

IMPROVED MEASUREMENT AND A PREDICTIVE MODEL FOR THERMAL CONDUCTIVITY OF SAND-BENTONITE MIXTURES

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ABSTRACT

In this article, an improved thermal probe method for the measurement of thermal conductivity is proposed. Using this method, the thermal contact resistance between the probe and the specimen can be minimized and the total number of measurements when evaluating the relationship between thermal conductivity and these factors: dry unit weight of bentonite, water content, and fraction of sand or crushed granite is also reduced. A predictive model for sand-bentonite-based buffer material including two sub-models is presented. The first sub-model (the matrix model) is modified from the de Vries and Campbell model (1985), which is aimed at predicting the thermal conductivity of the matrix phase (representing bentonite, water, and void) at different densities and water contents. The second sub-model (the micromechanics model) predicts the overall thermal conductivity of a particulate-matrix composite. By assigning the sand or crushed granite as the particulate and the bentonite-water-void as the matrix, micromechanics models can be applied and the predictive results agree with the experimental data of the sand-bentonite mixture.

Key words: Buffer material, sand-bentonite mixture, thermal conductivity, micromechanics, de Vries and Campbell model.

1. INTRODUCTION

Bentonite-based buffer materials are employed as engineered barriers in high-level nuclear waste depository systems (Geneste 1990; Børgesson 1994; Gera *et al.* 1996; JNC 1999; NEA 2003). The buffer materials provide the mechanical support, thermal dissipation and hydraulic sealant for the waste canisters. In order to keep the maximum temperature in the buffer material below a certain value, the batch design of the buffer material should be considered (Børgesson 1994).

The thermal properties of porous media are affected by a variety of parameters. According to Farouki (1986), the thermal conductivity of soils is mainly influenced by the composition, structure, packing, density, porosity, grain, and pore-size distribution, as well as by contacts and binding effects. Previous research reveals that there are three major parameters that affect the thermal conductivity of bentonite-sand (or crushed granite) buffer materials: the proportion of bentonite/sand, the dry unit weight (or void ratio) of the bentonite, and the water content. There are many models, such as pure empirical regression models, numerical models, or semi-empirical models with theoretical basis, are widely used to describe the thermal conductivity of soils or buffer materials (Farouki 1986; JNC

soils or buffer materials (Farouki 1986; JNC 1999). Due to the complexity of porous media (such as soils or compacted powders), these models are either complicated or containing too many parameters.

The development of the thermal conductivity prediction model depends on the accuracy of the source data gathered from carefully conducted laboratory experiments. The methods for thermal conductivity measurement can be categorized as steady state methods and transient methods. The steady state methods apply a stationary heat flow through the sample to create a constant thermal gradient in the specimen for a long period of time. There is a problem in partially water-saturated materials since the thermal gradient will cause moisture migration within the sample (Yong *et al.* 1997). Therefore, the transient state methods are fairly common for moisture samples as a result of the brief measurement time. The thermal probe method has been adapted for the bentonite-sand (crushed granite) buffer materials (Børgesson 1994). However, the sample preparation and the probe installation may cause too much experimental uncertainty. In this article, a modified thermal probe method based on ASTM D5334 (2000) is developed to measure the thermal conductivity of the buffer material with various densities in a single sample. Based on the experimental results, a prediction model for sand-bentonite-based buffer materials is proposed.

2. MEASURING TECHNIQUES

The thermal probe or needle probe method is a rapid and convenient method for measuring the thermal conductivity of soils and soft rock. The probe is inserted into the soil sample. The proposed method is referred to herein as the needle probe method. Using this method, the thermal conductivity of bentonite-based buffer material can be measured with different densities in a single sample. The concept is to put the thermal probe inside the buffer material sample during compaction. When a desired unit

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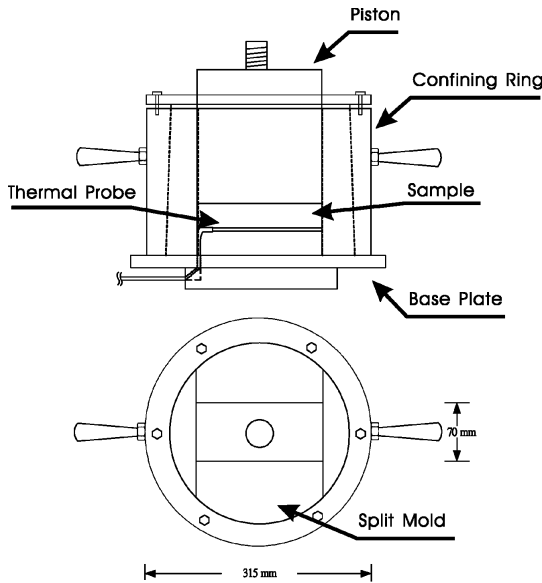


Fig. 1 The compaction mold

weight is reached, the thermal conductivity is measured. By repeating the process, the relationship between the thermal conductivity and unit weight is established. This method requires two apparatus – the compaction mold and the line-source measurement system.

2.1 The Compaction Mold

The schematic layout of the compaction mold is shown in Fig. 1. The mold is composed of the following components:

1. An inner split mold

The four-piece steel split mold forms a cubic space of 18 cm \times 7 cm and is 19 cm in height. A thermal hardening process is applied over the surface of the steel mold to achieve a higher stiffness to resist the scratching from the compaction process. The outside shape of the split mold when assembled is a cylindrical cone with a transition in diameter from 25.3 cm at the bottom and 24.3 cm at the top.

2. A confining ring

The ring is made of steel without applying the thermal process. The outer diameter is 31.5 cm. The inner shape of the ring is fitted into the split inner mold. The rationale for this design is that it is easier to remove the inner mold from the ring after the sample is compacted and the mold will not slip out when the compaction force exerts a lateral force on the sample. The height of the ring is 18 cm, less than that of the inner split mold. When the ring is put on the inner split mold assembly, there is a space (about 0.5 cm) between the ring and the base plate. This is to ensure the confining of the ring on the inner split mold.

3. A compaction piston

The cubic piston is made of steel and the thermal hardening process is also applied to it. The dimension of the piston is 17.9 cm \times 6.9 cm and is 19 cm in height. A threaded rod is installed on top to connect it to the loading frame.

4. A base plate

The base plate is also made of thermally-hardened steel. The base plate has an upper deck and a lower deck (side-view of which is shown in Fig. 1). A hole is drilled on the upper deck, while a groove is made at the same place on the lower deck. The

wire of the thermal probe goes through the hole to connect to the logging system and power supply.

5. A loading system

A servo-controlled loading system is used to apply the compaction loading. During the compaction process, the applied loading and displacement of the piston are continuously recorded by a load cell and a linear variable displacement transducer (LVDT). The capacity of the loading system is 1 MN. The maximum compaction pressure is around 100 MPa. The compaction method used in this research is a uniaxial static compaction (Tien *et. al.* 2004). By controlling the distance from the base plate and the piston, the volume of the sample is determined and the unit weight can be calculated.

2.2 The Thermal Probe Method

The probe should be thin to avoid disturbing the specimen. It consists of a heater, emitting thermal energy at a constant rate, and a temperature-sensing element, which is a thermal couple in this study. The rate of temperature rise in the probe depends on the thermal conductivity of the surrounding medium.

The theory of the thermal probe method is based on the solution of the line heat source placed in a semi-finite, homogeneous, and isotropic medium (Carslaw and Jaeger 1959). The testing equipment used in this research is based on ASTM D5334-Standard Test Method for Determination of Thermal Conductivity of Soils and Soft Rocks by Thermal Needle Probe Procedure. The required equipments are listed below:

1. Thermal Needle Probe

The thermal probe (Fig. 2), which is 15 cm long and 0.3 cm in diameter, consists of a nichrome heater wire and a T-type thermocouple (made of copper and constantan wire). The thermocouple is used for its durability and the temperature ranges from -200°C to 400°C . The heater and the thermocouple are placed in a stainless tube and magnesium oxide (MgO) powder is filled in the gaps. A hydrostatic pressure of 210 MPa is applied on the probe to compress the tube and the filling of magnesium oxide to form the probe. Therefore, the strength and stiffness of the thermal probe is strong enough to suffer compaction pressure. To protect the wires during compaction, a metal mesh tube with TEFLON liner covers the connecting wire all the way through the hole on the base plate. The position of the probe in the sample is kept in the centre by first putting half of the sample in the mold, bending the flexible wire to lay the probe on the surface, and then using the other half of the sample (Fig. 3). The resistance of the nichrome heater wire is checked with an Ohm meter before and after each experiment.

2. Constant Current Source

An adjustable linear power supply is used to provide a constant current to the thermal probe, resulting in a constant thermal energy output. A 9 ~ 12 Volts of input voltage to a thermal probe with resistance 60 ~ 70 Ohm is suitable for general soils.

3. Temperature Readout Unit

An Agilent 34970A data acquisition/switch unit with HP 34901A 20-channel armature multiplexer is used for logging temperature and time. While using the T-type thermocouple with

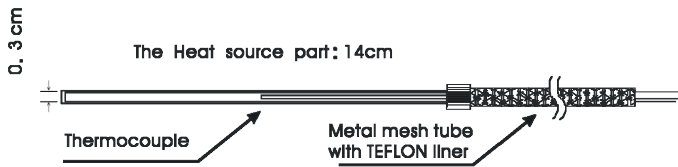


Fig. 2 The thermal probe

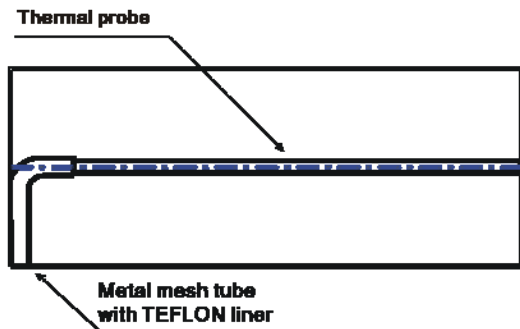


Fig. 3 The position of the thermocouple inside the sample

this system, an ice-bath should be used to create a known reference temperature (0°C) in order to prevent the internal junction error. The resistance of the nichrome heater and the voltage of the power supply is calibrated and recorded to ensure the quality of the experiment. The calculation of thermal conductivity is described in ASTM D5334 (2000).

2.3 Experimental Procedures

1. Determination of unit weight and volume fraction

The determination of unit weight and volume fraction of sand-bentonite mixture is based on the following assumptions:

- The water in the sand-bentonite mixture is incompressible.
- The compressibility of sand (or crushed granite) is negligible because of its high stiffness in comparison with bentonite powder.
- The volume change of sand-bentonite mixture during compaction is caused by the densification of bentonite powder only.

Tien and Wu (2004) described the relationship between dry unit weight, volumetric fraction, and weight fraction in the compaction process of the bentonite-sand (or crushed granite) buffer material.

With these assumptions, the overall dry unit weight can be determined by measuring the distance from the piston to the base plate. The final stage of experiment shows that the volume of the sample equals 18 cm × 7 cm × 7 cm. This prismatic shape meets the geometry requirement specified in ASTM D5334.

2. Pouring the mixtures into the mold

Half of the powdery mixture of bentonite and sand (crushed granite) is filled into the mold and tamped to form a flat surface. The flexible wire of the thermal probe is immersed in the sample near the mold (as shown in Fig. 3). After tamping the first layer, the thermal probe is bent to lay on the surface and temporarily fixed to it. The other half of the powder is then filled onto it and tamped.

3. Applying the initial contact pressure

The piston is lowered onto the surface of the sample and then a contact force of 0.5 kN is applied. The settlement of the sample at this point is largely due to the loose powdery state of

the sample. The loading rate is slow (0.1 kN/sec) to allow the particles to be rearranged. The readings from the load cell will be stable when the settlement of the sample stops.

4. Compacting to the designated unit weight

Displacement-control is applied at a displacement rate of 0.1 mm/min. After the piston reaches the designated position and being held, the compaction stress will relax. The compaction stress was monitored by a load cell. Once the reading of compaction stress becomes stable, it means that the pore water pressure dissipation and clay particle packing rearrangement are completed. The required time for stabilization period is around 5 ~ 10 minutes. Therefore, a 30 minutes period in the sequence is sufficient for the consolidation and time dependent deformation.

5. Thermal conductivity measurement

The measurement of the thermal conductivity is next performed. First, the data logging system begins to record the temperature from the thermocouple at an interval of 0.5 sec. Then, the power supply is turned on to generate the heat and the time is recorded at the start of the measurement. The heating time for the measurement is 1200 sec. After the measurement, the sample needs to be cooled down to the room temperature for the next stage of measurement. A time interval of 40 minutes is sufficient for the temperature to drop, and this time interval can overlap with the compacting and stabilizing time at the next stage.

These two sequences are repeated until the final stage of unit weight is reached and the thermal conductivity measurement is applied. After all measurements are completed, the compaction force is released at a rate of 1 kN/sec. The sample is then taken out and cut into slices to measure the water content for comparison with the state before the experiment. The electrical resistance of the nichrome heater wire is also examined to see if the heater was damaged during the compaction procedure.

By embedding the thermal probe into the specimen, there is no gap or clearance between the probe and the specimen. Comparing to the drilled hole method suggested by ASTM D5334, the method in this study minimized the influence from thermal contact resistance (Tien *et al.* 2006). This method is suitable for determining the thermal conductivity of compactable materials at different dry unit weight in one specimen, while traditional methods need to perform the test on different specimens with different dry unit weights.

3. MATERIALS

The materials for the bentonite-sand (crushed granite) buffer material in this study are discussed below.

3.1 Bentonite

The MX-80 bentonite (Na-bentonite), from Wyoming, USA, is used in this study. It is also known as Black Hill bentonite (B.H. bentonite). The chemical composition and basic physical properties are shown in Table 1 and Table 2. The water content of bentonite powder is adjusted by using following procedure:

1. Moisture chamber curing

In order to acquire the bentonite with higher water content rather than open-pack water content, the bentonite is poured over a 30 cm × 40 cm flat punched pan with non-woven textile liner. The thickness of the bentonite layer is 1 cm. The pan is covered

Table 1 Typical chemical compositions of MX-80 bentonite (values given in %, w/w)

	SiO ₂	Al ₂ O ₃	Fe ₂ O ₃	K ₂ O	CaO	Na ₂ O	MgO	TiO ₂
MX-80 bentonite	64.5	18.5	3.45	0.39	1.18	1.94	2.48	0.29

Table 2 Physical index properties of MX-80 bentonite

	G_s	w_n (%)	A_c	LL (%)	PL (%)	PI (%)
MX-80 bentonite	2.65	9.69	6.2	497	33	344

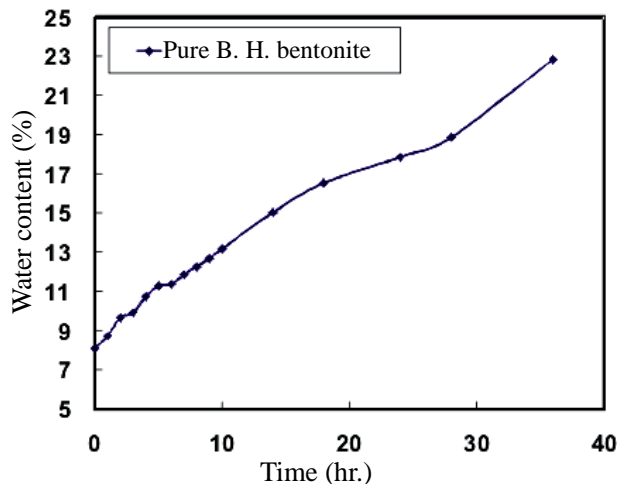


Fig. 4 The change of water content of B.H. Bentonite over time in a moisture chamber

with another sheet of non-woven textile and put into a moisture chamber with the relative humidity (RH) at 100% and temperature at 38°C. The change of water content in time is shown in Fig. 4. After this relationship is established, the bentonite sample with different water content can be prepared by putting it in the moisture chamber at the corresponding time intervals.

2. Oven drying

In order to acquire the bentonite powder with lower water content rather than open-pack water content, the bentonite is poured over a 65 cm × 100 cm flat stainless steel pan. The thickness of the bentonite layer is 1 cm. The pan is put into an oven with a constant temperature of 105°C. The change of water content over time is shown in Fig. 5. After this relationship is established, the bentonite sample with different water content can be also prepared and put in the oven at corresponding time intervals.

The bentonite prepared with both methods is first sieved with a #100 sieve to remove the clotted blocks, and then stored in re-sealable plastic buckets at least one week to ensure the moisture equilibrium throughout the powder. The water content should be checked before each test.

3.2 Sand (Crushed Granite)

The sand was crushed from granite of Kinmen Island, Taiwan. The gradation of crushed granite is passing through #10 and

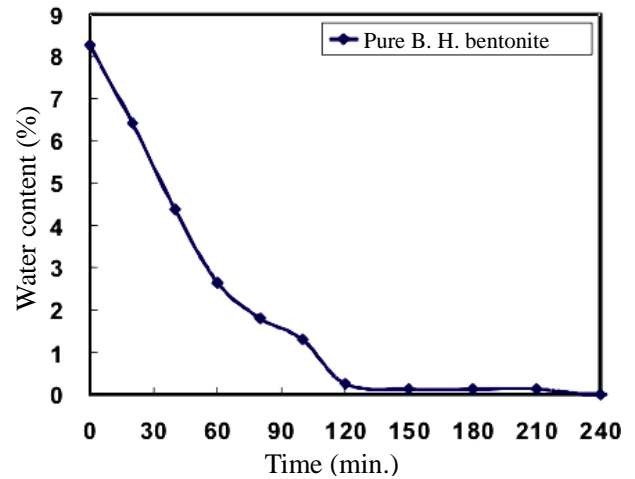


Fig. 5 The change of water content of B.H. Bentonite over time in an oven

retaining on #20. The specified gravity (G_s) and void ratio (e_g) of crushed granite is 2.67 and 0.01, respectively. Table 3 shows the mechanical properties of the intact Kinmen granite (Tien 2002).

3.3 The Mixtures for Sample Preparation

Since the thermal conductivity of sand-bentonite is affected by three major factors: dry unit weight of matrix (clay), water content of matrix (clay), and the fraction of sand (crushed granite), an effort is made in this study to develop a predictive model for the thermal conductivity based on the three factors.

1. Dry unit weight

The dry unit weight for buffer material suggested by various researchers is within a range of 16 kN/m³ ~ 17 kN/m³, while the backfill material is between 14 kN/m³ ~15 kN/m³ (JNC 1999; Wardrop 1985). In this article, we take the advantage of the proposed method to deal with the dry unit weight issue. The designated dried densities of the matrix at which thermal conductivity is to be measured are 14 kN/m³, 15 kN/m³, 16 kN/m³, 17 kN/m³, and 18 kN/m³.

2. Water content

The moisture content in the clay barrier around the canister might be zero due to the vaporization of water due to the heat dissipated by the canister, or saturated with the inflow of groundwater flow. Therefore, the water contents (w) by weight of the bentonite powder in this study are 0%, 3.84%, 8.13%, 12.73%, 16.12%, and 20.04%, respectively.

3. Sand (crushed granite) weight fraction

With increasing of the weight fraction, G_g (the ratio of weight of sand over weight of sand-bentonite mixture) of the crushed granite, the overall thermal properties would be gradually changed for the difference in thermal conductivity between bentonite and sand particles. Mixtures with G_g at 0%, 12.5%, 25%, 37.5%, and 50%, were implemented for study in this article.

4. RESULTS OF THE EXPERIMENTS

4.1 Thermal Conductivity of Pure Bentonite

The thermal conductivity of pure bentonite with different dried densities and different water contents are shown in Figs. 6 and 7.

As expected, the thermal conductivity increases as the water

content increases. The thermal conductivity also rises when the dry unit weight of bentonite increases. This trend is consistent with many other studies (Börgesson 1994; JNC 1999; Ould-Lahoucine 2002). In the relationship between thermal conductivity and water content, a sudden rise in thermal conductivity occurs around the water content range of 8% ~ 12% (Fig. 6). This shows that the water not only contributes to its higher thermal conductivity, but also interacts with the powder to form water bridges among particles, which is referred as water-island theory proposed by Philip and de Vries (1957). At the micro level, the water bridges provide the effective path for thermal conduction and increase the thermal conductivity at the macro level (Tarnawski 2000).

4.2 Thermal Conductivity of Sand-Bentonite Mixture

The result of sand-bentonite mixture with different crushed granite fractions are shown in Fig. 8. Note that the weight fraction has been transformed in to volumetric fraction in order to be able to apply micromechanics models. The relationship of weight fraction and volumetric fraction of the crushed granite in buffer materials with different bentonite dried densities is listed in Table 4. The readers are referred to Tien et al. (2004) for additional details.

5. PREDICTION MODELS

Efforts toward the development of prediction models for the thermal conductivity of soils were made by many researchers. Most of the models rely on the fraction analysis of the constituents in soils. A discussion of the validity of these models is provided by Farouki (1986).

The prediction model for buffer material is simpler than common soils due to the simplicity of the constituents—clay, water, granite, and void. In this approach, we try to reduce the number of constituents. Sand-bentonite buffer material is considered to be a statistically isotropic two-phase composite material. The matrix phase (continuous phase) is the bentonite with water and void, while the particular phase (discrete phase) is crushed granite. Using this definition, two-phase micromechanics models are applicable.

The proposed approach for predicting the thermal conductivity of sand-bentonite mixtures consist of two sub-models, matrix model and micromechanics model. Figure 9 is a schematic view of the proposed model.

They are described in the following.

5.1 Matrix Model

The matrix model is adopted to predict the thermal conductivity of the pure bentonite block (matrix phase). Since the thermal conductivity of pure bentonite block is related to the unit weight and the water content, models concerning these two variables could be used in a matrix model.

By investigating the experimental results of pure bentonite, the phenomenon that thermal conductivity would raise in an extraordinary pattern that when the water content comes to a certain level, the thermal conductivity would be raised dramatically is observed (Fig. 10). Among the various prediction models for soil, it is essential to choose a model that can describe this behavior. Campbell (1985) fitted the relationship between water content and thermal conductivity of sand, silt loam, and forest litter with

the following equation (McInnes 1981):

$$k = A + B\theta_v - (A - D) \exp[-(C\theta_v)^E] \tag{1}$$

where θ_v is the volumetric water content, and $A, B, C, D,$ and E are material parameters related to soil properties. By regressions and curve-fitting with data from de Vries (1963), the equations for these coefficients are listed below:

$$A = 0.65 - 0.78 \gamma_d + 0.60 \gamma_d^2 \tag{2}$$

$$B = 1.06 \gamma_d \tag{3}$$

Table 3 Typical mechanical properties of Kinmen granite

Unconfined strength (MPa)	Bulk modulus (GPa)	Poisson's ratio
147	21	0.25

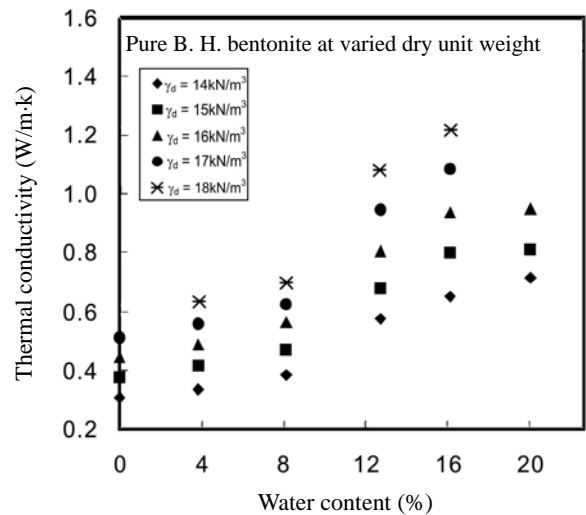


Fig. 6 The thermal conductivity of B.H. Bentonite with respect to water content

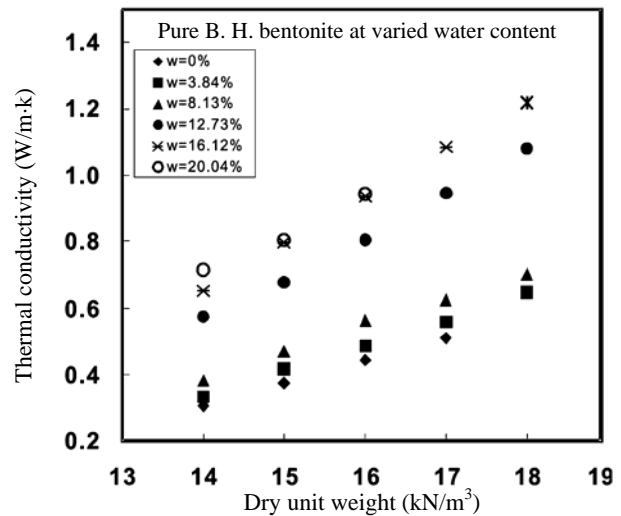
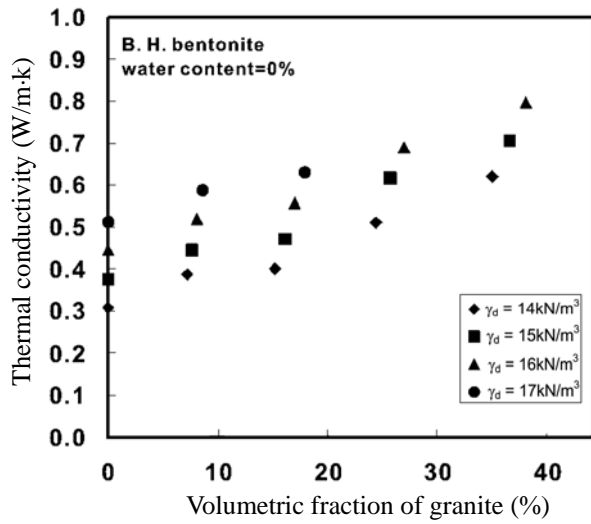
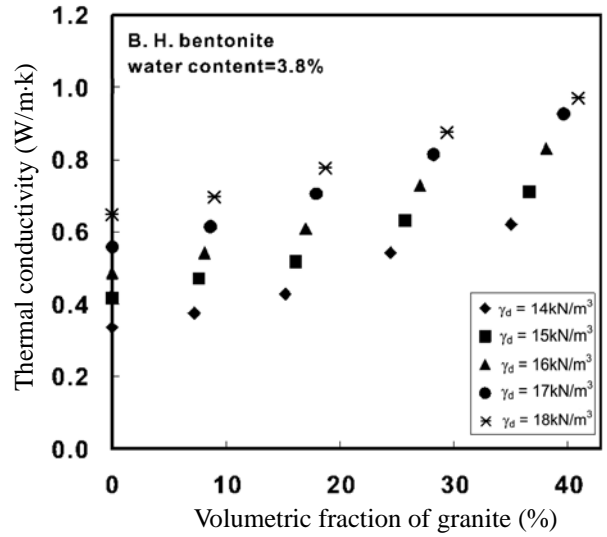


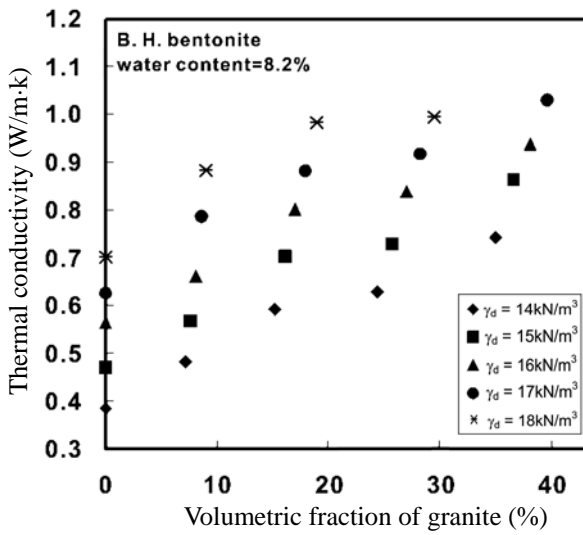
Fig. 7 The thermal conductivity of B.H. Bentonite with respect to dry unit weight



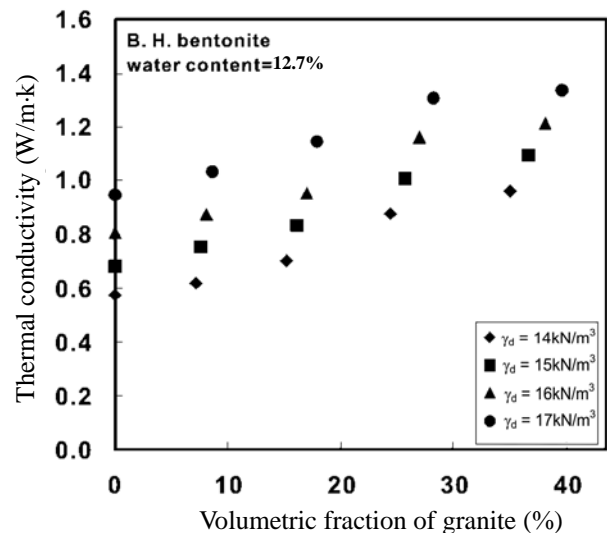
(a)



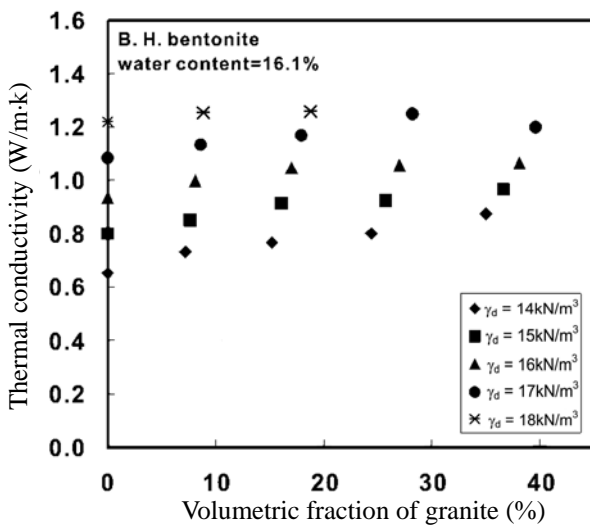
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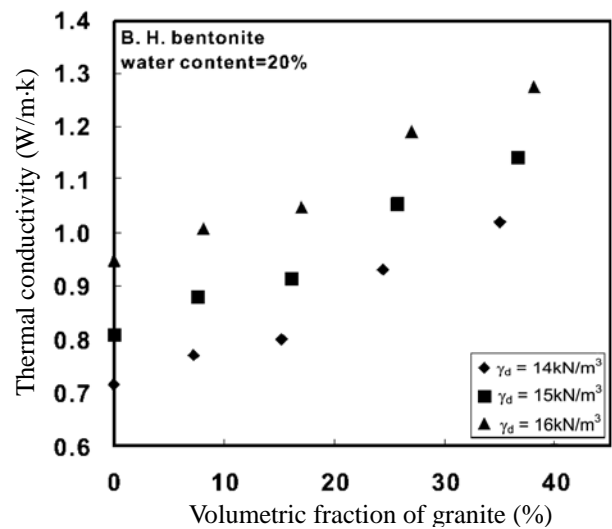
(c)



(d)



(e)



(f)

Fig. 8 The thermal conductivity of buffer material with different water content, clay dry unit weight, and volumetric fraction of granite

Table 4 The relationship of weight fraction and volumetric fraction of crushed granite in sand-bentonite mixture with different clay dried densities

Clay dry unit weight (kN/m ³)	Volumetric fraction of granite (%)				
	Weight fraction of granite (%)	12.5	25	37.5	50
14		7.2	15.2	24.4	35.0
15		7.6	16.1	25.7	36.6
16		8.1	17.0	27.0	38.1
17		8.6	17.9	28.2	39.6
18		9.0	18.8	29.4	40.9

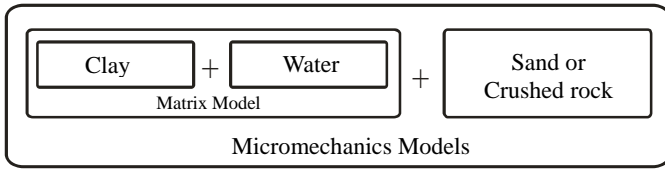


Fig. 9 Schematic view of the proposed model

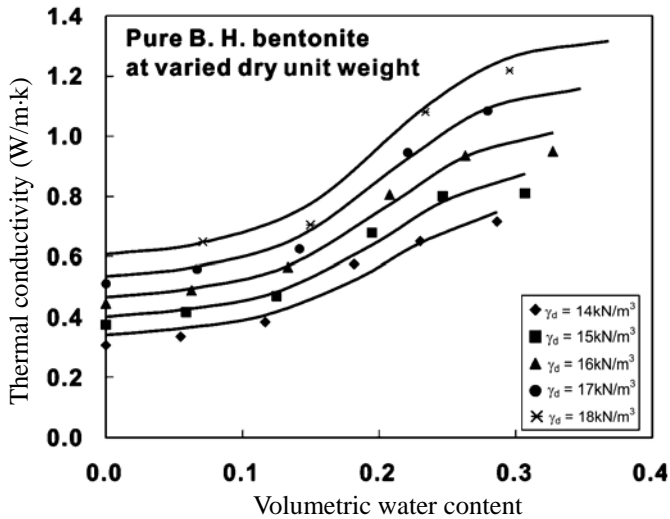


Fig. 10 The predicted and experimental data

$$C = 1 + 2.6/(mc^{0.5}) \tag{4}$$

$$D = 0.03 + 0.1 \gamma_d^2 \tag{5}$$

$$E = 4 \tag{6}$$

where mc is the clay content and γ_d is the dry unit weight of the clay.

When the volumetric water content reaches zero, the parameter D represents the thermal conductivity relating to the dry unit weight of soil. When saturation, the later term in Eq. (1) becomes zero, and the thermal conductivity is represented with $(A + B\theta_v)$. Parameter A represents the influence of the mineral ingredients on thermal conductivity, while parameter B represents the influence from water. Parameter E determines the pattern of the curve, while parameter C determines the water content where thermal conductivity begins to increase rapidly, and is highly correlated with clay content.

5.2 Micromechanics Models

The micromechanics model is used to predict the overall thermal conductivity of the sand-bentonite mixture. Once the thermal conductivity of the pure bentonite in the sand-bentonite mixture is determined by the matrix model, the micromechanics models can be adopted, given the fraction of particulate (sand or crushed granite). Micromechanics models are developed to deal with the overall properties of composite materials with the consideration of volume fraction, orientation, packing, and shape of the particulate phase (Christensen 1979; Jones 1975).

A composite is statistically isotropic when its overall property is independent of the choice of a coordinate system. Examples include a random mixture of two phases, a matrix containing spherical type particles, a randomly oriented elongated particles, and porous media (Hashin 1983).

From the viewpoint of micromechanics, the bentonite-sand mixture can be regarded as a type of composite material. Bentonite can be treated as a matrix and the sand or crushed rock can be treated as a particulate (also called inclusion). The sand in the sand-bentonite mixture is a nearly equal-dimension-shaped particulate and is distributed randomly, thus the sand-bentonite mixtures can be regarded as a two-phase, statistically isotropic composite materials. By using the volumetric fraction and the thermal conductivity of each phase, the overall thermal conductivity can be calculated using micromechanics models.

Micromechanics models are suitable for predictions of the thermal conductivity of composite materials (Nemat-Nasser and Hori 1993; McLaughlin 1977). The self-consistent-scheme (Budiansky 1965) and differential scheme (Tarn 1980) are used in this article to predict the thermal conductivity of sand-bentonite mixtures.

5.3 Implementation of the Proposed Model

The steps for implementing the model are described below.

1. Matrix model

The first step is to establish a matrix model for bentonite at a specified dry unit weight and water content. Based on the experimental data of this study, the parameters for the modified de Vries and Campbell model of pure bentonite block are established as followed (after Campbell 1985):

$$A = 0.55 - 0.78 \gamma_d + 0.60 \gamma_d^2 \tag{7}$$

$$B = 0.259 \gamma_d \tag{8}$$

$$C = 14.5 \tag{9}$$

$$D = -0.067 + 0.2 \gamma_d^2 \tag{10}$$

$$E = 4.1 \tag{11}$$

The experimental data for pure bentonite block and prediction curve of the modified de Vries and Campbell Model is shown in Fig. 10. The comparison of the predictions and the experimental data is shown in Fig. 11. The R^2 equals 0.9847, indicating a good correlation.

2. Micromechanics model

Self-consistent scheme and differential scheme are used here for comparison.

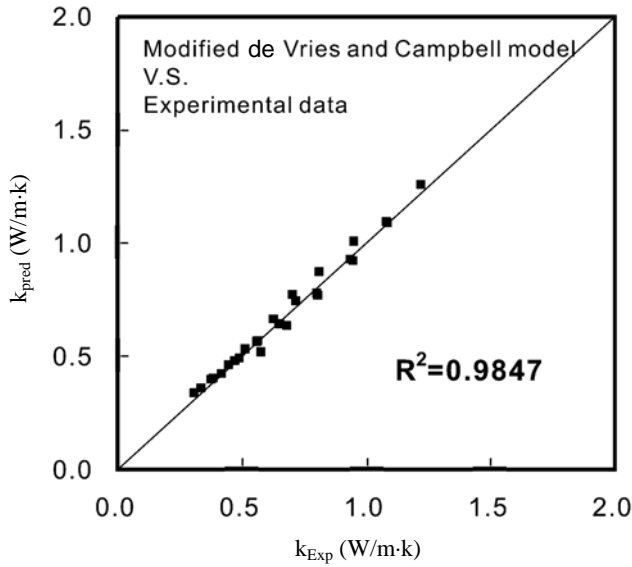


Fig. 11 The correlation of predicted and experimental data

3. Self-consistent scheme

The self-consistent scheme has a very simple geometric interpretation. Specifically, each phase of the composite is alternatively viewed as being lumped as a single ellipsoidal inclusion in an infinite matrix of the unknown effective properties of the problem. Equation (12) is a general equation for self-consistent scheme.

$$\frac{C_1(k - k_1)}{2k + k_1} + \frac{C_2(k - k_2)}{2k + k_2} = 0 \tag{12}$$

By substituting k_1 with the predicted value from matrix model, and k_2 with thermal conductivity of the inclusions, while C_1 and C_2 represent the volumetric fraction of clay part and sand, respectively, the equation can be solved with Mathematica or Microsoft Excel.

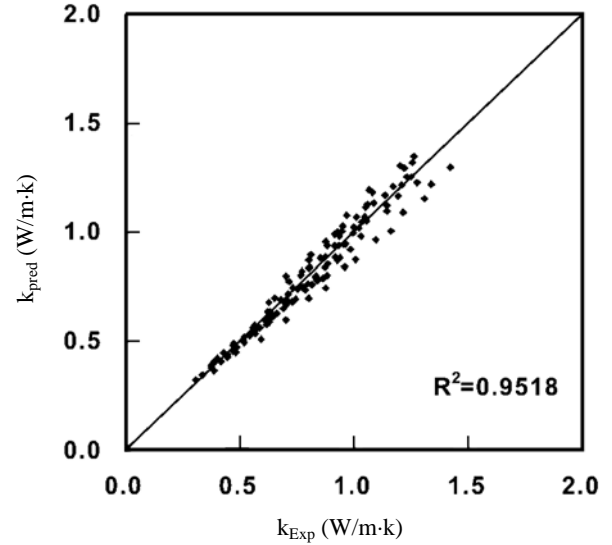
4. Differential scheme

Differential scheme for thermal conductivity in composite material was derived by Tang (1980). Equation (13) is a general equation for differential scheme.

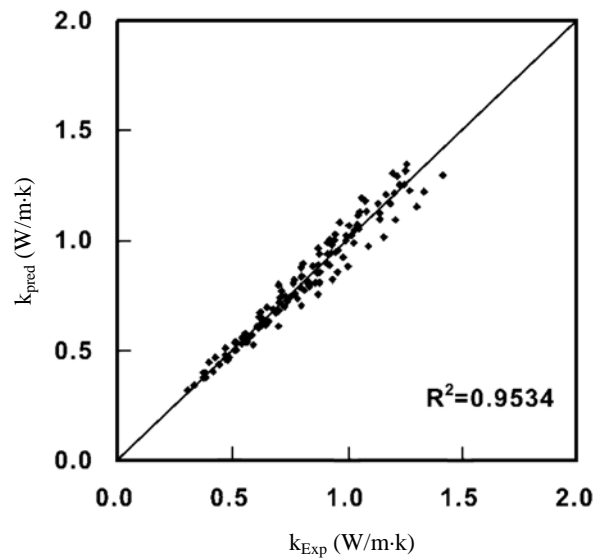
$$\left(\frac{(k_2 - k)}{(k_2 - k_1)} \right)^3 - \frac{k \times C_2^3}{k_2} = 0 \tag{13}$$

With differential scheme, the properties of each phase could not be interchanged. Therefore, k_2 in the following equation should be substituted with the higher value, which is the thermal conductivity of sand. k_1 is substituted with the predicted thermal conductivity by matrix model.

The thermal conductivity of granite is said to be 1.69 W/m.K (Touloukian 1970). Table 5 shows the thermal conductivity of sand-bentonite mixtures predicted by the matrix model with Differential Scheme/Self-Consistent Scheme. The predicted results are shown to match well with the experimental data, as shown in Fig. 12. The difference between adapting Self-Consistent Scheme and Differential Scheme is small due to the



(a)



(b)

Fig. 12 The correlation of prediction and experimental data, (a) Differential scheme, (b) Self-consistent scheme

thermal conductivity prediction of the clay from the well-defined matrix model. Thus, the proposed approach of combining these two sub-models yields a new comprehensive model for the sand-bentonite mixture, which considers dry unit weight, water content, and sand fraction simultaneously.

6. CONCLUSIONS

In this article, a method for measuring the thermal conductivity of sand-bentonite-based buffer materials with different densities, water contents, and fractions of sand (crushed granite) is established. By placing the thermal probe in the sand-bentonite powder while being compacted in a specially designed mold, in which the volume can be controlled by the position of the compacting piston, the measurement can be conducted in a single

Table 5 The thermal conductivity (measured and predicted) of sand-bentonite-based buffer materials with varied conditions

Dry unit weight of clay (kN/m ³)	Water content w-wt (%)	Thermal conductivity of pure bentonite		Crushed granite weight fraction (%)											
				12.5%			25.0%			37.5%			50.0%		
		$K_{m,e}$	$K_{m,p}$	K_e	K_D	K_S	K_e	K_D	K_S	K_e	K_D	K_S	K_e	K_D	K_S
14	0.00	0.307	0.319	0.386	0.362	0.376	0.400	0.419	0.446	0.511	0.499	0.536	0.620	0.610	0.651
	3.84	0.335	0.342	0.375	0.386	0.399	0.428	0.444	0.469	0.542	0.524	0.558	0.621	0.634	0.672
	8.17	0.384	0.399	0.482	0.445	0.456	0.592	0.505	0.525	0.628	0.585	0.611	0.742	0.691	0.721
	12.73	0.576	0.558	0.619	0.606	0.612	0.700	0.666	0.677	0.876	0.741	0.756	0.959	0.838	0.854
	16.11	0.652	0.696	0.732	0.743	0.746	0.767	0.799	0.805	0.801	0.868	0.876	0.875	0.954	0.963
	20.04	0.716	0.771	0.771	0.816	0.819	0.801	0.869	0.874	0.932	0.934	0.940	1.021	1.014	1.021
15	0.00	0.375	0.376	0.445	0.424	0.436	0.472	0.488	0.511	0.616	0.573	0.603	0.706	0.686	0.719
	3.84	0.417	0.403	0.471	0.452	0.463	0.518	0.516	0.537	0.631	0.601	0.628	0.711	0.713	0.742
	8.17	0.470	0.480	0.567	0.531	0.539	0.702	0.595	0.611	0.729	0.678	0.698	0.863	0.784	0.806
	12.73	0.679	0.688	0.752	0.738	0.742	0.832	0.798	0.805	1.006	0.871	0.880	1.094	0.961	0.971
	16.11	0.801	0.833	0.852	0.879	0.881	0.915	0.933	0.936	0.926	0.997	1.002	0.968	1.074	1.079
	20.04	0.810	0.892	0.881	0.936	0.937	0.915	0.986	0.989	1.055	1.047	1.050	1.143	1.119	1.122
16	0.00	0.445	0.437	0.519	0.491	0.501	0.557	0.559	0.578	0.690	0.648	0.672	0.797	0.761	0.787
	3.84	0.488	0.469	0.543	0.523	0.532	0.609	0.592	0.608	0.729	0.679	0.701	0.831	0.790	0.813
	8.17	0.564	0.571	0.661	0.626	0.632	0.801	0.693	0.704	0.838	0.776	0.791	0.938	0.879	0.822
	12.73	0.805	0.835	0.874	0.884	0.886	0.952	0.940	0.944	1.161	1.001	1.012	1.213	1.087	1.092
	16.11	0.935	0.977	0.998	1.019	1.020	1.046	1.068	1.070	1.057	1.124	1.127	1.065	1.190	1.192
	20.04	0.949	1.025	1.009	1.065	1.066	1.049	1.110	1.112	1.191	1.163	1.165	1.275	1.224	1.225
17	0.00	0.512	0.502	0.588	0.560	0.569	0.631	0.632	0.647	-	-	-	-	-	-
	3.84	0.559	0.539	0.615	0.598	0.605	0.706	0.669	0.682	0.815	0.757	0.774	0.926	0.867	0.884
	8.17	0.626	0.676	0.786	0.733	0.737	0.882	0.800	0.808	0.917	0.881	0.890	1.030	0.978	0.988
	12.73	0.946	1.000	1.032	1.043	1.045	1.144	1.093	1.095	1.306	1.150	1.152	1.336	1.215	1.219
	16.11	1.085	1.130	1.135	1.166	1.166	1.170	1.206	1.207	1.250	1.251	1.252	1.200	1.303	1.304
18	3.84	0.648	0.614	0.695	0.675	0.681	0.774	0.748	0.759	0.873	0.836	0.849	0.963	0.941	0.953
	8.17	0.702	0.795	0.883	0.851	0.854	0.983	0.916	0.921	0.995	0.991	0.997	-	-	-
	12.73	1.080	1.179	1.208	1.213	1.214	1.230	1.251	1.252	1.419	1.294	1.294	-	-	-
	16.11	1.219	1.290	1.255	1.316	1.316	1.261	1.345	1.345	-	-	-	-	-	-

$K_{m,e}$: experimental data of matrix; $K_{m,p}$: predicted data from matrix model.

K_e : experimental data;

K_D : predicted by matrix model with Differential Scheme;

K_S : predicted by matrix model with Self-Consistent Scheme.

sample under different compaction level. Using this method, the amount of specimens due to different densities can be reduced and the consistency of measurement results made it suitable for powdery materials in which the relationship between the thermal conductivity and the unit weight needs to be established.

A new approach for predicting the thermal conductivity of

sand-bentonite buffer material is proposed. The matrix model deals with the water content and the unit weight of pure bentonite, while the micromechanics model deals with the overall thermal conductivity. This approach yields good results, as reflected in an excellent agreement between the predictions and the experimental results.

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