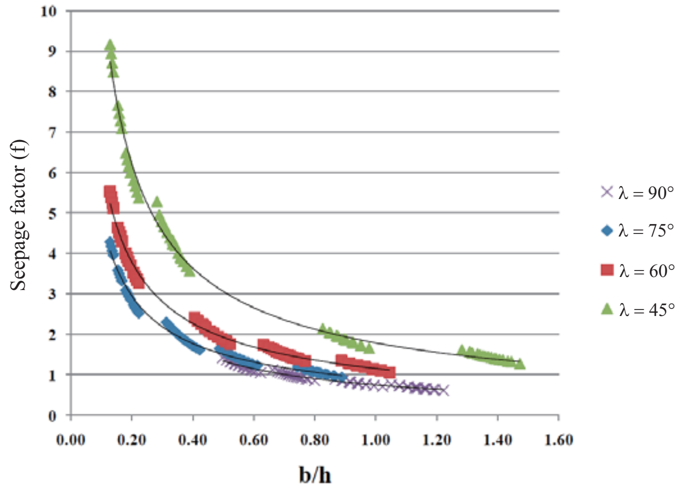

Fig. 8 Changes in seepage factor by *b*/*h* at different **λ** for oblique core dams

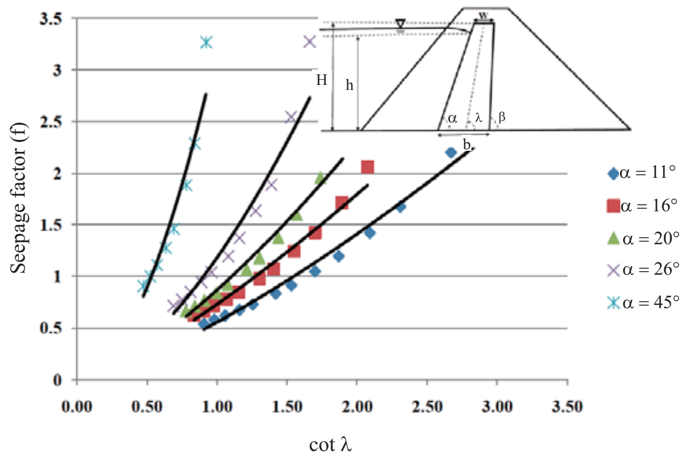

Fig. 9 Changes in seepage factor by cotan**λ** at different **α** for oblique core dams

Table 1 Limit of different parameters in models for performing the analyses

Width of crest (*W*)	Height of upstream water (*h*)	Angle of central axis of core (λ)	Angle of upstream slope (α)	Angle of downstream slope (β)
5 ~ 12 m	20 ~ 50 m	13° ~ 90°	11° ~ 80°	14° ~ 90°

$$f = (2.27 - 0.006W - 0.004h - 0.38\tan\ \alpha) \times H^{(-0.361)} \times \left(\frac{c}{h}\right)^{(0.3947\tan\alpha + 0.015h - 1.3591)} \quad (9)$$

In addition, it can be shown in the dams with oblique core, Eq. (10), the amount of *f* is obtained precisely. The differences between Seep/W results for all cases are in average less than 6 percent and the maximum error is 17 percent.

$$f = (0.4 \cot^2 \lambda + 1.1 \cot \lambda + 0.4) \times \left(\frac{b}{h}\right)^{(-0.054 \cot \lambda - 0.71)} \quad (10)$$

$$b = W + H(\cot \alpha - \cot \beta) \quad (11)$$

In Eqs. (9), (10) and (11), f is the seepage factor, λ is the angle of central axis of the core with the horizon which equals to $\alpha + \beta/2$, h is the height of upstream water of the core, β is the angle of downstream slope, α is the angle of upstream slope of oblique core dam, W is the length of dam core crest and H is the height of dam core.

4. COMPARISON OF THE RESULTS

Table 2 and Fig. 10 compare the results from the suggested equation (Eq. (9)) and the results obtained from the equations recommended by previous researchers. Comparison of discharge rate calculated based on proposed equation with the results from numerical simulations and results of Schaffernak (1917) equation shows that the proposed equation have better proportionality with the results obtained from software. In addition, comparing the suggested equation with Stello (1987) method shows the high compatibility of the two equations with each other and the results from numerical simulations. In addition, as in this research, the correlation for the discharge rate passing through the body of the dam has been presented, it does not have the error resulting from the readings through design chart; which is an advantage over Stello (1987) method.

In addition, in Table 3 and Fig. 11, the results of Eq. (10) have been compared with the results from numerical simulations.

Table 2 Comparing the result of proposed method and previous methods for vertical core or homogenous dams

α, β (degrees)	W (m)	H (m)	h (m)	K $\left(\frac{m}{sec}\right)$	Seepage $\left(\frac{m^3}{sec}\right)/m$			
					Schafferank (1917)	Stello (1987)	Proposed formula (Eq. 9)	Seep/W
45.0	5	40	36	1.00E-06	3.605E-05	1.152E-05	1.315E-05	1.354E-05
18.4	6	50	45	1.00E-05	1.686E-03	5.850E-05	8.984E-05	7.050E-05
26.6	7	45	40	1.00E-06	1.019E-04	7.200E-06	9.762E-06	8.533E-06
14.0	8	47	45	1.00E-05	2.043E-03	4.725E-05	7.852E-05	6.167E-05
14.0	9	48	43	1.00E-04	2.191E-02	3.870E-04	6.649E-04	5.110E-04
18.4	10	42	40	1.00E-05	1.413E-03	6.000E-05	7.820E-05	6.737E-05
45.0	5	46	43	1.00E-05	4.370E-04	1.462E-04	1.582E-04	1.727E-04
18.4	4	41	39	1.00E-04	1.332E-02	5.460E-04	7.836E-04	6.113E-04
26.6	6	40	36	1.00E-06	8.999E-05	6.300E-06	8.744E-06	7.852E-06
45.0	9	45	41	1.00E-06	4.100E-05	1.230E-05	1.437E-05	1.502E-05
18.4	8	47	42	1.00E-05	1.612E-03	5.040E-05	7.893E-05	6.407E-05
14.0	5	43	38	1.00E-06	1.952E-04	3.230E-06	5.417E-06	4.479E-06
11.3	7	44	39	1.00E-05	2.506E-03	2.925E-05	4.695E-05	3.746E-05
30.5	10	42	35	1.00E-03	8.342E-02	6.300E-03	8.360E-03	7.553E-03
15.5	12	46	42	1.00E-04	1.900E-02	4.620E-04	6.944E-04	5.562E-04
20.8	7	49	46	1.00E-05	1.427E-03	6.900E-05	1.056E-04	8.723E-05
11.3	9	50	45	1.00E-05	2.827E-03	3.465E-05	6.193E-05	4.430E-05
45.0	7	52	50	1.00E-06	5.132E-05	1.650E-05	1.809E-05	2.049E-05
14.0	10	46	45	1.00E-03	1.989E-01	4.950E-03	7.973E-03	6.408E-03
11.3	5	20	19	1.00E-03	1.100E-01	1.615E-03	1.863E-03	2.086E-03
45.0	6	182	140	1.00E-05	1.428E-03	3.640E-04	5.885E-04	4.481E-04
15.5	9	25	22	1.00E-03	1.080E-01	2.090E-03	2.654E-03	2.690E-03
20.8	5	64	60	1.00E-03	1.832E-01	1.020E-02	1.635E-02	1.162E-02
20.8	5	64	62	1.00E-03	1.806E-01	1.240E-02	1.775E-02	1.285E-02
11.3	7	52	50	1.00E-03	2.777E-01	4.750E-03	8.330E-03	5.786E-03
18.4	9	44	40	1.00E-04	1.507E-02	5.000E-04	7.429E-04	6.222E-04
26.6	10	43	40	1.00E-03	9.822E-02	7.800E-03	1.005E-02	8.917E-03
35.0	11	49	45	1.00E-03	7.733E-02	1.125E-02	1.369E-02	1.271E-02

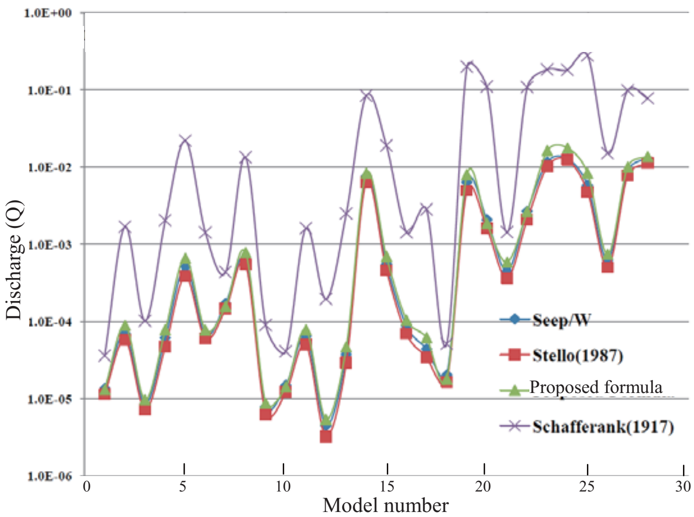

Fig. 10 Comparion of the results of proposed method and previous methods for vertical core dams

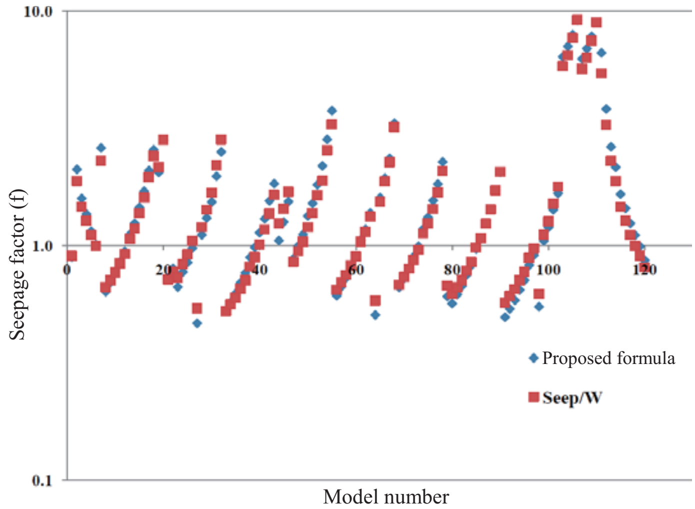

Fig. 11 Comparion of the results of proposed method and results of the software for oblique core

Table 3 Comparing the methods presented for calculating the seepage factor

λ	*b*/*h*	Seepage factor (*f*)	
		Seep/W	Proposed formula (Eq. (10))
64.6	1.194	0.903	0.891
52.0	0.639	1.881	2.110
55.4	0.806	1.459	1.591
57.6	0.906	1.273	1.360
60.2	1.017	1.107	1.151
62.4	1.106	0.997	1.011
50.0	0.528	2.289	2.605
52.1	3.139	0.661	0.635
49.9	3.050	0.708	0.695
47.7	2.961	0.764	0.761
45.1	2.850	0.839	0.852
42.9	2.750	0.917	0.943
39.5	2.583	1.064	1.113
37.5	2.472	1.178	1.241
34.8	2.306	1.372	1.456
32.5	2.139	1.597	1.702
29.9	1.917	1.956	2.089
27.8	1.694	2.403	2.563
66.8	0.350	2.158	2.049
64.6	0.250	2.830	2.820
55.4	2.300	0.715	0.725
53.2	2.217	0.773	0.799
38.5	5.250	0.728	0.664
35.2	5.083	0.833	0.770
33.2	4.972	0.914	0.848
30.5	4.806	1.047	0.973
28.2	4.639	1.194	1.110
25.6	4.417	1.419	1.311
23.4	4.194	1.675	1.532
20.5	3.806	2.200	1.977
18.4	3.417	2.831	2.510
49.2	4.528	0.523	0.527
46.9	4.439	0.560	0.574
44.7	4.350	0.595	0.624
42.1	4.239	0.653	0.694
39.9	4.139	0.708	0.762
36.5	3.972	0.811	0.888
34.5	3.861	0.889	0.981

5. CONCLUSIONS

Based on the results obtained from numerical analysis, new formulas have been presented for calculating the seepage passing through embankment dams. The mentioned formulas have the capability that consider parameters such as width of dam crest, height of dam, height of the upstream water of the core, slope of upstream and downstream of the core and the slope of central core. Comparing the results obtained from the equations suggested with the Seep/W shows that except in limited cases, in other cases, the errors of the equations are in acceptable range.

In addition, comparing the results of the method suggested by Stello (1987) and the results from numerical simulations for vertical dams shows around 20 percent errors in average. On the other hand, since in Stello (1987) method, in order to calculate the discharge rate passing through body of the dam, it is necessary to use design chart, the possibility of increase in error exists. Comparing the results of suggested equation for the dam with the vertical core, and the results of Schaffernak (1917) shows that the suggested equation is very much closer to the results from numerical simulations. Although, some analysis as compared with Eqs. (9) and (10) have 20 percent errors, in comparison to the other available method, the proposed equations are more accurate. Also to forecasting seepage in primary steps of dam design, this value of error is acceptable and will be offset by using safety factor in next steps of dam design.

It should be mentioned that none of the previous researchers have given any equations for the oblique core. Nevertheless, in this research, by using numerical analysis on the oblique core dams, the correlation has been presented for calculating the discharge rate passing through the body of the dam. Comparing the results of this equation with the results from numerical simulations show that the results have high compatibility. The equation presented in this area has maximum 17 percent and in average 6 percent difference from the results from numerical simulations.

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