EFFICIENT RELIABILITY-BASED DESIGN OF DRILLED SHAFTS IN SAND CONSIDERING SPATIAL VARIABILITY

Zhe Luo [1] and C. Hsein Juang [2]

ABSTRACT

This paper presents an efficient approach for reliability-based design (RBD) of drilled shafts considering the effect of spatial variability of soil property. The spatial averaging technique is employed herein to simplify the modeling of soil spatial variability for practical application. Specifically, the effect of the spatial correlation of soil property between the tip resistance zone and the side resistance zone on the results of RBD of drilled shafts is investigated. The reliability analysis is realized herein using the first-order reliability method (FORM) that is implemented in a spreadsheet. The developed approach is illustrated in a design of drilled shafts under drained compression in loose sand. The reliability analysis shows that neglecting the spatial effect overestimates the probability of failure for both ultimate limit state and serviceability limit state requirements and can yield unduly conservative design. This efficient approach may be adapted for other loading conditions and is applicable to RBD of other geotechnical structures.

Key words: Reliability-based design, ultimate limit state, serviceability limit state, first-order reliability method, spatial variability, spatial correlation, drilled shafts.

1. INTRODUCTION

Drilled shafts, also known as bored piles, have been extensively used in geotechnical practice as a foundation system for buildings, bridges, towers, *etc*. The procedures for designing drilled shafts vary depending on the soil profile and the types of applied load, which may include torsion, lateral loading, uplift, compression and earthquake loading (*e.g.*, Kulhawy 1991; O'Neill and Reese 1999; Nusairat *et al*. 2004; Brown *et al*. 2010). In recent years, the methods of drilled shafts design adopted by the geotechnical community have been in a transition from allowable stress design (ASD) to load and resistance factor design (LRFD), a category of reliability-based design (RBD). In this regard, significant progress has been made and systematic RBD approaches have been reported. For example, Phoon *et al*. (1995) developed a RBD methodology for drilled shafts as foundations for transmission line structure. Paikowsky (2004) developed a LRFD methodology for design of deep foundations. Brown *et al*. (2010) documented detailed procedures for LRFD design of drilled shafts. Efforts to improve the current RBD framework have also been in progress. Wang *et al*. (2011a) proposed an expanded reliability-based design approach for the design of drilled shafts based on Bayes' theorem.

In the current framework of RBD, given a target reliability index, the load and resistance factors are calibrated for various levels of variation of the design soil parameters. The geotechnical engineers generally select partial factors based on the estimated variation of the soil parameters. Therefore, the uncertainty of soil parameters has a significant influence on the design decision. Traditional RBD procedure generally deals with the spatial constant condition and the effect of spatial variability of soil property is generally ignored. Recent studies (*e.g.*, Fenton and Griffiths 2008; Griffiths *et al*. 2009), however, suggest that the results of reliability analysis could be affected by the effect of spatial variability of soil properties. Ignoring the effect of spatial soil variability could have an adverse consequence.

The traditional reliability-based design generally neglects the effect of spatial variability of soil property. In a reliability analysis considering the spatial effect, the uncertainty of soil property is generally described by means of sample statistics (*e.g.*, mean and standard deviation) under a certain assumption of the type of distribution (*e.g.*, normal or lognormal distribution) and the scale of fluctuation. The scale of fluctuation is the maximum distance within which the spatially random parameters are correlated (Vanmarcke 1983). Typical values of the vertical and horizontal scales of fluctuation for various soil parameters can be found in Phoon and Kulhawy (1999). The vertical scale of fluctuation typically ranges from 0.5 m to 3 m depending on the geological condition and composition of soil in the field (Suchomel and Mašín 2010). In recent years, the influence of the spatial variability on the RBD of various geotechnical problems has been reported (*e.g.*, Fenton and Griffiths 2003; Fenton *et al*. 2005; Griffiths and Fenton 2009; Luo *et al*. 2011; Luo *et al*. 2012a,b). It has been concluded that the spatial variability has a significant influence on the design decision using RBD.

In this paper, an efficient approach for the reliability-based design of drilled shafts considering the effect of spatial variability is developed. As an example to illustrate the new approach, the design of drilled shafts in loose sand under drained compression is considered. The axial compression capacity of drilled shafts consists of tip resistance and side resistance, and the design approaches summarized by Kulhawy (1991) are adopted as

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the basis for reliability analysis in this study. To make the new approach more practical, the first-order reliability method (FORM) is employed to perform the reliability analysis and the entire approach is implemented in a spreadsheet. Here, the spatial variability of soil property is dealt with using a spatial averaging technique (Vanmarcke 1983). Then, the effect of the spatial correlation of soil property between the tip resistance zone and the side resistance zone on the results of RBD of drilled shafts is investigated. The developed approach is shown to be effective with illustrated examples.

2. RELIABILITY-BASED DESIGN OF DRILLED SHAFTS

An illustrative example for a reliability-based design (RBD) of drilled shafts documented in Phoon *et al.* (1995) is re-analyzed in this study. The schematic diagram of a drilled shaft under drained compression loading (F) in loose sand is shown in Fig. 1(a). In this example, the water table is set at the ground surface. The diameter and depth of the shaft are denoted as B and D in Fig. 1(a) respectively. Other design parameters regarding soil and structure properties are listed in Table 1. In the RBD framework, B and D are determined to meet the target reliability index or the corresponding probability of failure through trial-and error.

The requirements of both ultimate limit state (ULS) and serviceability limit state (ULS) should be both satisfied in the RBD. For either ULS or SLS requirement, the drilled shaft is considered a failure if the compression load exceeds the shaft compression capacities. In this study, the compression load F is set as the 50-year return period load F_{50} for both ULS and SLS design (F_{50} = 800 kN as per Phoon *et al.* 1995). The ULS compression capacity (denoted as Q_{ULS}) is determined with the following equation (Kulhawy 1991):

$$Q_{\text{ULS}} = Q_{\text{side}} + Q_{\text{tip}} - W \tag{1}$$

where Q_{side}, Q_{tip} and W = side resistance, tip resistance, and effective shaft weight, respectively. Considering that the cohesion term is neglected in the design in loose sand, the Q_{side}, and Q_{tip} can be computed as:

$$Q_{\text{side}} = \pi BD(K/K_0)_n K_0\, \sigma'_{vm} \tan\phi' \tag{2}$$

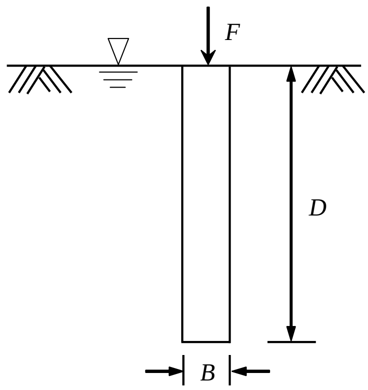
$$Q_{\text{tip}} = 0.25\pi B^2 \Big[0.5B(\gamma - \gamma_w)\, N_\gamma\, \zeta_{\gamma s}\, \zeta_{\gamma d}\, \zeta_{\gamma r} + (\gamma - \gamma_w)\, DN_q\, \zeta_{qs}\, \zeta_{qd}\, \zeta_{qr} \Big] \tag{3}$$

where $(K/K_0)_n$ = nominal operative in-situ horizontal stress coefficient ratio; σ'_{vm} = mean vertical effective stress along the shaft depth; ϕ' = soil effective stress friction angle; N_γ, N_q = bearing capacity factors defined as (Vesic 1973):

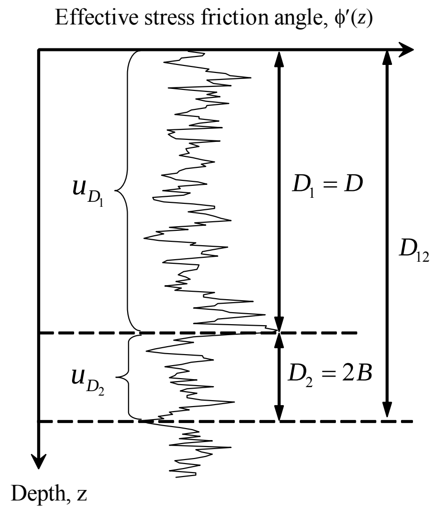
$$N_q = \tan^2(45° + \phi'/2)\exp(\pi \tan\phi') \tag{4}$$

$$N_\gamma = 2(N_q + 1)\tan\phi' \tag{5}$$

and $\zeta_{\gamma s}$ and ζ_{qs} = shape correction factors; $\zeta_{\gamma d}$ and ζ_{qd} = depth correction factors; and $\zeta_{\gamma r}$ and ζ_{qr} = rigidity correction factors for respective bearing capacity factors. Detailed methods for computing the bearing capacity factors and correction factors are



(a) Schematic diagram of drilled shaft



(b) Spatial variability of soil parameter

Fig. 1 Design example of drilled shaft considering the spatial variation of soil property with depth

Table 1 Input parameters for a drilled shaft design as shown in Fig. 1(a)

Parameter	Symbol	Value
Shaft diameter	B	1.2 m
Shaft depth	D	4.2 m
Effective stress friction angle	ϕ'	32°*
Total unit weight of soil	γ	20.0 kN/m^3*
At-rest coefficient of horizontal soil stress	K_0	1.0*
Nominal operative in situ horizontal stress coefficient ratio	$(K/K_0)_n$	1.0*
Concrete unit weight	γ_{con}	24.0 kN/m^3*
50-year return period load	F_{50}	800 kN**
Allowable displacement	y_a	25 mm**
curve-fitted parameters	a	4.0
	b	0.4

Note: * Mean values of input parameters adopted by Wang *et al.* (2011a)
** Load and allowable displacement used by Phoon *et al.* (1995) and Wang *et al.* (2011a)

documented in Kulhawy (1991). Then the SLS compression capacity (denoted as Q_{SLS}) is determined with the following equation (*e.g.*, Wang *et al*. 2011a):

$$Q_{SLS} = 0.625a\left(\frac{y_a}{B}\right)^b Q_{ULS} \tag{6}$$

where a = 4.0 and b = 0.4 are curve-fitted parameters for the load- displacement model, y_a = allowable displacement = 25 mm. The probability of ULS failure (p_f^{ULS}) and the probability of SLS failure (p_f^{SLS}) are defined as $P_r(Q_{ULS} < F)$ and $P_r(Q_{SLS} < F)$, respectively. The reliability-based design can be realized by meeting the target probability of ULS failure (p_f^{ULS}) and the target probability of SLS failure (p_f^{SLS}), respectively.

3. CORRELATION BETWEEN SPATIAL AVERAGES

In the traditional reliability analysis and design, soil parameters are generally modeled as spatial constant variables, which are represented by their statistics (*e.g.*, mean value and standard deviation) under lognormal assumption (*e.g.*, Phoon and Kulhawy 1999) or normal/truncated normal assumption (*e.g.*, Most and Knabe 2010). In recent years, Griffiths and his colleagues have performed a series of study on the effect of spatial variability of soil property on the reliability analysis for various geotechnical problems (Fenton and Griffiths 2008). It is also reported that the negligence of spatial variability in the reliability-based design in geotechnical engineering can lead to either an overestimation or underestimation of the failure probability in a given design. The reader is referred to Wang *et al*. (2011b) and Luo *et al*. (2012a,b) for further discussions on this issue.

In lieu of the computationally intensive approach such as random field modeling, the spatial variability of soil properties in the reliability analysis may be simplified and represented with spatial averages (*e.g.*, Most and Knabe 2010; Luo *et al*. 2011; Luo *et al*. 2012a,b). In this study, the spatial effect of soil property on the reliability-based design of drilled shafts is investigated using the spatial averaging technique (Vanmarcke 1977). The variance reduction factor for the spatial average such as the effective stress friction angle ϕ' over a depth interval Δz is obtained through the integration of exponential autocorrelation function (Vanmarcke 1983):

$$\Gamma^2(\Delta z) = \frac{1}{2}\left(\frac{\theta}{\Delta z}\right)^2\left[\frac{2\Delta z}{\theta} - 1 + \exp\left(-\frac{2\Delta z}{\theta}\right)\right] \tag{7}$$

where θ is the scale of fluctuation and the exponential correlation structure is assumed for Eq. (7). The reduced variance is then computed as the product of variance reduction factor and the variance of the soil parameter of concern.

As reflected in Eqs. (2) ~ (5), the compression capacity of a drilled shaft is closely correlated with the effective stress friction angle ϕ'. The effect of the spatial variability of ϕ' on reliability analysis has been widely reported in the literature (*e.g.*, Fenton *et al*. 2005; Griffiths *et al*. 2009; Suchomel and Mašín 2010). In this regard, ϕ' is treated as a spatial variable in this study and an example of the spatial variability of ϕ' is shown in Fig. 1(b). The side resistance Q_{side} is computed from the spatial average of ϕ'_{side} [denoted as u_{D_1} in Fig. 1(b)] over the depth interval D_1 which is equal to the depth of shaft D. The tip resistance Q_{tip} can be computed from the average value of soil strength parameters between the base and a depth of 2B beneath the base of the shaft (O'Neill and Reese 1999). Therefore, Q_{tip} is calculated from the spatial average of ϕ'_{tip} [denoted as u_{D_2} in Fig. 1(b)] over the depth interval D_2 which is equal to 2B. Then, the coefficient of correlation between the two adjacent spatial averages (u_{D_1} and u_{D_2}) can be readily obtained with the following equation (derived from Vanmarcke 1977):

$$\rho_{u_{D_1},u_{D2}} = \frac{D_{12}^2\,\Gamma^2(D_{12}) - D_1^2\Gamma^2(D_1) - D_2^2\,\Gamma^2(D_2)}{2D_1\,D_2\,\Gamma(D_1)\,\Gamma(D_2)} \tag{8}$$

where $\Gamma^2(D_1)$, $\Gamma^2(D_2)$, and $\Gamma^2(D_{12})$ are the variance reduction factor for the spatial averages over D_1, D_2 and D_{12}, respectively. Traditional reliability-based design of drill shafts simply assumes the correlation between the soil properties (u_{D_1} and u_{D_2}) that were used to compute shaft and toe resistances (Q_{side} and Q_{tip}) are perfect correlated, *i.e.*, $\rho_{u_{D_1},u_{D_2}}$ = 1.0. Due to the effect of spatial variability, the correlation between the spatial averages of soil properties (u_{D_1} and u_{D_2}) are actually partially correlated ($0 < \rho_{u_{D_1},u_{D_2}} < 1.0$). The effect of this correlation may be investigated using Eq. (8). Within the current RBD framework, the design of a drilled shaft can be performed by considering the spatial correlation of u_{D_1} (spatial average for side resistance) and u_{D_2} (spatial average for tip resistance).

4. EFFICIENT RELIABILITY-BASED DESIGN APPROACH CONSIDERING SPATIAL VARIABILITY

4.1 First-Order Reliability Approach That Incorporates Spatial Correlation

In a reliability analysis involving multiple input variables, approximate methods such as the first-order reliability methods (FORM) are commonly used. Various techniques are documented in the literature for solving reliability index and the corresponding probability of failure using FORM (*e.g.*, Hasofer and Lind 1974; Ditlevsen 1981; Haldar and Mahadevan 2000; Baecher and Christian 2003; Low 2005; Ang and Tang 2007). The efficient spreadsheet solution of FORM has also been proposed (Low and Tang 1997) and applied for reliability analysis in various geotechnical problems.

In this study, a simple spreadsheet-based approach that combines the FORM and the spatial correlation between spatial averages is developed for RBD of drilled shafts. Figure 2 shows the layout of the spreadsheet solution for a reliability analysis of ULS failure with the consideration of spatial variability of ϕ'. The ϕ'_{tip} for tip resistance and ϕ'_{side} for side resistance are modeled as a lognormal distribution. A separate spreadsheet solution similar to the one shown in Fig. 2 for reliability analysis of SLS failure is also developed. The spreadsheet that implements the proposed approach and formulation is available from the authors upon request. Comparing with existing previous spreadsheet solutions (for example, Low and Tang 1997), the improvement herein is the implementation of spatial parameters as shown in the lower-left corner in Fig. 2. The variance reduction factors for two spatial averages [u_{D_1} and u_{D_2} as shown in Fig. 1(b)] are determined with Eq. (7) for a certain specified scale of fluctuation θ. The coefficient of correlation ($\rho_{u_{D1},u_{D2}}$) between u_{D_1} and u_{D_2} is calculated with Eq. (8). Then, the relevant cells in the correlation matrix are set to be $\rho_{u_{D1},u_{D2}}$ as shown in Fig. 2. The reliability analysis of a drilled shaft can be realize using this spreadsheet solution considering spatial variability of φ′.

4.2 Parametric Study

It is advisable to investigate the influence of spatial effect of soil property on the RBD of a drilled shaft. To this end, a series of parametric analyses are conducted for a given design of drilled shaft with B = 1.2 m and D = 4.2 m. For these analyses, only the spatial variability of φ′ are considered in order to assess the effect of the spatial correlation. The parameter φ′ is modeled as a lognormal distributed variable; all other input parameters are treated as constant parameters and the values listed in Table 1 are used in the analysis. In this parametric study, the following ranges of parameters are analyzed:

COV = 0.10, 0.15, 0.20

θ = 0.5 m, 1 m, 3 m, 10m, 25 m, 50 m, 100 m

where COV = coefficient of variation and θ = the scale of fluctuation. For each pair of COV and θ, a single run of FORM with the spreadsheet setup for ULS failure or SLS failure is performed respectively. Figure 3(a) shows how the computed probability of ULS failure (p_f^{ULS}) varies with the COV and θ of φ′. It is observed that for each level of COV, p_f^{ULS} increases significantly with θ especially when θ is smaller than 10 m. Recall that θ in the vertical direction typically ranges from 0.5 m to 3 m depending on the geological history and composition of the soil deposit (Suchomel and Mašín 2010). Note that the solution of θ = 100 m is close to that of the traditional reliability analysis in which soil property is modeled as spatial constant. In Fig. 3(a), at the COV of 0.2, the computed p_f^{ULS} is 0.0048 for θ = 3 m, while p_f^{ULS} is 0.0267 for θ = 100 m. It is apparent that the predicted p_f^{ULS} is significantly overestimated if the spatial variability is neglected in the ULS design.

Similarly, Fig. 3(b) shows how the computed probability of SLS failure (p_f^{SLS}) varies with the COV and θ of φ′. Significant increase of p_f^{SLS} with θ (especially at θ < 10 m) is also observed regardless of COV. Therefore, it is implicated that both p_f^{ULS} and p_f^{SLS} will be much overestimated if the spatial effect is neglected. The RBD can be too conservative without considering the effect of spatial variability of soil parameters. This has an important implication in engineering practice as the engineer might be less inclined to use RBD if it produces unduly over conservative design.

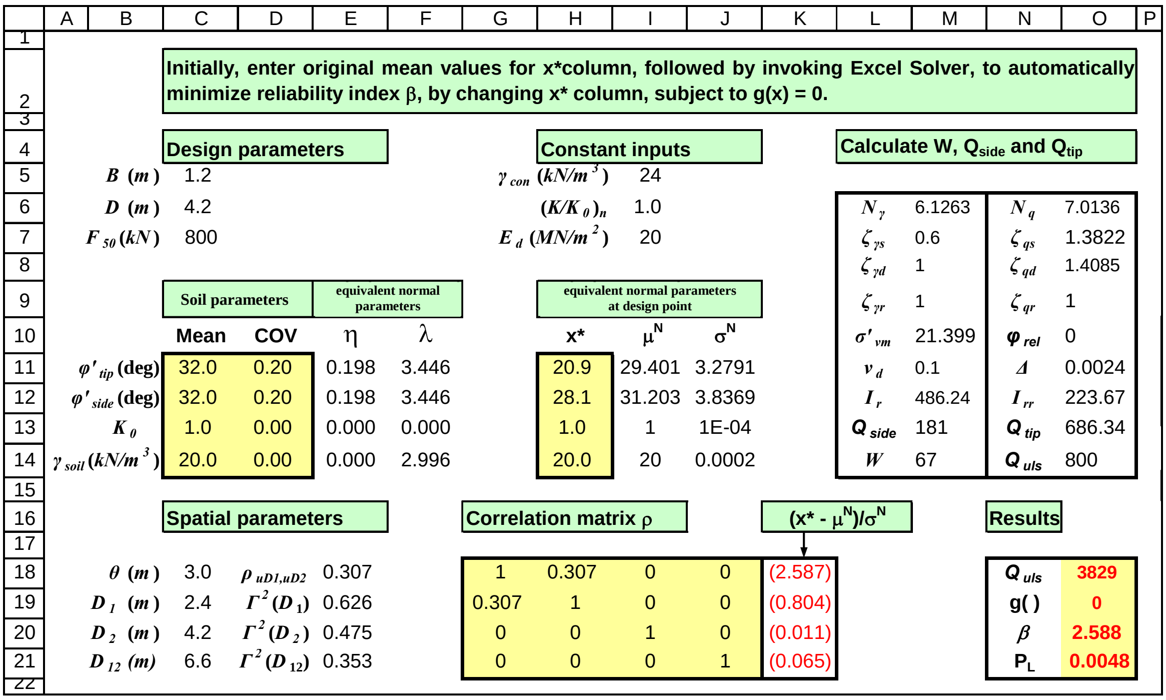
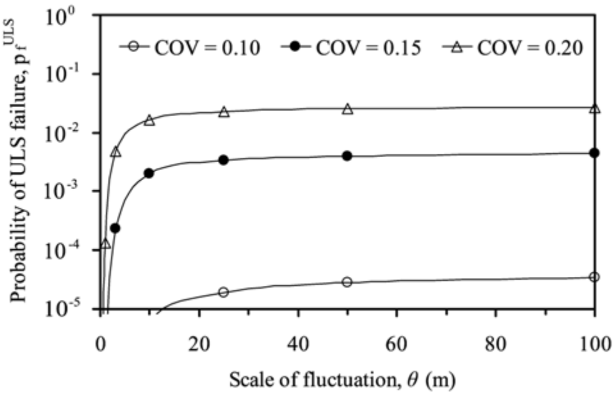
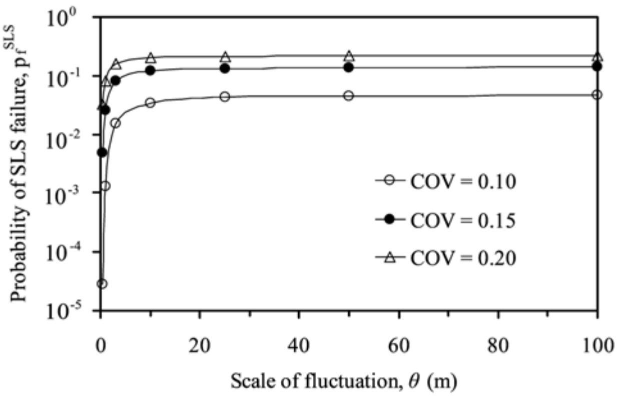


Fig. 2 Layout of the spreadsheet for reliability-based design against ULS failure



(a) Ultimate limit state



(b) Serviceability limit state

Fig. 3 Effect of scale of fluctuation on probability of failure at various levels of COV

4.3 Reliability-Based Design Considering Spatial Correlation

The effect of spatial variability of soil property on the reliability-based design of drilled shafts is examined with the developed spreadsheet solution in this study. The reliability-based design of drilled shafts is generally realized through determining *B* and *D* of a shaft that satisfies the target reliability index or the corresponding probability of failure for both ULS and SLS requirement. To be able to compare results with a recent RBD study that did not consider spatial variability (Wang *et al*. 2011a), the target reliability indices against ULS and SLS failure are set to 3.2 and 2.6, respectively. Thus, the corresponding target probability of ULS failure (p_T^{ULS}) and target probability SLS failure (p_T^{SLS}) are 0.00069 and 0.0047, respectively. The possible shaft diameter *B* depends on the size of augers that are available and *D* is determined to be the minimum shaft depth that meets the target reliability indices through trial-and-error (or through a simple cost optimization from all solutions that satisfy the target reliability requirement). In this study, a series of possible *B* and *D* values for the candidate designs used by Wang *et al*. (2011a) is adopted herein:

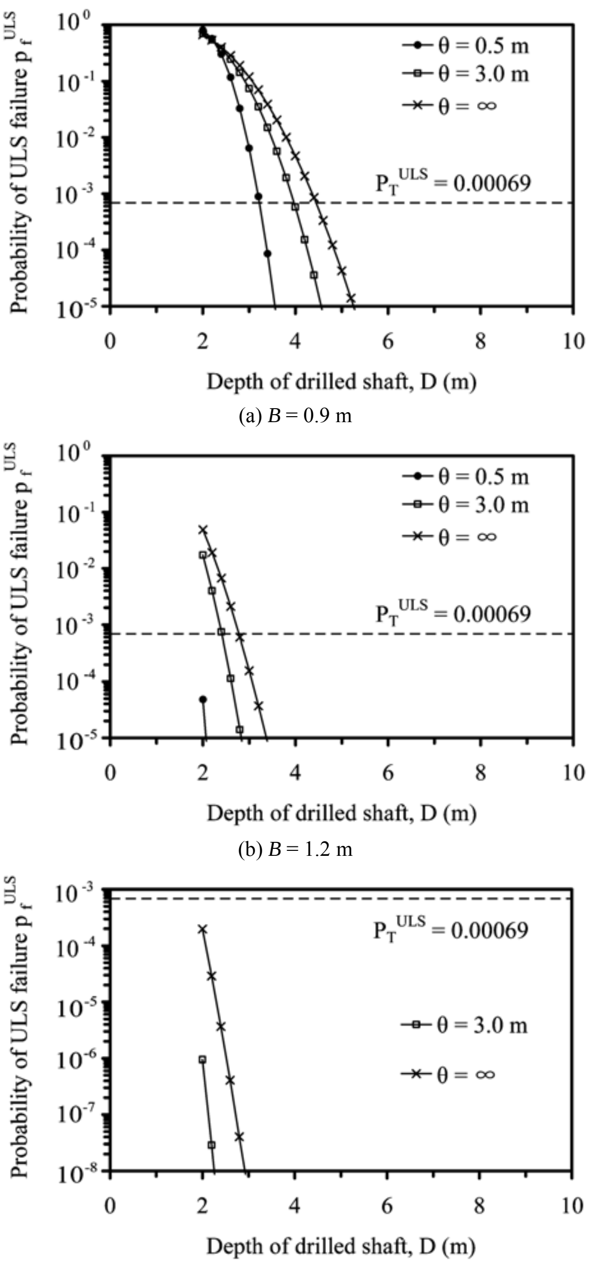
B = 0.9, 1.2, 1.5m

D = 2, 2.2, 2.4, …, 8m

For each combination of *B* and *D*, the spreadsheet-based FORM such as the one shown in Fig. 2 is performed to calculate the reliability index and probability of failure for both ULS and SLS. In those reliability analyses, only the effective stress friction angle is modeled as lognormally distributed random variable with a mean value of 32° and COV of 7% (note: These are the statistics used by Wang *et al*. 2011a). All other parameters are treated as constants and the detailed input values are shown in Table 1 and Fig. 2. It should also be noted that the values of all other input soil and structural parameters are the same as those adopted by Wang *et al*. (2011a). The reliability analysis is first performed under the assumption of spatial constant condition. Thus, the scale of fluctuation in the spreadsheet such as Fig. 2 is set to be a very large number (*e.g.*, $\theta = 10^6$) and with Eq. (7) it results in unity value for all variance reduction factors. Accordingly, the coefficient of correlation between spatial averages, as determined by Eq. (8), is 1.0, which indicates the spatial constant condition.

Figure 4 shows the computed probability of ULS failure (p_f^{ULS}) at various combinations of *B* and *D* values with three levels of spatial variability of ϕ'. The case of spatial constant condition ($\theta = \infty$) is denoted using symbol "χ". Given the target reliability index against ULS failure of p_T^{ULS} = 0.00069, the minimum feasible shaft depth *D* is 4.6 m for *B* = 0.9 m, as shown in Fig. 4(a). The minimum *D* values for *B* = 1.2 m and 1.5 m are 2.8 m and 2.0 m respectively, as shown in Figs. 4(b) and 4(c). Similarly, the computed probability of SLS failure (p_f^{SLS}) at various combinations of *B* and *D* values is shown in Fig. 5. Given the target probability of SLS failure of p_T^{SLS} = 0.0047, the minimum *D* values are 6.2 m, 4.4 m and 3.4 m for *B* = 0.9 m, 1.2 m and 1.5m, respectively, as shown in Figs. 5(a), 5(b) and 5(c). It should be noted that the feasible designs without considering the effect of spatial variability obtained using spreadsheet-based FORM is identical with those presented by Wang *et al.* (2011a) for both USL and SLS requirements. It should be noted that the solutions presented by Wang *et al.* (2011a) were obtained using Monte Carlo simulation method. The results presented in this paper validate the effectiveness and correctness of the spreadsheet-based FORM solution.

The emphasis of this paper is to study the effect of spatial variability of soil property on the reliability-based design of drill shafts. To this end, the aforementioned reliability analysis using the developed spreadsheet solution is repeated herein for various combinations of *B* and *D* values at two specified scales of fluctuation: θ = 0.5 m and θ = 3 m (Note: These are the typical bounds of the vertical scale of fluctuation). The results are also shown in Figs. 4 and 5 for ULS and SLS requirements, respectively. The effect of spatial variability is seen to have a significant influence on the reliability-based design of drilled shafts. For instance, as illustrated in Fig. 4(a), the minimum feasible *D* values that meets p_T^{ULS} = 0.00069 at *B* = 0.9 m are 3.4 m and 4 m for θ = 0.5 m and θ = 3 m, respectively, as opposed to the

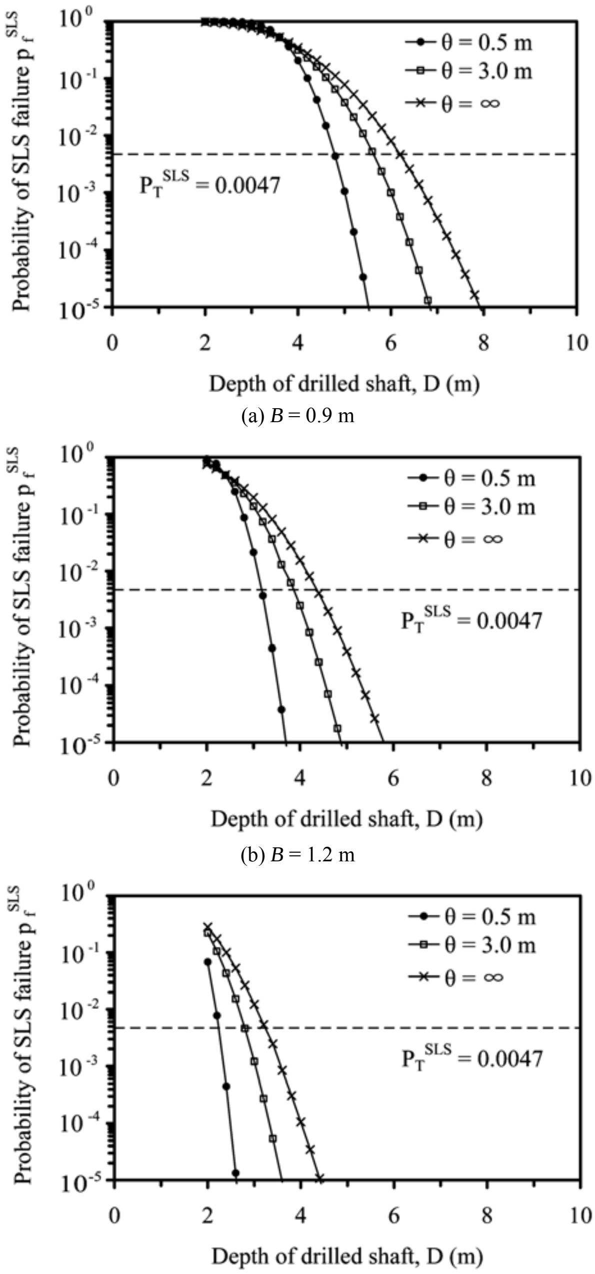


(a) B = 0.9 m

(b) B = 1.2 m

(c) B = 1.5 m

Fig. 4 Effect of scale of fluctuation on probability of ULS failure (COV of ϕ′ = 7%)



(a) B = 0.9 m

(b) B = 1.2 m

(c) B = 1.5 m

Fig. 5 Effect of scale of fluctuation on probability of SLS failure (COV of ϕ′ = 7%)

4.6 m for θ = ∞. Similarly, in Fig. 5(a), the minimum feasible D values that meets p_T^{SLS} = 0.0047 at B = 0.9m are 4.8m and 5.8 m for θ = 0.5 m and θ = 3 m, respectively, comparing with the 6.2 m for θ = ∞. Based on Figs. 4 and 5, it is concluded that the reliability-based design of drilled shafts will be more conservative than necessary if the spatial effect is simply neglected. For each level of θ and B, the minimum feasible shaft depth D for both ULS and SLS requirement is further summarized in Table 2. It is apparent that at the same θ and B level, the minimum feasible D value that meets the SLS requirement is larger than that meets the ULS requirement. Therefore, in this case study the SLS requirement dominates the design of drilled shafts given the target reliability indices against ULS and SLS failure are 3.2 and 2.6, respectively.

4.4 Final Design Based on Target Reliability Indices and Minimum Cost Requirement

With the feasible design candidates that meet the ULS and SLS requirements, the final design may be determined based on the minimum cost requirement. The feasible design candidates for three θ levels are summarized in Table 2. As aforementioned, the SLS requirement dominates the design and as shown in Table 2, there are three candidate designs for each θ level. The procedure for selecting the final design based on the minimum cost introduced by Wang and Kulhawy (2008) is adopted in this study to demonstrate the effect of spatial variability on the final design of drilled shafts. For each candidate design, the shaft depth is first selected as the one based on SLS requirement. Then, the total cost for each design is computed as the product of the shaft

Table 2 Final design based on minimum cost requirement (COV of ϕ' = 7%)

θ (m)	*B* (m)	*Required D* (m)		Unit cost* (USD / 0.3 m in depth)	Total cost (USD)	Final design
		ULS	SLS			
0.5	0.9	3.4	4.8	77.5	1240	*B* = 0.9 m, *D* = 4.8 m; or *B* = 1.2 m, *D* = 3.2 m
	1.2	2.0	3.2	116.0	1240	
	1.5	2.0	2.4	157.0	1260	
3.0	0.9	4.0	5.8	77.5	1500	*B* = 1.5 m *D* = 2.8 m
	1.2	2.6	4.0	116.0	1550	
	1.5	2.0	2.8	157.0	1465	
∞	0.9	4.6	6.2	77.5	1600	*B* = 0.9 m** *D* = 6.2 m**
	1.2	2.8	4.4	116.0	1700	
	1.5	2.0	3.4	157.0	1780	

Note: * Data from R. S. Means Co. (2007)
** Identical with the suggested design by Wang *et al.* (2011a) under spatial constant condition

depth and the unit cost. For each θ level, the final design is the one with the minimum total cost. As shown in Table 2, when the spatial variability is ignored (θ = ∞), the final design is *B* = 0.9 m and *D* = 6.2 m, which is identical with the design suggested by Wang *et al*. (2011a). For θ = 0.5 m, the final design is determined to be *B* = 0.9 m and *D* = 4.8 m, or equivalently *B* = 1.2 m and *D* = 3.2 m, since the total cost for both designs is approximately the same; for θ = 3 m, the design *B* and *D* values are 1.5 m and 2.8 m, respectively. When the spatial variability is considered in design, the total cost is reduced as the scale of fluctuation decreases. The decision of auger selection for shaft diameter *B* is also influenced by the scale of fluctuation in the design based on the minimum cost requirement.

4.5 Discussion: Reliability-Based Design at Higher Variation of Soil Property

The previous reliability analyses are performed using COV of ϕ' at 7%. As reported by Phoon *et al.* (1995), the COV of ϕ' for loose sand can be as high as 15 ~ 20%. To examine the influence of spatial variability at higher variation of soil property, the aforementioned procedures are repeated using COV of ϕ' at 15%. The computed probabilities of failure for ULS and SLS requirements at various levels of scales of fluctuation are shown in Figs. 6 and 7, respectively. The same target probability of ULS failure (p_T^{ULS} = 0.00069) and target probability SLS failure (p_T^{SLS} = 0.0047) are employed to determine the minimum feasible *D* values, and the results are summarized in Table 3. As shown in Table 3, the reliability-based designs of drilled shafts are seen to be overly conservative if the spatial effect is neglected, as larger shaft depths than necessary are required at the spatial constant condition (θ = ∞). This is exactly what was observed with results shown in Table 2 except that the effect of spatial variability is more profound with higher COV of ϕ' (15% versus 7%).

Table 3 Final design based on minimum cost requirement (COV of ϕ' = 15%)

θ (m)	*B* (m)	*Required D* (m)		Unit cost* (USD / 0.3 m in depth)	Total cost (USD)	Final design
		ULS	SLS			
0.5	0.9	4.6	6.2	77.5	1600	*B* = 1.5 m *D* = 3.0 m
	1.2	2.8	4.4	116.0	1700	
	1.5	2.0	3.0	157.0	1570	
3.0	0.9	6.0	8.0	77.5	2070	*B* = 0.9 m *D* = 8.0 m
	1.2	4.0	5.8	116.0	2245	
	1.5	2.8	4.4	157.0	2300	
∞	0.9	7.4	9.4	77.5	2430	*B* = 0.9 m *D* = 9.4 m
	1.2	5.2	7.2	116.0	2785	
	1.5	3.6	5.6	157.0	2930	

Note: * Data from R. S. Means Co. (2007)

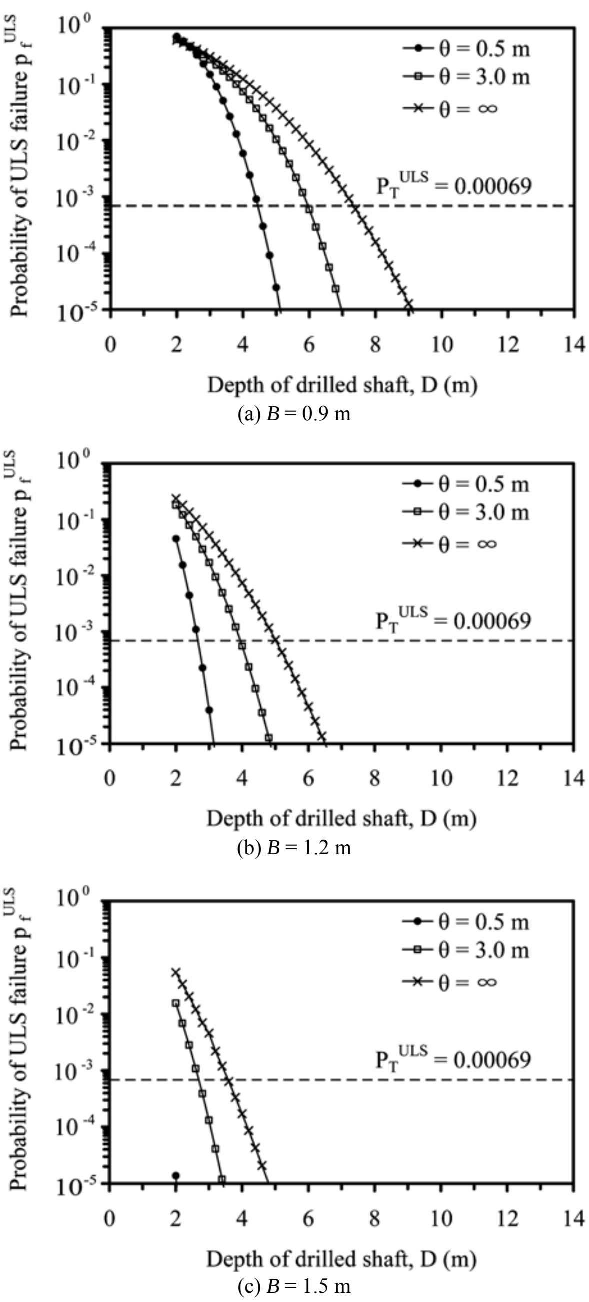


Fig. 6 Effect of scale of fluctuation on probability of ULS failure (COV of ϕ' = 15%)

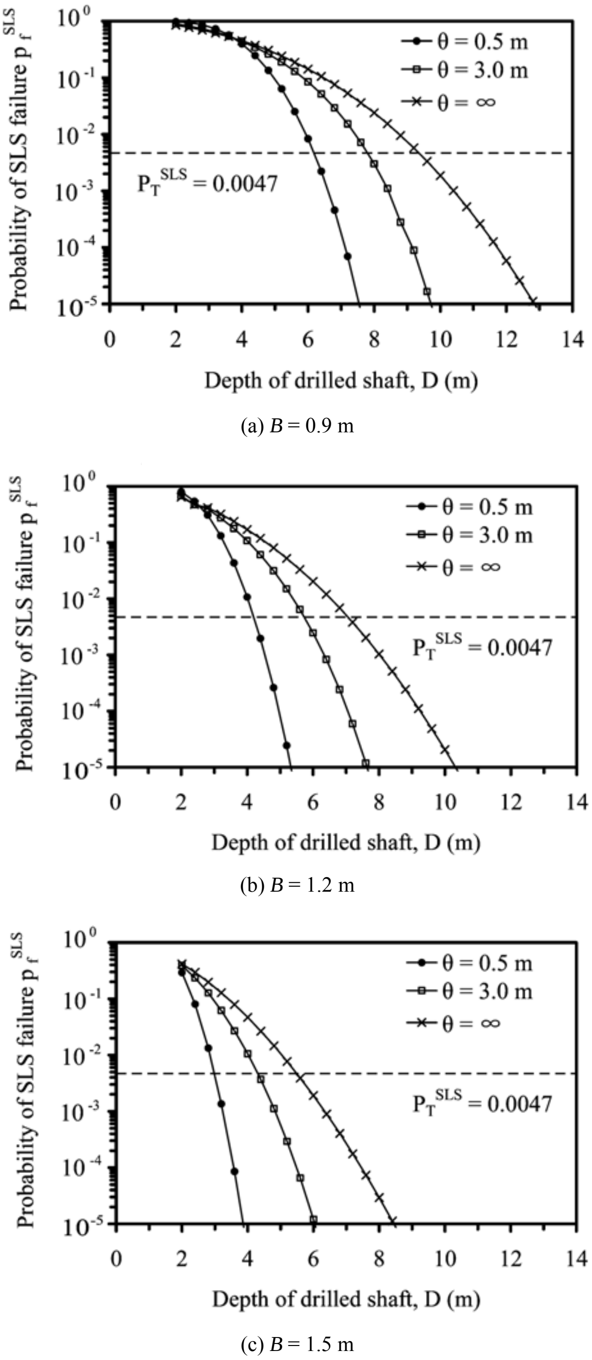


Fig. 7 Effect of scale of fluctuation on probability of SLS failure (COV of ϕ′ = 15%)

As in the previous analysis (COV of ϕ′ = 7%), larger D values are required to meet SLS requirement than ULS requirement. Therefore, the SLS requirement also dominates the design of drilled shafts using RBD at higher COV level (COV of ϕ′ = 15%). Finally, comparing with lower level of COV (7% as in Table 2), the minimum feasible D values are larger at higher COV (15% as in Table 3) for the same level of B value and scale of fluctuation, as expected. Of course, even at the spatial constant assumption ($\theta = \infty$), the increase in the required D value at the same B value is significant as COV of ϕ′ increases from 7% (Table 2) to 15% (Table 3). Thus, it is important to have an accurate estimate of both the COV of ϕ′ and the scale of fluctuation.

With the feasible design candidates for the three θ levels in Table 3, the final designs are also determined based on the minimum cost requirement and shown in Table 3. As in the previous analysis (COV of ϕ′ = 7%), the total cost of drilled shaft at final designs at higher COV level (COV of ϕ′ = 15%) can also be reduced if the spatial effect is considered.

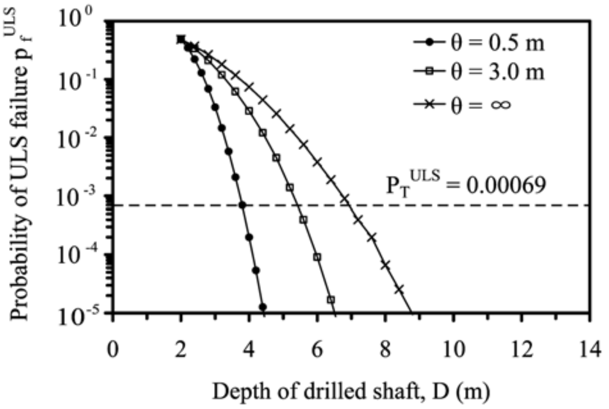
4.6 Discussion: Reliability-Based Design Under Larger Axial Compression Load

In the previous example, a compression load of 800 kN was used so that the consistency with the illustrated example by Phoon *et al*. (2005) can be maintained. In this section, the effect of the spatial variability of ϕ′ on the reliability-based design of drilled shafts is further investigated using a different load level, F_{50} = 1200 kN. The shaft diameter B is set to be 1.2 m and the COV of ϕ′ is assumed to be 15%. The same procedures as described previously are applied and the computed probabilities of failure for ULS and SLS requirements at various levels of shaft length D for three scales of fluctuation are shown in Fig. 8. It is observed in this new case study that the SLS requirement also dominates the design of drilled shafts. Based on the results from the SLS requirement (Fig. 8b), the minimum feasible D values are 6.0 m, 8.0 m and 9.8 m for θ = 0.5 m, 3 m and ∞, respectively. Based on further comparison of Fig. 8(a) with Fig. 6(b), and Fig. 8(b) with Fig. 7(b), it is obvious that larger D values are required if the axial compression load increase from 800 kN to 1200 kN, as expected. The effectiveness of the developed approach is further demonstrated using a larger axial compression load. This approach based on spreadsheet solution can also be easily adapted for other loading conditions and soil profiles.

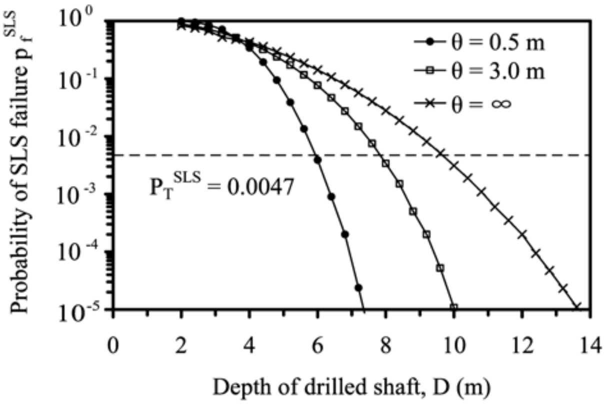
5. SUMMARY AND CONCLUDING REMARKS

In this paper, an efficient approach for the reliability-based design (RBD) of drilled shafts subjected to drained compression in loose sand with the consideration of spatial variability of soil property is developed. The spatial averaging technique is adopted to simplify the modeling of spatial variability. The effect of the spatial correlation of soil property between the tip resistance zone and the side resistance zone on the design of drilled shafts is investigated. The proposed approach is realized with the use of first-order reliability method (FORM) implemented in a spreadsheet, a practical engineering tool. When the spatial variability is ignored in RBD for both ULS and SLS requirements, this spreadsheet solution yields results that are virtually identical to those obtained with Monte Carlo simulation by Wang *et al.* (2011a) that did not consider the spatial variability.

It is observed from the results of the parametric study that the traditional reliability analysis that neglects the spatial effect overestimates the probability of failure for both ULS failure and SLS failures. Thus, the design of drilled shafts using RBD without considering the effect of spatial variability of soil parameters can be overly conservative. When the typical range of the scale of fluctuation of ϕ′ (0.5 ~ 3 m) is considered, the minimum required shaft depth (D) that meets the target reliability index against ULS and SLS failure at the same level of shaft diameter (B) and COV of ϕ′ is significantly reduced, compared to the results under the spatial constant condition (θ = ∞). In addition, the influence of spatial variability on the decision of final design



(a) Ultimate limit state



(b) Serviceability limit state

Fig. 8 Effect of scale of fluctuation on probability of ULS and SLS failure under axial compression load (F_{50}) of 1200 kN (B = 1.2 m and COV of ϕ' = 15%)

based on the minimum cost requirement is demonstrated. It is shown that under the same reliability requirement (either ULS or SLS requirement), the total cost is reduced as the scale of fluctuation decreases. Finally, the determination of auger size for shaft diameter B is also affected by the spatial variability in the design based on the minimum cost requirement.

The same finding regarding the effect of spatial variability on the RBD of drilled shafts in loose sand is observed regardless of the level of COV of ϕ' that was assumed in the reliability analysis. Thus, it is important to have an accurate estimate of both the COV of ϕ' and the scale of fluctuation at a given site.

The proposed approach for the RBD of drilled shafts considering spatial variability is illustrated using the deterministic procedure developed by Kulhawy (1991) for the scenario of drained compression in loose sand. This simplified approach is shown to be effective and efficient, especially with a spreadsheet implementation. The proposed approach may be adapted for RBD of drilled shafts for other soil types and loading conditions, including the scenario of spatial variability of multiple soil parameters. In fact, the approach is shown to be effective and efficient, especially with a spreadsheet implementation, it has the potential as a tool for general geotechnical applications of RBD.

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