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CHARACTERIZATION OF MODEL UNCERTAINTIES FOR CANTILEVER RETAINING WALLS IN SAND

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ABSTRACT

Because of our geotechnical heritage that is steeped in empirical calibrations, model uncertainties can be significant. Even a simple estimate of the average model bias is crucial for reliability-based design. If the model is conservative, it is obvious that the probabilities of failure calculated subsequently will be biased, because those design situations that belong to the safe domain will be assigned incorrectly to the failure domain, as a result of the built-in conservatism. This paper presents a critical evaluation of model factors for limit equilibrium analysis of cantilever retaining walls in sand. A total of 20 tests were collected from the literature to calibrate a theoretical model. It is important to note that there is no unique way of defining the model factor. Detailed statistical analyses showed that developing a regression equation using the theoretical embedment depth as the predictor variable can produce a significantly higher coefficient of determination, in contrast to other plausible methods. In addition, it was also shown that reasonable variations in theoretical details produce less effect on the degree of model uncertainty than variations in the definition of the model factor. This highlights the importance of choosing the definition of the model factor carefully.

A practical reliability-based design approach for cantilever walls in cohesionless soils that can consistently account for the modelling error and uncertainties associated with soil friction angle and wall friction is presented. The proposed approach is very simple to use, as it is almost identical to the US Army Corps of Engineers (1996) approach. The key difference is that a range of rigorously calibrated factors of safety is applied on the friction angle to achieve a consistent reliability index, rather than the original empirically prescribed single value of 1.5.

Key words: Limit equilibrium analysis, model uncertainties, cantilever retaining walls, reliability calibration, reliability-based design.

1. INTRODUCTION

Free embedded cantilever walls are commonly used to retain relatively low heights of cohesionless soils. Traditionally, these walls are designed based on limit equilibrium analysis (*e.g.*, US Army Corps of Engineers 1996). The wall is assumed to rotate as a rigid body about some point O in its embedded length (Fig. 1(a)). This assumption implies that a net active pressure acts from the top of the wall to point O. The earth pressure is then assumed to vary linearly from the net active pressure at point O to the net passive pressure at the base of the wall (Fig. 1(b)). By considering force and moment equilibrium, the point of rotation (z) and depth of embedment (d) can be solved. For design purposes, a conservative value of d is desired. The US Army Corps of Engineers method (1996) achieves this by dividing the tangent of the soil friction angle on the passive side by FS = 1.5. For reliability analysis, it is however necessary to compute a realistic value for d at incipient failure. If the model is conservative, it is obvious that the probabilities of failure calculated subsequently will be biased, because those design situations that belong to the safe domain will be assigned incorrectly to the failure domain, as a result of the built-in conservatism. In a study on model uncertainties of laterally loaded drilled shafts, Phoon and Kulhawy (2003) noted that reasonable predictions of fairly complex soil-structure interaction behavior still can be achieved through empirical calibrations, although many geotechnical calculation models are "simple". Because of our geotechnical heritage that is steeped in such empiricisms, model uncertainties can be significant and cannot be brushed aside. Even a simple estimate of the average model bias is crucial for reliability-based design. The development of a fully rigorous reliability-based design that can handle the entire range of geotechnical design problems is currently impeded by the scarcity in statistics on model uncertainties. With the possible exception of foundations, insufficient test data are available to perform robust statistical assessment of the model error in many geotechnical calculation models. This paper hopes to encourage more research on this critical aspect by presenting a critical evaluation of model factors for limit equilibrium analysis of cantilever retaining walls in sand.

It is incorrect to assume that a realistic depth of embedment can be obtained using FS = 1. Such a purely analytical approach is flawed because errors introduced by model idealizations are ignored. A purely empirical approach is feasible but the result cannot be extended beyond the range of the database with confidence. The most robust approach is to calibrate a sound theoretical model using test data. In this study, the limit equilibrium method outlined above with FS = 1 is calibrated using data from 20 tests to: (a) determine the optimal method for introducing model factor such that uncertainties are minimized and (b) quantify the model factor in probabilistic terms for downstream reliability-based design (RBD). To illustrate the practical usefulness of this characterization exercise, simplified RBD equations for cantilever walls in cohesionless soils are developed in the last section of this paper. Further details on reliability calibration using the proposed model factor are given elsewhere (Liu 2000).

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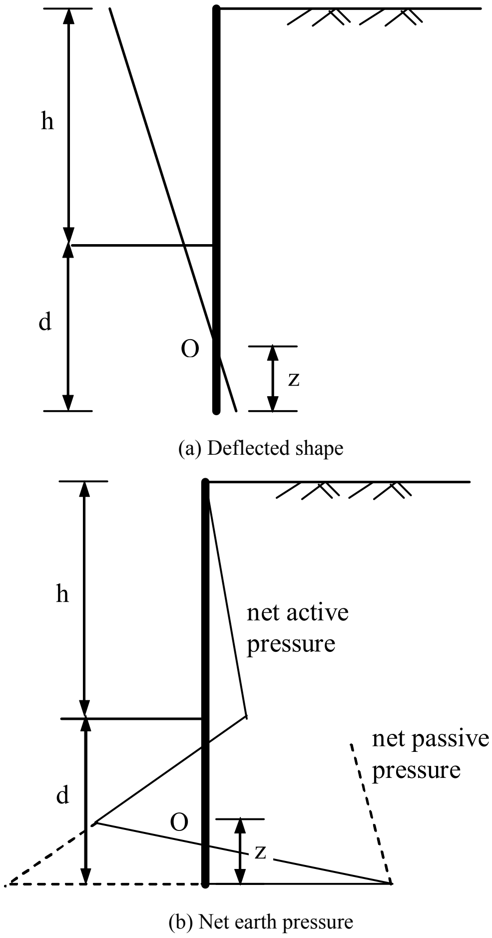


Fig. 1 Idealized limiting condition for cantilever wall

2. DATABASE

Rowe (1951) was among the first to use large-scale laboratory tests to study sheet pile walls embedded in sands. His tests showed that for this type of structure, conditions at collapse were well represented by the idealized theoretical distribution of effective stresses with the wall friction $\delta = 2/3\phi$, in which ϕ is the soil friction angle. Thereafter, several important works on model tests of cantilever retaining walls were published (*e.g.*, Bransby and Milligan 1975; Lyndon and Pearson 1984; King and McLoughlin 1992). In addition, Fourie and Potts (1989) reported their numerical analysis of this problem by finite element method

With the reference to several publications and centrifuge model tests, Bica and Clayton (1992) proposed an empirical equation for preliminary design of walls embedded in granular soil:

$$\left(\frac{d}{h}\right)_f = \frac{2}{3}\exp\left[-\frac{(\phi_{ps}-30)}{18}\right] \tag{1}$$

in which $(d/h)_f$ is the ratio at failure and ϕ_{ps} is the plane strain soil friction angle. Equation (1) provides an average fit to the experimental data. However, the uncertainty about this equation was not given. More importantly perhaps, it is difficult to extrapolate a purely empirical equation beyond its range of calibration. For example, this equation does not include the wall friction, which is known to be influential.

In this study, a total of 20 tests were collected from the literature to calibrate a theoretical model. Details of the theoretical model are given in the next section. This section focuses on the interpretation of the test data. Because of varying test geometries, the observed embedment depth at failure is normalised by the retained soil height. The ratio $(d/h)_f$ would be compared with that determined from theory. The plane strain soil friction angle (ϕ_{ps}) is more representative of the boundary conditions imposed during the tests. The triaxial compression values ϕ_{tc} given in some tests are correlated to the plane strain value using Matsuoka and Nakai (1982) equation:

$$\phi_{ps} = \frac{9}{8}\phi_{tc} \tag{2}$$

Wall friction is commonly expressed in terms of δ/ϕ. The denominator refers to a generic soil friction angle and is assumed to be independent of the test type used to evaluate ϕ. Test details are given below with emphasis on soil strength parameters, wall friction angle, and point of failure. A summary of these tests are given in Table 1.

2.1 Rowe (1951)

Five model tests of cantilever sheet pile walls embedded in both loose and dense sands were performed by Rowe (1951). The sand in loose state was obtained by pouring the material from a height of about 3-ft above the placement level. Sand in dense state was obtained by vibrating the sand during filling with a Westool-Stewart high frequency vibrator. The tests were conducted by dredging the sand from the outside in stages. In the paper, no clear information on the way shear strength parameters were obtained was given. Bica and Clayton (1989) assumed the parameters to be the triaxial compression values and correlated them to the plane strain values using Eq. (2). These plane strain values of soil friction angle are used in this study. The wall friction angle (δ) is taken as 2/3 of soil friction angle as assumed by Rowe (1951).

2.2 Bransby and Milligan (1975)

Bransby and Milligan (1975) reported the results of eight model tests on cantilever sheet pile walls with different flexibilities and roughness in sand. The smooth condition of the wall was achieved by rubbing the surfaces of the wall successively with finer grades of emery paper. For the rough state, sand grains were glued to the wall surface. The friction angle of the soil was taken to be 49° for the dense sand and 35° for the loose sand. No information was given on the wall friction angle; in this study, it is assumed to be zero for the smooth wall and to be equal to the soil friction angle for the rough wall.

In simulating the excavation procedure, all the tests were conducted by dredging, except test 9, which was conducted by backfilling. In test 9, the wall and the soil conditions were the same as that in test 6, but at failure, the value of d/h was much lower and the deflection in the wall was much larger. Bransby and Milligan (1975) suggested that the final layers of sand placed in the backfilled test might undergo very little deformation and exert a pressure closer to the 'at rest' pressure than the fully active pressure. Hence, test 9 may not be representative of failure by excavation.

Table 1 Summary of test data for cantilever retaining walls

Authors	Test No.	Wall flexibility log (ρ)	Soil friction angle (ϕ_{ps}°)	Wall friction (δ/ϕ)	$(d/h)_f$
Rowe (1951)	1	−6.2	33.8 [a]	2/3 [b]	0.538
	2	−6.2	37.2 [a]	2/3 [b]	0.449
	3	−6.2	43.9 [a]	2/3 [b]	0.324
	4	−6.2	46.1 [a]	2/3 [b]	0.286
	5	– *	50.6 [a]	2/3 [b]	0.194
Bransby and Milligan (1975)	6	−5.95	49	0 [c]	0.235
	7	−6.93	49	0 [c]	0.205
	8 [f]	−5.53	49	0 [c]	– *
	9 [f]	−5.95	49	0 [c]	– *
	10	−5.95	49	1.0 [c]	0.19
	11	−6.07	35	0 [c]	0.63 [d]
	12 [f]	−5.57	35	0 [c]	0.604 [d]
	13	−6.21	35	0 [c]	0.622 [d]
Lyndon and Pearson (1984)	14	– *	38	1.0 [c]	0.414
	15 [f]		38	1.0 [c]	0.514
Bica and Clayton (1992)	16	– *	47	0.5 [b]	0.26
	17		36	0.5 [b]	0.48 [e]
King and McLoughlin (1992)	18	−6.84	49.5	0	0.517 [e]
	19	−6.84	49.5	0.358	0.333 [e]
	20	−6.84	49.5	1	0.257 [e]
	21	−6.84	40	0	0.760 [e]
	22	−6.84	40	0.395	0.517 [e]
	23	−6.84	40	1	0.333 [e]
Shen *et al.* (1998)	24	−6.77	48.4 [a]	0.2 [c]	0.455 [e]

*. Not clearly reported
a. ϕ_{ps} determined from ϕ_{tc} using Eq. (2)
b. δ/ϕ assumed by original authors
c. δ/ϕ assumed for this study
d. d/h correspond to the points where the tests stopped.
e. d/h are the averaged values of the reported range.
f. Tests not considered for subsequent calculations.
Note: The wall flexibility number is given by $\rho = H^4/EI$, in which H and EI are height and flexural stiffness of wall, respectively (Rowe 1951). The flexibility numbers computed by Rowe (1951) were based on feet for H, lbf/in^2 for E and in^4/ft for I. In this study, the units of H, E, and I are meters, N/mm^2 and mm^4/mm, respectively (Bransby and Milligan 1975). The flexibility numbers presented in this table (ρ) are related to those from Rowe (1951) (ρ_{Rowe}) by: $\log_{10}(\rho) = \log_{10}(\rho_{\mathrm{Rowe}}) - \log_{10}(0.92 \times 10^{-3}) \approx \log_{10}(\rho_{\mathrm{Rowe}}) - 3.0$

The effect of wall flexibility was also clearly displayed in the test results. For example, the wall in test 8 was much more flexible than tests 6 and 7 with other conditions kept almost constant. There were clear failure points in tests 6 and 7, but test 8 underwent very large deflection without failure. This shows that the wall in test 8 cannot be taken as effectively rigid and is not suitable for analyzing the full plastic limit state of the soil. For the same reason, test 12 is not included in subsequent calculations. For tests 11 and 13 in loose sand, the deflection of the wall and the settlement of the sand behind the wall became so large that the tests were discontinued. The values of $(d/h)_f$ for these two tests were taken at the point where the tests were stopped.

2.3 Lyndon and Pearson (1984)

Lyndon and Pearson (1984) performed several centrifuge tests on rigid structures in cohesionless soils and two of them were reported. The prototype was a two dimensional large diameter bored secant piled cantilever retaining wall. The friction angles in plane strain tests were 38° for each test. The authors incorporated slots onto the surface of the walls to mobilize the maximum wall friction angle, which is assumed to be equal to soil friction angle in this study.

During the test, successive excavation before the wall was continued until the wall failed catastrophically. For the wall in test 14, the displacement characteristics were associated with rotation about the toe; for the wall in test 15, the principal movement was translation combined with a minor rotation. Lyndon and Pearson (1984) suggested that this might have been due to an observed initial slight backward lean of the wall under K_0 conditions. The behavior in test 15 might not be suitable for analyzing overturning of the wall and is not considered in subsequent calculations.

2.4 Bica and Clayton (1992)

A simplified experiment partially modeling the free embedded cantilever wall was reported by Bica and Clayton (1992). In the tests, only the segment of the wall situated below the excavation level was modeled. The effect of the soil above the excavation level from the active side was simulated by equivalent forces, according to the assumption of linear earth pressure distribution. Direct shear tests were used to evaluate the plane strain value of soil friction angle. It was found that ϕ_{ps} = 47° for the dense sand, and 36° for the loose sand. The wall was coated with fine sand and according to Bica and Clayton (1992), the wall friction angle was assumed to be one-half of the plane strain value of the soil friction angle.

At failure, a well-defined point was obtained for the dense sand. For the loose sand, there was no clear failure point and a range of plausible values of $(d/h)_f$ was given by the authors. The upper and lower bounds of this range correspond to the points where the horizontal displacement at the top of the wall is equal to 6% and 12% of the height of retained soil, respectively. For subsequent calculations, the average value of $(d/h)_f$ is used.

2.5 King and McLoughlin (1992)

King and McLoughlin (1992) described six centrifuge model studies of cantilever retaining wall, in both loose and dense sands. Three surface conditions of the wall were considered: a) a natural milled surface as having intermediate roughness; b) a smooth surface, achieved by coating the wall in silicone grease and placing a single latex rubber sheet over both faces and c) a rough surface, achieved by applying double sided adhesive tape to the faces of the wall and coating the outer surfaces with sand. The soil friction angle and wall friction angle were determined from plane strain and direct sliding tests respectively.

During the test, soil in front of the wall was excavated in stages by stopping and restarting the machine. It is not possible to find the precise failure point for this operation. A range of *h* was given in the paper, corresponding to the last stable excavation depth, and the removal of a further 0.5m of soil in front of the wall, which resulted in collapse. Therefore, for the excavation depth at failure, the mean values of the ranges are used.

2.6 Shen *et al*. (1998)

Shen *et al*. (1998) reported two centrifuge model tests, in which the behavior of the walls was identical. The soil used was dry Toyoura sand and the triaxial compression friction angle of soil was 43°. This is correlated to the plane strain value by Eq. (2). The wall was simulated by aluminum plate and its surface can be taken as relatively smooth. Compared with those smooth walls previously studied, which were all specially greased to ensure the smoothness, the wall friction angle in this test is assumed to be 0.2 of the plane strain value of soil friction angle.

In the test, in-flight excavation was simulated by the drainage of a heavy zinc chloride solution. When the desired height of sand bed was reached, the soil surface was leveled by means of vacuum. As excavation proceeded, the wall demonstrated a continuous deformation with an increasing deformation rate. When the excavation depth reached 5 m, the deformation rate of the retaining wall accelerated rapidly and finally collapsed as the excavation proceeded to the depth of 6 m. Thus, the failure excavation depth should be in the range of 5 ~ 6 m; the mean value of 5.5 m is taken in this study to calculate $(d/h)_f$.

3. THEORETICAL MODEL

The database shown in Table 1 would be analysed in conjunction with a theoretical model. As noted above, a purely statistical analysis of the database is not robust in the sense that the average empirical relationship so determined may not be applicable to design conditions not found in the database. From a probability point of view, it is also important to ensure that model uncertainty is random. Systematic variations in $(d/h)_f$ caused by variations in the key parameters governing the problem (*e.g.*, ϕ and δ/ϕ) are largely explainable from simple equilibrium considerations and should not be conveniently lumped under model uncertainty. This separation between systematic and random components is very important and is not easy to perform even in the presence of a theoretical model. The reason is that unexplainable (hence "random" looking) features may be explainable by more sophisticated theoretical models (both constitutive and mechanical models). Nevertheless, supporting experimental data with a sound physical basis (even a fairly simple one) would eliminate a large part of the systematic variations.

Conventionally, limit equilibrium analysis assumes that the wall rotates as a rigid body about some point O in its embedded length, as shown in Fig. 1(a). Calculation of the position of point O requires both force and moment equilibrium, which is rather complicated (Fig. 2(a)). In practice, a simpler computational procedure shown in Fig. 2(b) may be used for design. The final embedment depth is determined by increasing the computed depth d_0 by 20% (Padfield and Mair 1984). In this study, the procedure illustrated in Fig. 2(a) is used because it satisfies both force and moment equilibrium. For the calculation of theoretical values of $(d/h)_f^T$, selection of proper values of K_a and K_p should be addressed first.

3.1 Earth Pressure Coefficient

The earth pressures coefficients, K_a and K_p, are commonly determined from Rankine's theory, Coulomb's theory or tables from Caquot and Kerisel (1948). The choice of the theory used usually does not lead to major differences in the values of K_a. The active earth pressure coefficient from Coulomb's theory is adopted in this study:

$$K_a = \frac{\cos^2(\phi - \alpha)}{\cos^2\alpha \cdot \cos(\alpha + \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\cdot \sin(\phi - \theta)}{\cos(\alpha + \delta)\cdot \cos(\alpha - \theta)}}\right]^2} \tag{3}$$

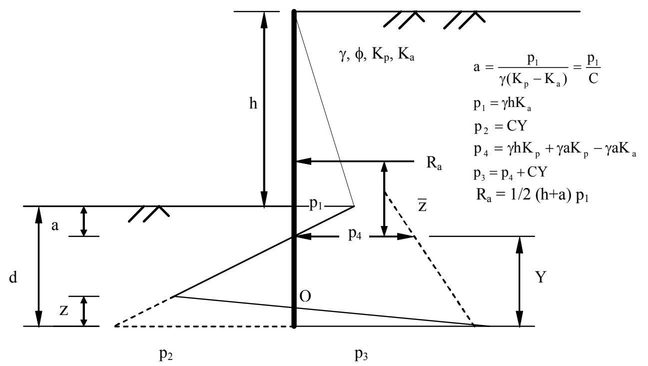
where α is the inclination of the back surface of the wall (equal to 0° in this study) and θ is the inclination of the ground surface (equal to 0° in this study).

For the passive earth pressure coefficient, the choice of theory can lead to significantly different results. To better represent the passive failure wedge, Caquot and Kerisel's tables are selected. To facilitate the analysis of the data, it is useful to derive an interpolation expression for K_p for soil and wall friction angles that are not given in the tables. Ramachandran (1988) selected a sixth order polynomial for K_p calculation for the case of $\delta/\phi = 0.5$, based on the tables. For a general value of δ/ϕ, the following sixth order polynomials are developed:

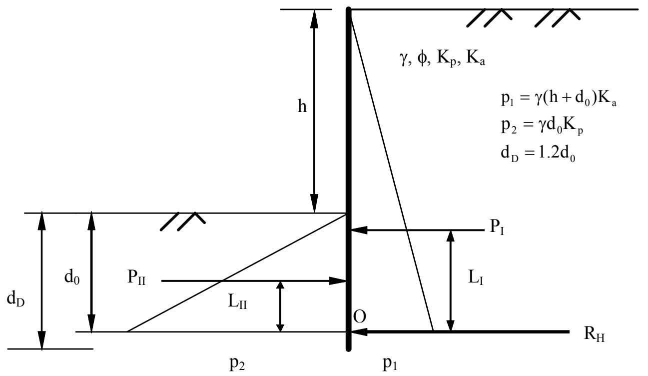
$$\ln\ (K_p) = C_0 + C_1 \tan(\phi) + C_2 \tan^2(\phi) \tag{4}$$

in which

$$C_0 = 2.6422\,(\delta/\phi)^6 - 7.6124\,(\delta/\phi)^5 + 8.3664\,(\delta/\phi)^4 - 4.4019\,(\delta/\phi)^3 + 1.1744\,(\delta/\phi)^2 - 0.214\,(\delta/\phi) - 0.0168 \tag{5a}$$



(a) Net effective earth pressure distribution



(b) Simplified earth pressure distribution

Fig. 2 Limit equilibrium analysis using: (a) net effective earth pressure distribution and (b) simplified earth pressure distribution

$$C_1 = 10.893\ (\delta/\phi)^6 - 26.36\ (\delta/\phi)^5 + 20.005\ (\delta/\phi)^4 - 3.8996\ (\delta/\phi)^3 - 1.4647\ (\delta/\phi)^2 + 1.7054\ (\delta/\phi) + 2.1346 \tag{5b}$$

$$C_2 = -22.58\ (\delta/\phi)^6 + 56.009\ (\delta/\phi)^5 - 48.714\ (\delta/\phi)^4 + 17.428\ (\delta/\phi)^3 - 2.3153\ (\delta/\phi)^2 + 1.0795\ (\delta/\phi) - 0.3544 \tag{5c}$$

Comparison with the tables from Caquot and Kerisel (1948) shows that the above equations are adequate to calculate K_p (Liu 2000).

3.2 Calculation Procedure

With the terms defined in Fig. 2(a) and K_a and K_p given by Eqs. 3 to 5, a general solution can be obtained for the case of walls embedded in dry cohesionless soils (Das 1995):

$$Y^4 + Y^3 A_1 - Y^2 A_2 - YA_3 - A_4 = 0 \tag{6}$$

in which

$$A_1 = \frac{p_4}{\gamma(K_p - K_a)} \tag{7a}$$

$$A_2 = \frac{8R_a}{\gamma(K_p - K_a)} \tag{7b}$$

$$A_3 = \frac{6R_a\left[2\bar{z}\gamma(K_p - K_a) + p_4\right]}{\gamma^2(K_p - K_a)^2} \tag{7c}$$

$$A_4 = \frac{R_a(6\bar{z}p_4 + 4R_a)}{\gamma^2(K_p - K_a)^2} \tag{7d}$$

Substituting Eq. (7) into Eq. (6), a simplified solution can be obtained for calculating $(d/h)_f^T$ [see Liu (2000) for details]:

$$\left(\frac{d}{h}\right)_f^T = \lambda + \frac{K_a}{(K_p - K_a)} \tag{8}$$

and λ can be obtained by the following equations:

$$\lambda^4 + \lambda^3 B_1 - \lambda^2 B_2 - \lambda B_3 - B_4 = 0 \tag{9}$$

in which

$$B_1 = \frac{K_a + K_p}{(K_p - K_a)} \tag{10a}$$

$$B_2 = \frac{4K_a K_p}{(K_p - K_a)^2} \tag{10b}$$

$$B_3 = \frac{5K_a^2 K_p + 5K_a K_p^2}{(K_p - K_a)^3} \tag{10c}$$

$$B_4 = \frac{K_a^3 K_p + 3K_a^2 K_p^2 + K_a K_p^3}{(K_p - K_a)^4} \tag{10d}$$

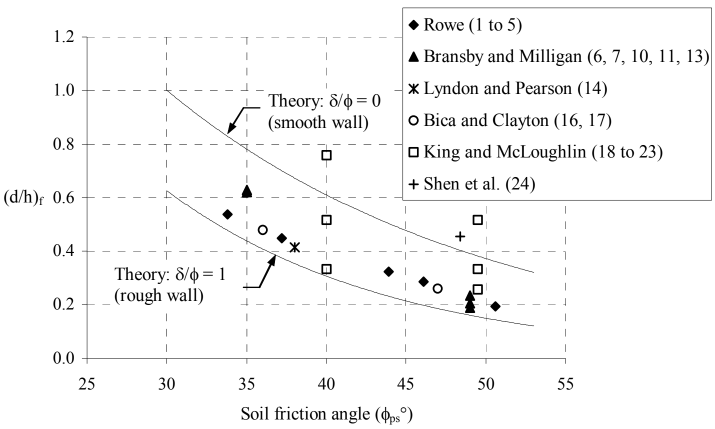
4. DEVELOPMENT OF MODEL FACTORS

An overview of the experimental data in comparison with results from the limit equilibrium method described in Section 3 is shown in Fig. 3. Most of the experimental data can be explained by theory. The remaining variations may be captured by applying a model factor to the theory. From the scatter shown in Fig. 3, it is unlikely that this model factor would be a unique number. In addition, there are many ways of defining the model factor. This section presents the optimal method for introducing the model factor such that uncertainties are minimized and quantifies the model factor in probabilistic terms for downstream reliability-based design.

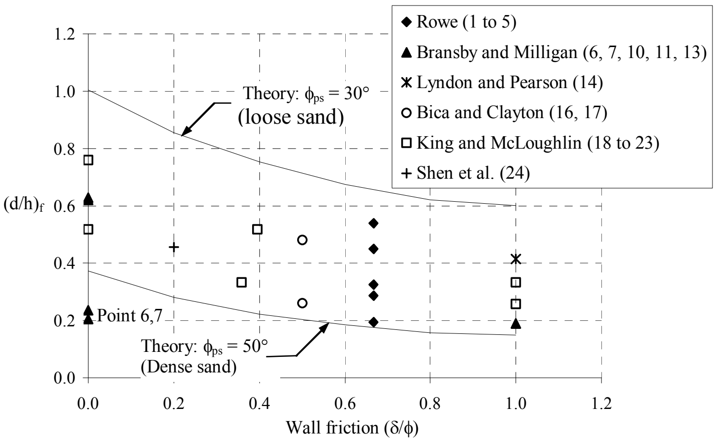
4.1 Correction on the Passive Earth Pressure Coefficient

It is commonly agreed that in the limit state, the active earth pressures are completely mobilized behind the wall, since the required deflection of the wall is very limited; on the other hand, the deflection of the wall for the full passive state is too large to achieve (Terzaghi 1936, 1941). To express the mobilized passive state for those tests, a correction factor can be applied on the theoretical passive earth pressure coefficient as follows:

$$(K_p)_m = CF_P \times K_p \tag{11}$$

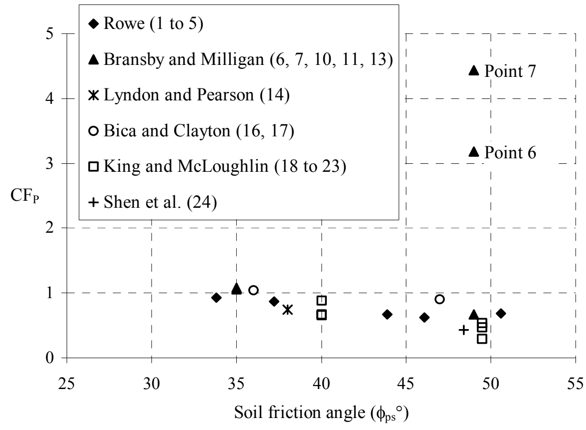


(a) Effect of soil friction angle

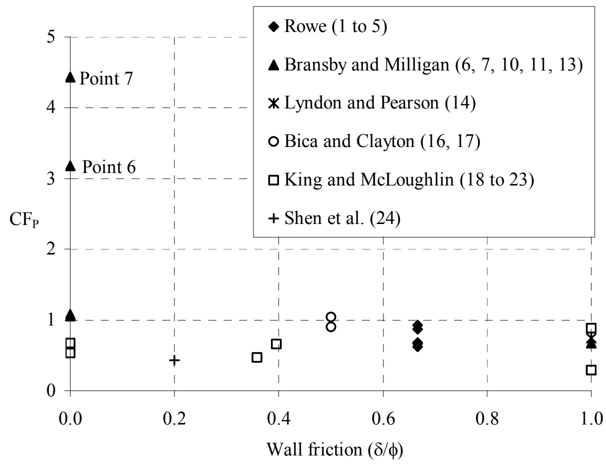


(b) Effect of wall friction

Fig. 3 Comparison of observed embedded depth at failure with limit equilibrium analyses



(a) Effect of soil friction angle



(b) Effect of wall friction

Fig. 4 Model factor defined as correction to theoretical passive earth pressure coefficient

in which $(K_p)_m$ is the mobilized passive earth pressure coefficient, $\mathrm{CF_P}$ is the correction factor, and K_p is the passive earth pressure coefficient computed from Eqs. (4) and (5) in accordance with the reported ϕ_{ps} and δ/ϕ. Note that this method of correcting K_p is also applied in design, *e.g.*, Civil Engineering Code of Practice No. 2 (1951).

The correction factors on K_p for the 20 tests are back-calculated from theory and are plotted against the parameters of ϕ_{ps} and δ/ϕ in Fig. 4. Clearly, tests 6 and 7 from Bransby and Milligan (1975) are outliers and should be excluded. Table 1 shows that the soil friction angle of these two tests are the same as that of test 10 and the values of $(d/h)_f$ for tests 6, 7, and 10 are almost the same. However, the wall surface of test 10 is rough, while that of tests 6 and 7 is smooth. Taken at face value, this appears to suggest that wall friction does not have an influence on embedment depth at failure, which is contrary to theoretical expectations. Figure 3 suggests that the assumption of $\delta/\phi = 0$ made by the authors for tests 6 and 7 based on qualitative descriptions may not be valid. It is tempting to assume $\delta/\phi \approx 0.5$ on hindsight, but this is speculative in the absence of more detailed information from the original authors.

Regression analysis by SPSS on the other 18 points shows that the influence from the parameter δ/ϕ is not significant, so it is not considered as one of the predictor variables. This is in agreement with Eq. (1) proposed by Bica and Clayton (1992). The following regression equation is obtained:

$$\mathrm{CF}_P = 1.981 - 1.673\ \phi_{ps}(\text{radians}) + \varepsilon \tag{12}$$

in which coefficient of determination, $R^2 = 0.621$ and model uncertainty is a zero-mean normal random variable (ε) with standard deviation, $s_\varepsilon = 0.143$.

4.2 Correction on the Soil Strength Parameter

It is also possible to apply a correction factor (CF_S) to the soil friction angle as follows:

$$\tan(\phi_m) = \frac{\tan(\phi_{ps})}{\mathrm{CF}_S} \tag{13}$$

in which ϕ_m is the mobilized value of the reported soil friction angle ϕ_{ps}. The mobilized wall friction angle (δ_m) is obtained by assuming that the ratio of δ_m/ϕ_m is the same as the reported δ/ϕ. This approach is similar to that discussed in the preceding section, although both active and passive earth pressures are modified in this case. In design, this strength-factored method is applied for a different reason, namely to introduce a safety factor at the main source of uncertainty (Padfield and Mair 1984).

Using the above definitions, values of CF_S for the 20 tests are calculated and their relations with ϕ_{ps} and δ/ϕ are plotted in Fig. 5. Tests 6 and 7 from Bransby and Milligan (1975) are again found to be outliers, although less distinct when viewed from Fig. 5(b). If both tests were excluded, the following regression equation is obtained:

$$CF_S = 0.493 + 0.792\phi_{ps}(\text{radians}) + \varepsilon \tag{14}$$

in which $R^2 = 0.482$ and $s_\varepsilon = 0.0892$.

4.3 Correction on $(d/h)_f^T$

In the previous sections, the correction factor is applied on one of the design parameters, such as K_p and ϕ_{ps}. Based on the coefficient of determination (R^2), it may be argued that CF_P is better than CF_S, because about 60% of the variations can be explained using Eq. (12), in contrast to about 50% from Eq. (14). This also demonstrates that the choice of definition can affect the degree of model uncertainty. Although CF_P is preferred, it is worthwhile to evaluate if there are other means of defining the model factor such that R^2 can be further improved.

One obvious alternative is to correct the theoretical embedment depth, $(d/h)_f^T$, directly. This procedure has the potential of explaining mismatches between observations and theory that arise from several factors. The previous approaches of attributing all mismatches to a single design parameter may be overly restrictive, although the parameters chosen (K_p or ϕ_{ps}) are very influential and there is a physical basis for the correction procedure. It is possible to correct the theoretical embedment depth in two different ways: (a) determine the ratio of observed $(d/h)_f$ over theoretical $(d/h)_f^T$ or (b) perform regression using $(d/h)_f^T$ as the predictor variable. In this study, the former method was found to produce a significantly lower R^2 (0.533) compared to the latter (0.809) (Liu 2000). Phoon and Kulhawy (2003) also demonstrated that the former method may produce ratios that are significantly correlated to the numerator and denominator, *i.e.,* $(d/h)_f$ and $(d/h)_f^T$, respectively. Statistically, such ratios do not form a homogeneous population and cannot be modelled as a single random variable.

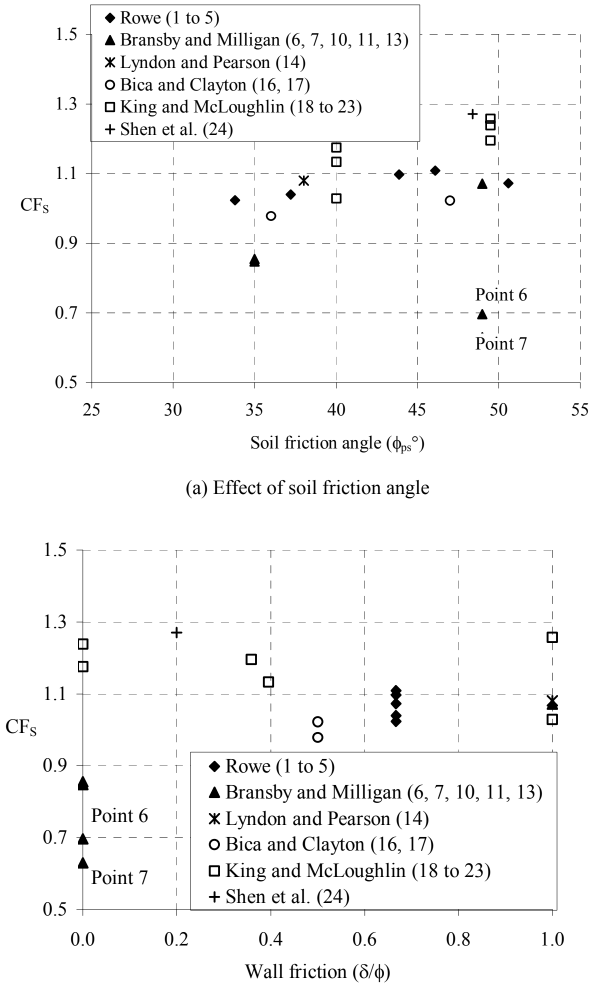
When all 20 tests are considered, the regression equation is given by:

$$\left(\frac{d}{h}\right)_f = 0.115 + 0.743\left(\frac{d}{h}\right)_f^T + \varepsilon \tag{15}$$

in which $R^2 = 0.683$ and $s_\varepsilon = 0.0947$. Deleting tests 6 and 7 has minimal effect on the estimates of the regression coefficients, but results in a noticeable increase in R^2. The recommended regression equation based on remaining 18 tests is:

$$\left(\frac{d}{h}\right)_f = 0.134 + 0.749\left(\frac{d}{h}\right)_f^T + \varepsilon \tag{16}$$

in which $R^2 = 0.809$ and $s_\varepsilon = 0.0722$. Detailed diagnostic checks show that the error term ε roughly satisfies constant variance, independence, and normality (Liu 2000). Both regression equations [Eqs. (15) and (16)] are compared with all 20 data points in Fig. 6.



(a) Effect of soil friction angle

(b) Effect of wall friction

Fig. 5 Model factor defined as correction to soil friction angle

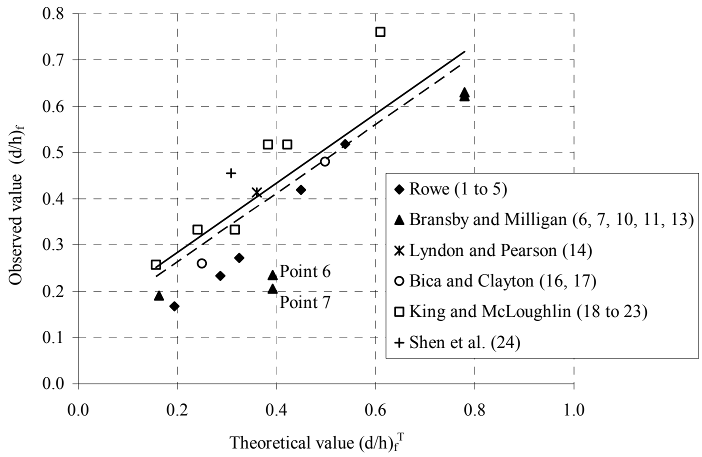


Fig. 6 Comparison between observed and theoretical embedment depth at failure [dashed line – Eq. (15), solid line – Eq. (16)]

4.4 Discussions

When Eq. (16) is applied, the corrected theoretical embedment depth decreases with increasing ϕ_{ps} and δ/ϕ as shown in Fig. 7. The proposed correction seems to produce physically sensible results. Overall, the corrected curves are less steep that the theoretical ones, indicating that the effect of ϕ_{ps} or δ/ϕ is not as influential as predicted from theory. On the average, it would also appear that theoretical solutions for smooth wall in loose sand are conservative, while the solutions for rough wall in dense sand are unconservative. Even when the model uncertainty (ε) is considered (depicted as $\pm$ one standard deviation), the above observations for these relatively extreme scenarios remain valid.

The model factors presented previously were developed using the plane strain value of soil friction angle and the net effective earth pressure distribution (Fig. 2(a)). It is interesting to see if the regression results, especially the value of R^2, are more dependent on these assumptions or the choice of the model correction procedure. To study this, the above assumptions are replaced by common alternatives, namely: (a) triaxial compression value of soil friction angle, ϕ_{tc} and (b) the simplified earth pressure distribution (Fig. 2(b)). For the calibration of each model factor, either ϕ_{tc} or ϕ_{ps} is selected for limit equilibrium analyses using the net effective or simplified earth pressure distribution. Results from these 4 scenarios are summarised in Table 2. Note that results in the upper left box were discussed in detail in Section 4. It is clear that the choice of the model correction procedure is more important than theoretical details. The net effective earth pressure distribution does not seem to furnish more precise predictions than the simplified earth pressure distribution. The application of ϕ_{tc} increases model uncertainty, unless the regression method [*e.g.*, Eq. (16)] is used to determine the model factor. Based on these observations, it would appear that the regression method is quite robust and results in relatively smaller residual uncertainty that needs to be assigned to the random model factor.

Table 2 Comparison of R^2 for different model factors calibrated under different theoretical assumptions

ϕ(°)	Model	Net effective earth pressure distribution	Simplified earth pressure distribution
ϕ_{ps}	CF_P	0.569	0.555
	CF_S	0.482	0.41
	CF_R[a]	0.533	0.496
	Regression[b]	0.809	0.811
ϕ_{tc}	CF_P	0.115	0.095
	CF_S	0.348	0.274
	CF_R[a]	0.247	0.203
	Regression[b]	0.814	0.814

a. $(d/h)_f = CF_R\,(d/h)_f^T$

b. Regression: $(d/h)_f = a + b(d/h)_f^T + \varepsilon$; a and b are regression constants; ε is error

5. SIMPLIFIED RELIABILITY-BASED DESIGN

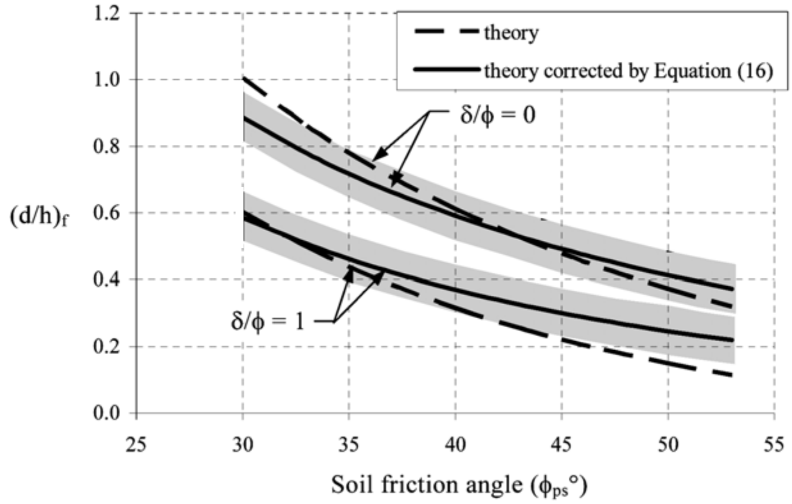
5.1 Probability Models

For the design of cantilever wall, the objective of RBD can be formally stated as:

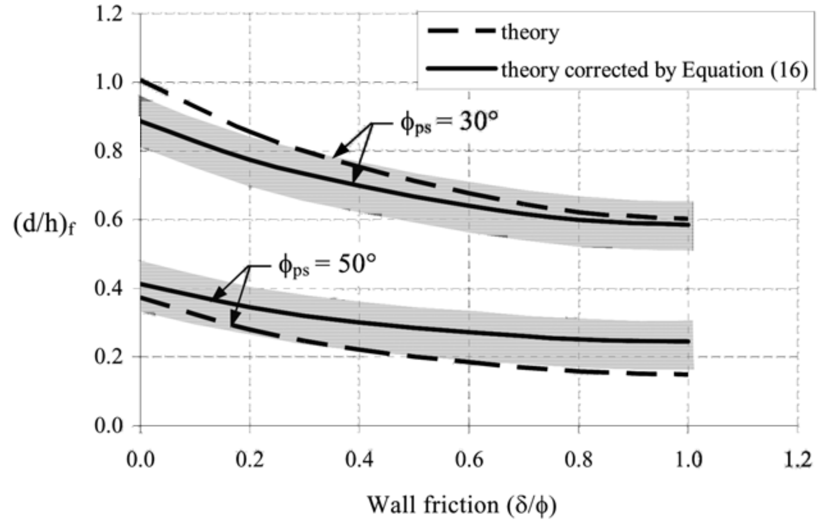
$$p_f = \Pr ob\left[\left(\frac{d}{h}\right)_D < \left(\frac{d}{h}\right)_O\right] \le p_T \tag{17}$$

in which p_f = probability of failure, $(d/h)_D$ = design embedment length, and p_T = acceptable target value. In this study, the design embedment length $(d/h)_D$ is computed based on the US Army Corps of Engineers method (1996), which entails the use of: (a) net pressure diagram (Fig. 1), (b) Coulomb's active and passive earth pressure coefficients, and (c) factored soil friction angle on the passive side = $\tan^{-1}[\tan(\phi)/\mathrm{FS}]$, in which ϕ = soil friction angle and FS = 1.5, for the usual loading case. The actual embedment length $(d/h)_O$ is uncertain because of the random model error in Eq. (16) and the uncertainties associated with the estimation of soil friction angle (ϕ) and normalised wall friction (δ/ϕ). Hence, there is a finite probability that $(d/h)_D$ is less than $(d/h)_O$ [left hand side of Eq. (17)], which needs to be controlled within an acceptable level p_T [right hand side of Eq. (17)]. It is noteworthy that the random model error has been ignored in many previous studies (*e.g.*, Smith 1985; Ramachandran 1988; Basma 1991; Valsangkar and Schriver 1991; Cherubini *et al.* 1992) although it obviously has a significant effect on the probability of failure p_f.

To evaluate the left hand side of Eq. (17), it is necessary to characterise the uncertainties in ϕ and δ/ϕ. In this study, a symmetric triangular probability density function is used for both variables (Fig. 8). This simple distribution was chosen because: (a) it has well-defined bounds that can be controlled to prevent physically unrealizable numbers from occurring (*e.g.*, ϕ, $\delta/\phi < 0$; $\delta/\phi > 1$), (b) it is one of the simplest non-uniform distribution that will realistically reflect that some numbers are more likely to occur than others, (c) it can be completely determined by the mean and standard deviation, which are readily evaluated in practice, and (d) more complicated distributions such as beta or truncated normal distribution require higher moments or more statistics, which are usually difficult to estimate reliably from limited data. Due to the lack of data on wall friction, only 3 fairly diffuse triangular distributions ranging from 0 to 2/3 (mean = 1/3), 1/6 to 5/6 (mean = 1/2) and 1/3 to 1 (mean = 2/3) are used to represent a relatively smooth, medium and rough wall condition, respectively (Fig. 8(b)). The material variables (ϕ and δ/ϕ) and the model error (ε) are assumed to be uncorrelated. Once the uncertainties have been characterised, p_f can be evaluated using standard techniques such as Monte Carlo simulation. To arrive at a reasonable choice for target probability of failure (p_T) that does not depart radically from prior experience, the p_f levels of existing US Army Corps of Engineers (1996) cantilever wall designs can be calculated over a wide range of conditions for use as a reference base for selection of p_T. A comprehensive study has been conducted (Liu 2000) and it was found that existing p_f levels vary widely from about 0.01% to 5%. If a reasonable target of $p_T \approx 0.1\%$ is selected for reliability-based design, this would imply that the existing constant FS = 1.5 is uneconomical for some designs and potentially unconservative for others. A target of $p_T \approx 0.1\%$ has also been recommended for reliability-based foundation design (Phoon *et al.* 1995). The final step is to adjust FS so that $p_T \approx 0.1\%$ is achieved over the full range of typical design scenarios.



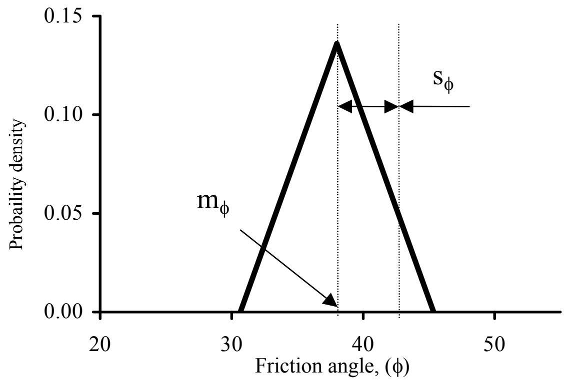
(a) Soil friction angle



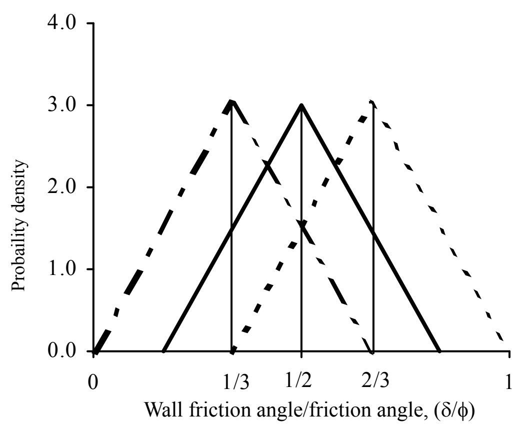
(b) Wall friction

Note: Shaded area represents ± one standard deviation about the corrected theoretical embedment depth

Fig. 7 Effect of correcting theoretical embedment depth using $(d/h)_f = 0.134 + 0.749(d/h)_f^T + \varepsilon$ [Eq. (16)] over a typical range of: (a) soil friction angle and (b) wall friction



(a) Soil friction angle (ϕ)



(b) Wall friction (δ/ϕ)

Fig. 8 Probability density functions for (a) soil friction angle (ϕ) and (b) normalised wall friction (δ/ϕ) (note: m_ϕ = mean soil friction angle, s_ϕ = standard deviation of soil friction angle)

5.2 Calibration of RBD Factors

The calibration of RBD factors can be briefly summarized as follows:

1. Perform a parametric study on the variation of p_f with respect to each deterministic and probabilistic parameter in the design problem. For example, the probabilistic parameters in the case of cantilever walls include the mean (m_ϕ) and standard deviation (s_ϕ) of the soil friction angle.
2. Partition the parameter space into several smaller domains. The reason for partitioning is to achieve greater uniformity in p_f over the full range of parameters. The parameters having a significant influence on p_f should be partitioned into smaller size than those less influential ones.
3. Select a set of representative points from each domain. Ideally, the set of representative points should capture the full range of variation in p_f over the whole domain.
4. The design embedment length $(d/h)_D$ is computed based on the US Army Corps of Engineers (1996) method using the set of parameter values associated each point and a trial value of FS. The p_f of these trial designs are then evaluated using Monte Carlo simulation.
5. Adjust the value of FS until the degree of uniformity in the p_f levels over the entire domain of interest is maximised. Repeat step (3) to step (5) for the other domains.

The above approach has been widely used for many applications (Phoon *et al*. 2000). Results of this calibration exercise are summarised in Table 3. The standard deviation of soil friction angle is taken to lie between only 1° and 3° based on an extensive study conducted elsewhere (Phoon and Kulhawy 1999). Note that the reliability-calibrated FS in Table 3 increases with the mean of soil friction angle (m_ϕ) and normalised wall friction ($m_{\delta/\phi}$). This trend is to be expected as the Coulomb's passive earth pressure coefficient prescribed by the US Army Corps of Engineers (1996) method becomes increasingly unconservative for higher values of m_ϕ and $m_{\delta/\phi}$. In addition, reliability-calibrated FS values are also sensitive to the degree of uncertainty in ϕ (represented by s_ϕ in Table 3). The FS used in another 2 common cantilever wall design methods have also been calibrated rigorously in a similar manner (Liu 2000).

Table 3 Reliability-calibrated factor of safety (FS) for US Army Corps of Engineers (1996) method

Mean value of soil friction m_ϕ (degrees)	Mean value of normalised wall friction $m_{\delta/\phi}$	Standard deviation of soil friction s_ϕ (degrees)	Reliability-calibrated factor of safety FS
35.0 - 37.5	1/3	1 - 2	1.38
		2 - 3	1.48
	1/2	1 - 2	1.43
		2 - 3	1.51
	2/3	1 - 2	1.46
		2 - 3	1.53
37.5 - 40.0	1/3	1 - 2	1.47
		2 - 3	1.56
	1/2	1 - 2	1.52
		2 - 3	1.60
	2/3	1 - 2	1.56
		2 - 3	1.63
40.0 - 42.5	1/3	1 - 2	1.57
		2 - 3	1.66
	1/2	1 - 2	1.63
		2 - 3	1.72
	2/3	1 - 2	1.68
		2 - 3	1.74
42.5 - 45.0	1/3	1 - 2	1.69
		2 - 3	1.78
	1/2	1 - 2	1.76
		2 - 3	1.84
	2/3	1 - 2	1.82
		2 - 3	1.87

Note: target probability of failure ≈ 0.1%

6. CONCLUSIONS

A total of 20 tests were collected from the literature to calibrate a theoretical model for the ultimate limit state of a cantilever wall in cohesionless soils. There is no unique way of defining the model factor. For the cantilever retaining wall, it is possible to define the model factor as a correction on the passive earth pressure coefficient, soil friction angle, or the theoretical embedment depth itself. Detailed statistical analyses showed that developing a regression equation using the theoretical embedment depth as the predictor variable can produce a significantly higher coefficient of determination, in contrast to other plausible methods. In addition, it was also shown that reasonable variations in theoretical details, such as triaxial compression versus plane strain soil friction angle or net effective versus simplified earth pressure distribution, produce less effect on the degree of model uncertainty than variations in the definition of the model factor. This highlights the importance of choosing the definition of the model factor carefully.

A purely statistical analysis of the database is not robust in the sense that the average empirical relationship so determined may not be applicable to design conditions not found in the database. From a probability point of view, it is also important to ensure that model uncertainty is random. Systematic variations in embedment depths caused by variations in the key parameters governing the problem (*e.g.*, soil friction angle and wall friction) are largely explainable from simple equilibrium considerations and should not be conveniently lumped under model uncertainty. This separation between systematic and random components is very important and is not easy to perform even in the presence of a theoretical model. The reason is that unexplainable (hence "random" looking) features may be explainable by more sophisticated theoretical models (both constitutive and mechanical models). Nevertheless, supporting experimental data with a sound physical basis (even a fairly simple one) would eliminate a large part of the systematic variations as demonstrated in this study.

A practical reliability-based design approach for cantilever walls in cohesionless soils that can consistently account for the modelling error and uncertainties associated with soil friction angle and wall friction is presented. The proposed approach is very simple to use, as it is almost identical to the US Army Corps of Engineers (1996) approach. The key difference is that a range of rigorously calibrated factors of safety is applied on the friction angle to achieve a consistent reliability index, rather than the original empirically prescribed single value of 1.5.

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