SECONDARY COMPRESSION BEHAVIOR IN ONE-DIMENSIONAL CONSOLIDATION TESTS

Toshihiko Takeda [1], Motohiro Sugiyama [2], Masaru Akaishi [2], and Huei-Wen Chang [3]

ABSTRACT

One-dimensional consolidation analysis is described for predicting the consolidation time curves of clay exhibiting secondary compression during primary consolidation. The constitutive soil model is based on the equation governing the secondary compression rate of the decrease in void ratio. This model uses four parameters, namely, C_c, C_c^*, C_α and c_v^*, that can be easily determined or assumed from incremental loading (IL) oedometric consolidation tests. In order to be certain of the correct proposed soil model, the consolidation time curves observed in oedometer specimens are compared against those predicted by the analysis. A satisfactory agreement between the computed behaviors and oedometer observations would indicate the correct assumption. In addition, it is shown that the void ratio rate ($\dot{e}_s$) due to secondary compression during primary consolidation varies by approximately $10^{2\sim3}$ times as much as its final value before the application of the next loading increment.

Key words: One-dimensional consolidation, secondary compression, clay, void ratio rate, finite difference analysis.

1. INTRODUCTION

Currently, Terzaghi's consolidation theory and the results of incremental loading (IL) oedometric consolidation tests are used to predict the consolidation settlement-time curve in a field. From the experimental data and the field observation, it is known well that the actual rate of consolidation differs from the rate predicted by Terzaghi's theory. The difference between the predicted and the observed values is attributed to the secondary compression effects. This secondary compression is noted to be proportional to the logarithm of time and can be observed after the completion of primary consolidation based on Terzaghi's theory in which a linear relationship between void ratio and effective stress is assumed.

Taylor and Merchant (1940) were the pioneers, to present a secondary compression model and since then the secondary compression has been a key to solving the consolidation problem. They and many other researchers recorded that the secondary compression occurs during the primary consolidation as well as after the end of primary consolidation. The literature on secondary compression is extensive and there is a brief review by Imai (1995) as well. In most studies, the one-dimensional consolidation analysis is assumed to involve two processes, namely, primary consolidation and secondary compression. Both processes are also assumed to begin simultaneously with consolidation. However, it is very difficult to differentiate the secondary compression behavior from the primary consolidation because of the total compression, namely, the sum of primary consolidation and secondary compression are recorded as one in the conventional IL oedometer test and the compressions are inseparable. Nevertheless, most studies (Wahls 1962; Mesri 1974; Murakami 1980; Akaishi 1980; Feng 2010 and others) have focused on the characteristics of secondary compression during the stage of primary consolidation.

The main area of debate (Ladd 1976; Mesri 1985; Leroueil 1988) has been whether creep is significant during primary consolidation. Leroueil *et al.* (1985) pointed out that the constitutive equations in the form of $F(e, \sigma', t) = 0$ encounter a major difficulty when an origin for time must be defined although secondary compression decreases logarithmically with time. A unique constitutive equation in the form of $F(e, \sigma', \partial e / \partial t) = 0$ was used to avoid this difficulty (Sekiguchi 1976; Leroueil 1985; Imai 1992). However, none of the existing constitutive equations of clays for one-dimensional consolidation provides a clear explanation of secondary compression behavior during primary consolidation.

It is the objective of this study; to clarify the secondary compression behavior during the primary consolidation, to provide one-dimensional consolidation analysis for a constitutive equation in the form of $F(e, \sigma', \partial e / \partial t) = 0$ and to compare the solution with oedometer test results.

2. OEDOMETER TESTS AND CLAY USED

The conventional IL oedometer tests based on JIS A 1217 (2009) were carried out on undisturbed clays. The undisturbed samples were carefully prepared by trimming it to fit into the consolidation oedmeter ring, which were 6 cm in diameter and 2 cm in height (initially). The rings were lubricated with silicon grease to reduce friction and filter papers were placed on the top and bottom of the sample. In all cases, drainage was permitted at both sides of the sample. A small setting stress, normally taken as 9.8 kPa, was applied, and after 60 minutes, the sample was submerged. The temperature of the constant temperature water-bath was maintained at 15 ± 2°C.

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[1] General Manager (corresponding author), Onoda Chemico Co., Ltd., Japan. (e-mail: t_takeda@chemico.co.jp).

[2.] Professor, Department of Civil Engineering, Tokai University, Japan (e-mail: sugi@keyaki.cc.u-tokai.ac.jp; redstones_3710812@yahoo.co.jp).

[3.] Professor, Department of Civil Engineering, National Central University, Taiwan (e-mail:hueiwen@ cc.ncu.edu.tw).

The stress increment ratio was kept at, $d\sigma / \sigma = 1$, and two incremental durations, 1 and 7 days, were used to study the secondary compression characteristics of the samples. In this study, σ is the vertical effective stress and the prime(′) for the effective stress is omitted.

All undisturbed Hitachi clay samples were obtained from 8 ~ 23 m below the ground surface, using thin-walled fixed-piston sampler. The natural water content varied from approximately 82% to 121%. The liquid limit range, the plastic limit range, the specific gravity range and the unit weight are 96% to 131%, 44% to 54%, 2.61 to 2.64 and 14.5 kN/m^3 to 15.5 kN/m^3, respectively.

3. LABORATORY INVESTIGATION

Figure1 shows the plot of the void ratio *e* versus the logarithm of vertical effective stress σ relationships, for clay samples taken from depths of 9 to 21 m. Two identical clay samples were used and subjected to two different incremental durations for consolidation. One sample was loaded for one day for each pressure increment, whereas the consolidation duration of the other sample was 7 days. The most probable value of the preconsolidation pressure p_c for all tests is approximately from 80 kPa to 100 kPa.

It can be seen from the legends accompanying Fig. 1 that there is no appreciable difference in the *e*－logσ relationships for the different pressure increment durations. It is clear that the *e*－logσ relationships are independent of these consolidation durations. The results in Fig. 1 are completely different from the corresponding consolidation test results in Crawford's paper (1964). The hypothesis which can explain clearly the reasons causing this discrepancy is unclear for the moment. However, the paper which becomes a similar result is published by Shirako *et al.* (2002) and Tsuchida *et al.* (1999). It is thought that clay that the influence due to the pressure incremental duration decreases exists when the pressure incremental duration is changed and tested from the overconsolidated stage where total strain amounts and the amounts of the secondary compression (or a coefficient of secondary compression) are small.

In Fig. 2, the coefficient of secondary compression, C_α, has being plotted against the vertical effective stress (consolidation pressure). The magnitude of C_α increases rapidly to a maximum value for the pressure higher than the preconsolidation pressure and then it decreases. As the duration of the pressure increment increases, the magnitude of C_α, which is indicated with a large symbols (open points) in Fig. 2, decreases.

The void ratio rate at the end of the consolidation, $\dot{e}_f$, can be calculated using C_α and consolidation duration. In Fig. 3, $\dot{e}_f$ in the range of normally consolidation is plotted against the void ratio at the end of consolidation. The magnitude of $\dot{e}_f$ varies with the void ratio and the consolidation duration. It has been shown in numerous studies that there is a unique *e*－logσ relationship in normally consolidated clay. From the author's oedometer test results, however, it is difficult to observe the constant $\dot{e}_f$ based on the relationships of *e*－logσ of normally consolidated clay.

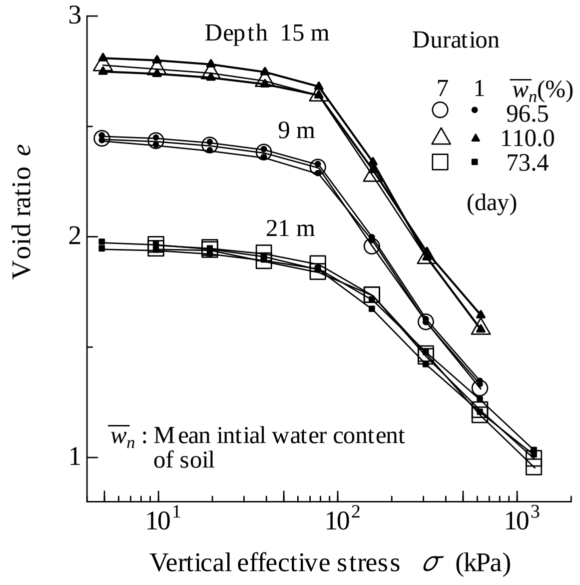

Fig. 1 Observed *e*－logσ curves for different increment durations

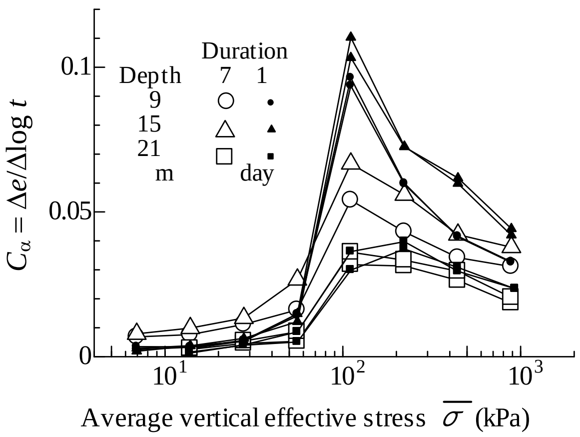

Fig. 2 Coefficient of secondary compression C_α vs average vertical effective stress

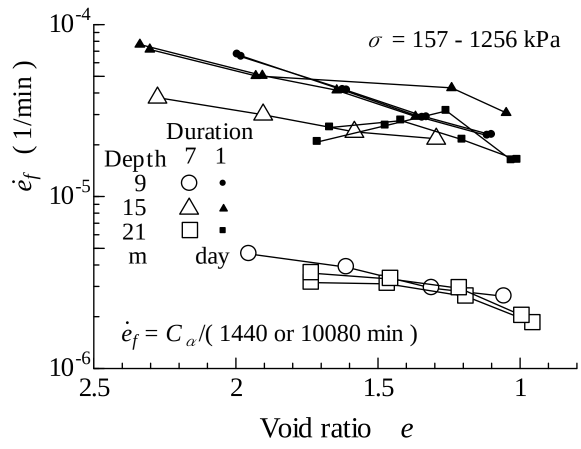

Fig. 3 Void ratio vs void ratio rate at end of consolidation

4. THEORETICAL CONSIDERATION

The one-dimensional consolidation process is assumed to involve two components, namely, primary consolidation and secondary compression. The void ratio rate $\dot{e}$ is also the sum of primary consolidation rate and secondary compression rate. Moreover, the $\dot{e}$ values due to both of them are assumed to begin simultaneously with the application of pressure. The new secondary compression in the next stage of loading would have a completely different behavior from the secondary compression in the previous stage. The continuity equation that governs the consolidation of saturated clay undergoing one-dimensional compression can be derived from the continuity condition expressions and Darcy's law, and can be expressed as follows.

$$\frac{\partial e}{\partial t} = (\dot{e}_p + \dot{e}_s) = (1+e_0)\frac{\partial}{\partial y}\left(\frac{k}{\gamma_\omega}\frac{\partial u}{\partial y}\right) \tag{1}$$

in which $\dot{e}_p$ and $\dot{e}_s$ are the void ratio rates of primary consolidation and secondary compression, respectively, e_0 is the initial void ratio, k is the permeability, γ_ω is the unit weight of water, t is the elapsed time, y is the vertical coordinate and u is the excess pore water pressure.

The model of secondary compression proposed in this study is expressed in terms of the relationship between the void ratio of clay under one-dimensional consolidation, e, the vertical effective stress, σ, and $\dot{e}_s$, as given by

$$e = e_0 - C_c^* \log(\sigma/\sigma_0) + C_\alpha \log(\dot{e}_s/\dot{e}_i) \tag{2}$$

$\dot{e}_s$ can be expressed by

$$\dot{e}_s = \dot{e}_i \times 10^{-\chi/C_\alpha} \tag{3a}$$

where

$$\chi = e_0 - C_c^* \log(\sigma/\sigma_0) - e \tag{3b}$$

in which C_c^* is the compression index defined by the primary compression, σ_0 is the initial effective stress, χ is the secondary compression and $\dot{e}_i$ is the secondary compression rate on the C_c^* curve, which varies depending on the position of the consolidation stratum with χ = 0 and vertical effective stress (see Eq. (7) and Fig. 4.)

The void ratio rate of primary consolidation, $\dot{e}_p$, showed as one-dimensional consolidation equation (Eq. (1)) can be expressed as

$$\dot{e}_p = -\frac{0.434 C_c^*}{\sigma}\cdot\dot{\sigma} = -m_p \cdot \dot{\sigma} \tag{4}$$

Substituting Eqs. (3a) and Eq. (4) into Eq. (1) then gives

$$\frac{\partial u}{\partial t} = c_v^* \frac{\partial^2 u}{\partial y^2} - \dot{e}_s / m_p \tag{5}$$

in which the coefficient of consolidation $c_v^* = k(1+e_0)/\gamma_\omega/m_p$, the coefficient of volume compressibility defined by the primary consolidation $m_p = 0.434 \cdot C_c^*/\sigma$.

Equation (5) can be solved by the finite difference method and is rewritten as,

$$u_{y,t+\Delta t} = u_{y,t} + M(u_{y-\Delta y,t} - 2u_{y,t} + u_{y+\Delta y,t}) - \dot{e}_s \times \Delta t / m_p \tag{6}$$

in which $M = c_v^* \cdot \Delta t / \Delta y^2 \le 1/2$.

To describe the one-dimensional consolidation of clay exhibiting secondary compression, Fig. 4 shows the unique relationship between void ratio, effective stress and void ratio rate. In Fig. 4, point A shows the initial effective stress σ_0 and the initial void ratio e_0 before the application of pressure. The compression index C_c^* is independent of secondary compression and is defined by the slope of the A-I line in Fig. 4. The difference in void ratio $e_0 - e_i$, immediately after the increase in effective stress, is related to primary consolidation. The void ratio rate at point I is denoted by $\dot{e}_i$. Secondary compression continues at a decreasing rate from I towards S and F. The void ratio rate at S due to secondary compression is expressed by Eq. (3a). The difference in void ratio $e_i - e$ leads to secondary compression at any time t. In the conventional IL oedometer test, the consolidation duration is equal to 1440 minutes and its total compression is used to determine $e - \log\sigma$ relations and C_c.

If the conventional compression index C_c is obtained from the void ratio change $e_0 - e_f$ in Fig. 4 (it is supposed that this method is applicable to not only a normally consolidated state but also an overconsolidated state), $\dot{e}_i$ can be calculated by

$$\dot{e}_i = \dot{e}_f \times 10(C_c - C_c^*)\log(\sigma/\sigma_0)/C_\alpha \tag{7}$$

in which $\dot{e}_f$ is the void ratio rate at point F.

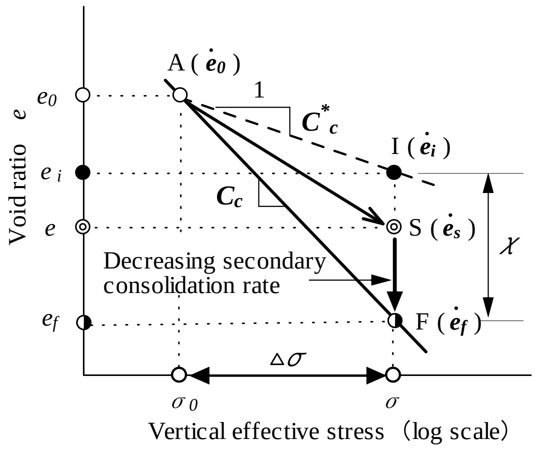

Fig. 4 Schematic diagram for $e - \log\sigma$ relations

4.1 Example of Calculation

To illustrate the one-dimensional consolidation behaviors of the proposed soil model, it has been used to calculate the void ratio and the void ratio rate-time curves for typical oedometer specimens. For a saturated clay sample loaded instantly under one-dimensional compression and drainage, the initial and the boundary conditions are

$$\left.\begin{aligned} & u(y=0\,,\,t>0)=0 \\ & u(H \ge y \ge 0\,,\,t=0)=u_0(=\Delta\sigma) \\ & \partial u/\partial y(y=H\,,\,t>0)=0 \\ & \dot{e}_s(=\dot{e}_i)(H \ge y \ge 0\,,\,t \le 0)=\dot{e}_f \end{aligned}\right\} \tag{8}$$

in which H is the length of the longest drainage path, u_0 is the initial excess pore water pressure, and the void ratio rate immediately before the application of loading $\dot{e}_s(=\dot{e}_i)$ which is assumed to be equal to one at the end of consolidation using Eq. (9). In an incremental loading (IL) Odedometric consolidation test, $t \le 0$ (t is less than or equal to 0) means the time just before the next loading after the previous loading. The material constants used in the calculations are listed in Table 1.

$$\dot{e}_f = 0.434C_\alpha\,/\,\text{(elasped time)} \tag{9}$$

Examples of numerical analysis of Eq. (6) for the special cases in which three types of C_c^* and C_c (= 1.5) are kept constant are shown in Fig. 5. The ratio of the compression indexes C_c^*/C_c is the ratio of primary consolidation to total consolidation, which is the sum of primary consolidation and secondary compression. The shape of consolidation-time curves depends on C_c^*/C_c. If this ratio is equal to 1, only secondary compression before loading occurs continuously in the primary consolidation. As it can be seen from the broken line in Fig. 5, therefore, the secondary compression is extremely small during primary consolidation and the consolidation-time curve approximately corresponds to Terzaghi's theory. It can also be seen from the dotted line and the solid line in Fig. 5 that the secondary compression during the primary consolidation increases as C_c^*/C_c decreases and the magnitude of secondary compression rate after the completion of primary consolidation, $C_\alpha = 0.05$, as used in this calculation. From these calculations, the rates of secondary compression, $\dot{e}_s$, inside the consolidation stratum during one-dimensional consolidation are shown in Fig. 6. It is important to note that the broken lines in Figs. 5 and 6 are calculated using $C_c^*/C_c = 1$ and the magnitude of secondary compression rate is approximately constant during primary consolidation. Solid lines representing the case of $C_c^*/C_c = 0.67$ as shown in Fig. 6, show the typical change in the rate of secondary compression immediately after the beginning of consolidation. It is important to realize that the rate of secondary compression increases with consolidation time and tends to converge at the same value at the later stage of consolidation. During the primary consolidation, the secondary compression expressed by the proposed model is not proportional to the logarithm of time.

Table 1 Soil parameters

C_c	C_c^*	c_v^*	C_α	e_0 a)	$\dot{e}_f$ b)
1.5	1 ~ 1.5	0.1	0.05	3	1.5×10^{-5}

Note: c_v^* (cm^2/min),

a) e_0 at σ_0 = 39.2 kPa,

b) elapsed time = 1440 min (1/min)

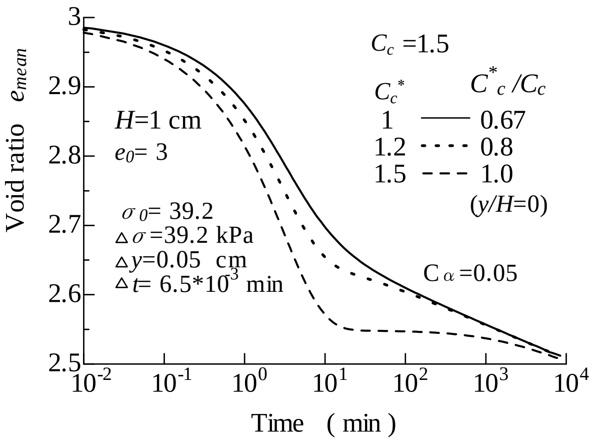

Fig. 5 Average void ratio-time curves

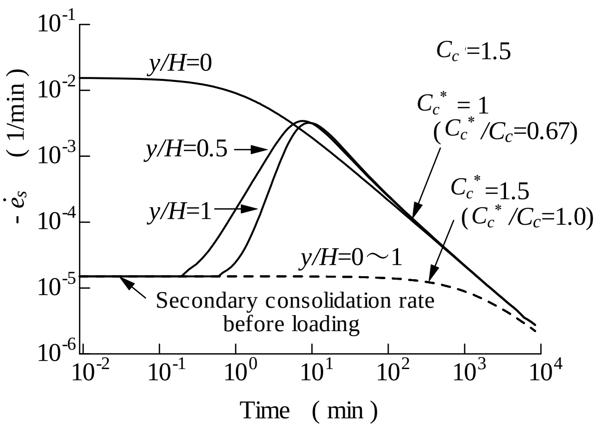

Fig. 6 Void ratio rate-time curves

4.2 Comparisons of Calculated and Experimental Results

To compare an experimental consolidation-time curve with the theoretical calculated ones, the values of C_c, C_c^*, C_α and c_v^* must be known. The compression index C_c is directly calculated by using the change of the void ratio in each loading stage and the coefficient of secondary compression C_α is easily determined from the final slope of the semi-logarithm plot in the conventional oedometer test results. The coefficient of consolidation c_v, maybe determined from the graphical solution for consolidation time curves, such as the square-root-of-time fitting method, is highly influenced by the secondary compression. It is very difficult to determine the values of c_v^* and C_c^*, which are independent of secondary compression effects.

In the present numerical analysis, the following simplified approach was adopted. The curve ruler method adopted by the Japanese Geotechnical Society (*e.g.* JIS A 1217(2009)) was used to determine the approximate values of c_v^* and C_c^*. To reduce the Secondary compression effects on c_v, the relation of the average degree of consolidation, U, and the time factor, T_v, up to U = 50% is fitted to the observed data. The first approximate value is obtained from the time t_{50} for U = 50% and the value of C_c^* is calculated using the consolidation settlement d_{50}, which corresponds to t_{50}. Using approximate values for soil indexes, the results of preliminary calculation for the consolidation-time curve were compared with the observed ones and the values of c_v^* and C_c^* were modified by a trial and error method to obtain a good fit of the calculated and observed curves with the curve fitting procedure. Figs. 7 and 8 show the calculated void ratio-time curves for the three specimens together with the observed values those were performed to two different loading durations. The close correlation of the calculated and observed curve is an indication of the reliability of the proposed model and the success of the above procedure.

Figure 9 shows the ratio of the compression index C_c^*/C_c used in the calculation. This provides the ratio of primary consolidation to the total at the end of consolidation period immediately before the application of the next loading increment. It can be seen from Fig. 9 that the ratio of compression index is independent of the magnitude of consolidation pressure and varies with the consolidation duration.

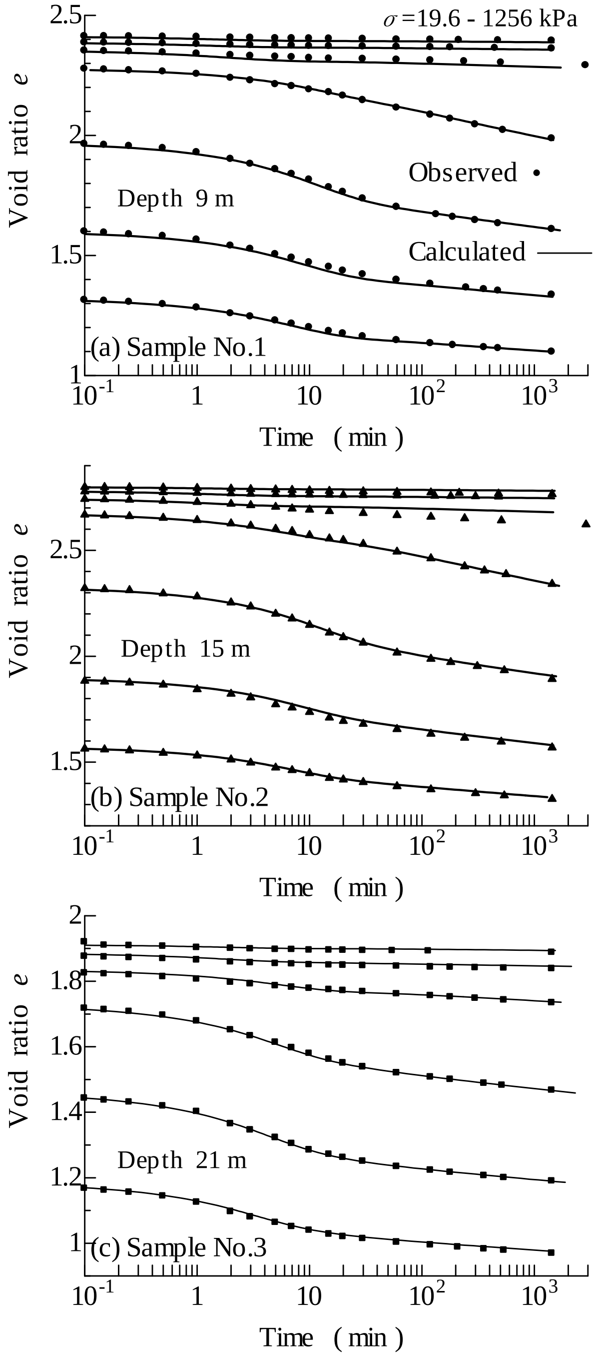

Fig. 7 Void ratio-time curves for the one-day loading duration

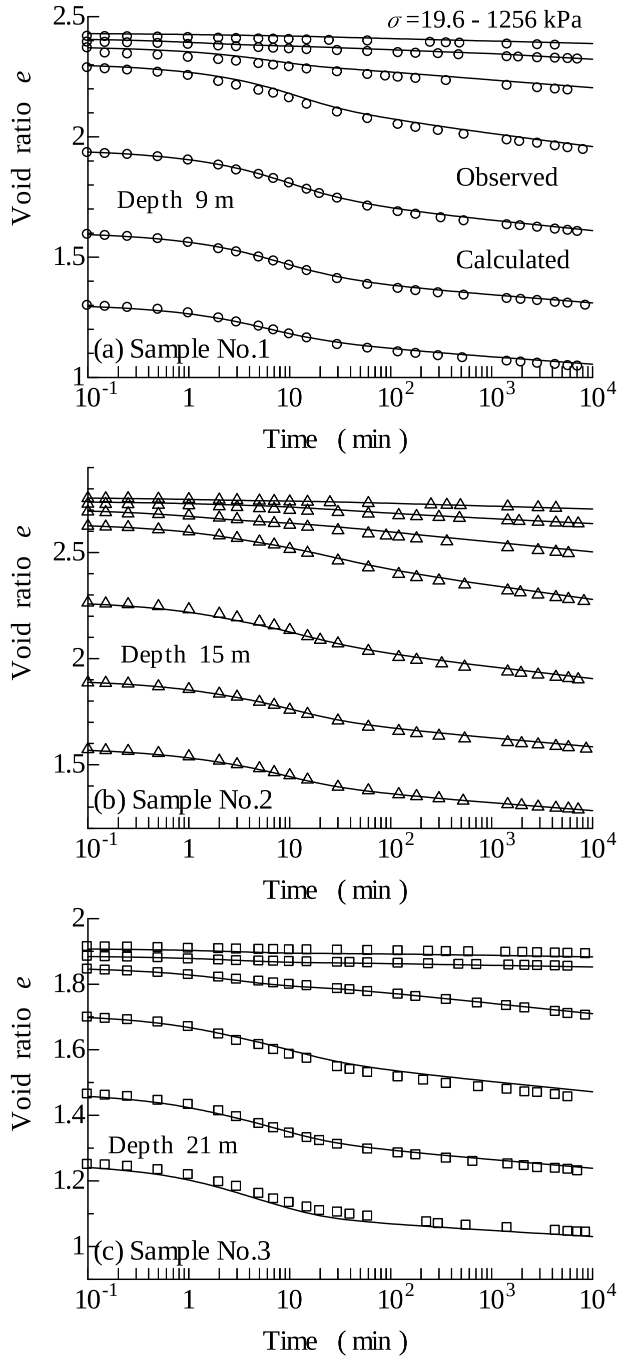

Fig. 8 Void ratio-time curves for the one-week loading duration

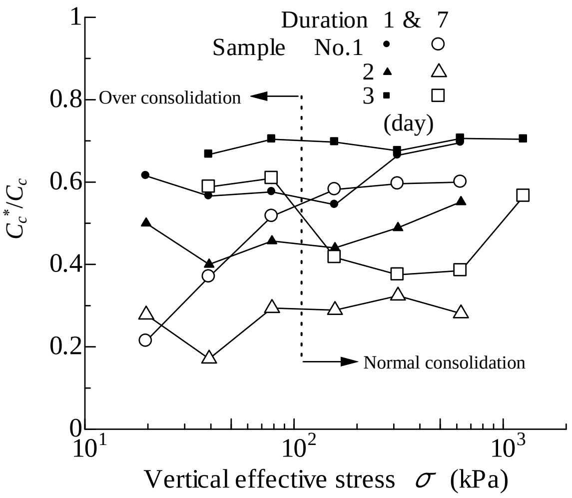

Fig. 9 Relationships between C_c^*/C_c and vertical effective stress

5. CONCLUSIONS

The main conclusions obtained from this experimental study and the numerical analysis are summarized as follows,

1. Relationship between the void ratio and the logarithm of vertical effective stress is independent of loading duration. The void ratio rate $\dot{e}_f$ of normally consolidated Hitachi clays at the end of consolidation varies with the decrease in the void ratio. It can be said that there is not always a unique relationship between $e-\sigma-\dot{e}$.
2. The void ratio rate $\dot{e}_s$ for secondary compression during primary and secondary consolidations may be expressed as a function of the magnitude of secondary compression generated during consolidation. Five parameters are required for this proposed model. They are the compression indexes C_c and C_c^*, the coefficient of consolidation c_v^*, the coefficient of secondary compression C_α and another parameter $\dot{e}_i$. Each parameter can be easily determined from the results of one-dimensional consolidation test and by trial and error.
3. The method of analyzing the results of one-dimensional consolidation of clay exhibiting secondary compression appears to give reliable predictions of the consolidation-time curves for laboratory consolidation tests.

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