

# RELIABILITY INDEX FOR SERVICEABILITY LIMIT STATE OF DRILLED SHAFTS UNDER UNDRAINED COMPRESSION

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## ABSTRACT

In recent years, reliability-based design (RBD) has gradually gained popularity in geotechnical engineering. Several RBD codes have been developed and implemented around the world that calibrate ultimate limit state (ULS) designs for a target ULS reliability index ( $\beta_{\text{uls}}$ ). However, the serviceability limit state (SLS) design still is considered using conventional deterministic approaches with an unknown SLS reliability index ( $\beta_{\text{sls}}$ ). This paper makes use of a relationship between  $\beta_{\text{sls}}$  and  $\beta_{\text{uls}}$  to infer the  $\beta_{\text{sls}}$  of drilled shafts under undrained compression from the  $\beta_{\text{uls}}$  that is specified already in the design codes. The values of  $\beta_{\text{sls}}$  are estimated for drilled shafts designed in accordance with three different design methods (*i.e.*, semi-empirical analysis using in situ and laboratory test data, analysis using static loading test results, and analysis using dynamic monitoring results) of the National Building Code of Canada (NBCC). The results indicate that, for the undrained compression capacity of drilled shafts designed in accordance with the NBCC, the designs automatically fulfill the corresponding SLS design requirements.

**Key words:** Serviceability limit state, ultimate limit state, drilled shafts, reliability-based design, undrained compression.

## 1. INTRODUCTION

Over the last two decades, reliability-based design (RBD) methodologies gradually have gained popularity in geotechnical engineering. Several RBD codes have been developed and implemented around the world, such as the load and resistance factor design (LRFD) code adopted by the American Association of State Highway and Transportation Officials (AASHTO 1997), the Canadian National Building Code (Becker 1996), Eurocode 7 [European Committee for Standardization (CEN) 2001], and the Japanese Geo-Code 21 (Honjo and Kusakabe 2002). A review of these RBD design codes shows that, although reliability principles are applied for ultimate limit state (ULS) designs to achieve a target ULS reliability index ( $\beta_{\text{uls}}$ ), the serviceability limit state (SLS) designs still are evaluated using conventional deterministic approaches with an unknown SLS reliability index ( $\beta_{\text{sls}}$ ). One exception is the RBD study for transmission line (and similar) structure foundations that was sponsored by the Electric Power Research Institute (EPRI) in North America (Phoon *et al.* 1995; Phoon *et al.* 2003a, 2003b; Phoon and Kulhawy 2008). However, in this EPRI study of SLS design, the limiting tolerable foundation settlement ( $y_{\text{lt}}$ ) still was considered to be deterministic. Recognizing that SLS design is an indispensable aspect in various design codes (*e.g.*, AASHTO 1997; CEN 2001) underscores the need of proper estimate of the  $\beta_{\text{sls}}$  values.

This paper makes use of a relationship between  $\beta_{\text{sls}}$  and  $\beta_{\text{uls}}$  to infer the  $\beta_{\text{sls}}$  of drilled shafts under undrained compression from the  $\beta_{\text{uls}}$  that is specified already in the design codes. The probabilistic distribution of  $y_{\text{lt}}$  and the uncertainties associated with the calculation models of drilled shafts are accounted for explicitly in this paper. First, the  $\beta_{\text{sls}}$  and  $\beta_{\text{uls}}$  relationship is

briefly described. Then, key variables in this relationship are explored, including the ratio ( $R$ ) of the SLS capacity ( $Q_{\text{sls}}$ ) to the ULS capacity ( $Q_{\text{uls}}$ ), coefficient of variation of the ULS capacity ( $\text{COV}_{Q_{\text{uls}}}$ ), and coefficient of variation of the design load  $F$  ( $\text{COV}_F$ ). Capacity herein refers to the maximum soil resistance mobilized when a foundation is loaded to reach either the ultimate limit state ( $Q_{\text{uls}}$ ) or serviceability limit state ( $Q_{\text{sls}}$ ). Then, the relationship is used to estimate  $\beta_{\text{sls}}$  for drilled shafts under undrained compression that are designed in accordance with the National Building Code of Canada (NBCC), as described by Becker (1996). Finally, the effects of  $y_{\text{lt}}$  on  $\beta_{\text{sls}}$  are discussed.

## 2. RELATIONSHIP BETWEEN $\beta_{\text{uls}}$ AND $\beta_{\text{sls}}$

In RBD, design quantities, such as the load ( $F$ ) and the capacity ( $Q$ ), are commonly modeled as lognormal random variables (Ang and Tang 1975). The basic reliability problem is to evaluate the probability of failure ( $p_f$ ) or  $\beta$  from some pertinent probabilistic characterizations of  $F$  and  $Q$ , which frequently include the mean ( $m_F$  and  $m_Q$ ), standard deviation ( $s_F$  and  $s_Q$ ), coefficient of variation ( $\text{COV}_F$  and  $\text{COV}_Q$ ), and even probability density functions.

For lognormally distributed  $F$  and ULS capacity  $Q_{\text{uls}}$ , the  $\beta_{\text{uls}}$  can be expressed as (*e.g.*, Barker *et al.* 1991; Becker 1996; Phoon *et al.* 2003a, 2003b):

$$\beta_{\text{uls}} = \Phi^{-1}(1 - p_{\text{fuls}}) = \frac{\ln \left[ \frac{m_{Q_{\text{uls}}}}{m_F} \sqrt{\frac{1 + \text{COV}_F^2}{1 + \text{COV}_{Q_{\text{uls}}}^2}} \right]}{\sqrt{\ln \left[ (1 + \text{COV}_{Q_{\text{uls}}}^2)(1 + \text{COV}_F^2) \right]}} \quad (1)$$

in which  $\Phi^{-1}$  = inverse standard normal probability distribution function,  $p_{\text{fuls}}$  = the probability of failure at ultimate limit state,  $m_{Q_{\text{uls}}}$  = mean of ULS capacity  $Q_{\text{uls}}$ , and  $\text{COV}_{Q_{\text{uls}}}$  = coefficient of variation of ULS capacity  $Q_{\text{uls}}$ . If the ratio of SLS capacity ( $Q_{\text{sls}}$ )

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to  $Q_{uls}$  is defined as  $R$ , the  $\beta_{sls}$  can be directly related to  $\beta_{uls}$  as (Wang and Kulhawy 2008a, b) :

$$\beta_{sls} = C_0 + C_1 \beta_{uls} \tag{2}$$

in which  $C_0$  and  $C_1$  = intercept and slope of this linear function, respectively, given by:

$$C_0 = \frac{\ln \left[ \frac{m_R}{\sqrt{1 + COV_R^2}} \right]}{\sqrt{\ln \left[ (1 + COV_R^2)(1 + COV_{Q_{uls}}^2)(1 + COV_F^2) \right]}} \tag{3}$$

$$C_1 = \sqrt{\frac{\ln \left[ (1 + COV_{Q_{uls}}^2)(1 + COV_F^2) \right]}{\ln \left[ (1 + COV_R^2)(1 + COV_{Q_{uls}}^2)(1 + COV_F^2) \right]}} \tag{4}$$

where  $m_R$  = mean of  $R$  and  $COV_R$  = coefficient of variation of  $R$ . Derivations of Eqs. (2) ~ (4) are referred to Wang and Kulhawy (2008b).

### 3. KEY VARIABLES IN THE $\beta_{uls}$ AND $\beta_{sls}$ RELATIONSHIP

The  $\beta_{sls}$  and  $\beta_{uls}$  relationship aforementioned shows that  $\beta_{sls}$  is a function of  $COV_F$ ,  $COV_{Q_{uls}}$ ,  $\beta_{uls}$ ,  $m_R$ , and  $COV_R$ . Each of them is discussed separately in this section. Table 1 summarizes the typical ranges of  $COV_F$  of various load effects for foundations. The  $COV_F$  of dead loads, live loads, and environmental loads lies in the ranges of 0.05-0.15, 0.2-0.6, and 0.3-0.5, respectively (Meyerhof 1993, 1995).

The values of  $COV_{Q_{uls}}$  depend on the design method used to calculate the  $Q_{uls}$ . Table 2 shows the values of  $COV_{Q_{uls}}$  adopted in NBCC (Becker 1996). When the  $Q_{uls}$  is calculated by semi-empirical analysis using in situ and laboratory test data, the value of  $COV_{Q_{uls}}$  is 0.40. In contrast, when the  $Q_{uls}$  is calculated by analyses using static loading test results or dynamic monitoring results, the values of  $COV_{Q_{uls}}$  are 0.25 and 0.30, respectively.

The values of  $\beta_{uls}$  are specified already in RBD codes, and Eq. (1) has been used as the basis in the RBD codes to achieve the target  $\beta_{uls}$  (e.g., Barker *et al.* 1991; Becker 1996; Phoon *et al.* 2003a; 2003b). Using a set of calibrated resistance factors ( $\psi$ ) which are reduction factors applied to the calculated resistance to account for its uncertainty, the RBD codes ensure that all ULS designs have a nominally consistent  $p_{fuls}$  or  $\beta_{uls}$ . Consider, for example, the National Building Code of Canada (Becker 1996, see Table 2), in which the proposed resistance factors are 0.4, 0.6, and 0.5 for three different design methods to achieve their respective target  $\beta_{uls}$  values of 3.4, 3.2, and 3.5. For reference, Table 3 correlates reliability indices for representative geotechnical components and systems and their corresponding probabilities of failure and expected performance levels. The reliability indices range from 1 to 5, corresponding to probabilities of failure varying from about 0.16 to  $3 \times 10^{-7}$ . A  $\beta_{uls}$  value larger than 3 is commonly adopted for ultimate limit state in RBD, and it corresponds to an expected performance level better than ‘‘above average’’.

A key variable in the relationship between  $\beta_{uls}$  and  $\beta_{sls}$  is  $R = Q_{sls}/Q_{uls}$  and its mean ( $m_R$ ) and coefficient of variation ( $COV_R$ ).

**Table 1 Coefficient of variation of load effects for foundations (after Meyerhof 1993 and 1995)**

Load Type	Coefficient of variation, $COV_F$
Dead loads	0.05 ~ 0.15
Live loads	0.2 ~ 0.6
Environmental loads	0.3 ~ 0.5

**Table 2 Summary of resistance factors for pile foundation in National Building Code of Canada (NBCC, after Becker 1996)**

Design method	Resistance factor, $\psi$	ULS reliability index, $\beta_{uls}$	Coefficient of variation of $Q_{uls}$ , $COV_{Q_{uls}}$
Semi-empirical analysis using in situ and laboratory test data	0.4	3.4	0.40
Analysis using static loading test results	0.6	3.2	0.25
Analysis using dynamic monitoring results	0.5	3.5	0.30

**Table 3 Relationship between reliability index ( $\beta$ ) and probability of failure ( $p_f$ ) (U.S. Army Corps of Engineers 1997, p. B-11)**

Reliability index $\beta$	Probability of failure $p_f = \Phi(-\beta)$	Expected performance level
1.0	0.16	Hazardous
1.5	0.07	Unsatisfactory
2.0	0.023	Poor
2.5	0.006	Below average
3.0	0.001	Above average
4.0	0.00003	Good
5.0	0.0000003	High

Note:  $\Phi(\ )$  = standard normal probability distribution function

Probabilistic characterization of  $R$  requires a load-displacement model that relates the foundation displacement to load capacity and probabilistic characterization of the limiting tolerable foundation settlement ( $y_{lt}$ ). A probabilistic load-displacement model for drilled shafts under undrained compression is described in next section, followed by probabilistic characterization of  $y_{lt}$  and closed-form approximations of  $m_R$  and  $COV_R$  in two respective sections.

### 4. LOAD-DISPLACEMENT MODEL FOR DRILLED SHAFTS UNDER UNDRAINED COMPRESSION

Accurate prediction of foundation movements is a difficult task, and most analytical attempts have met with only limited

success, primarily because they can not include all the important factors, such as the in-situ stress state, soil behavior, soil-foundation interface characteristics, and construction effects (Kulhawy 1994; Becker 1996, and Phoon *et al.* 2003b). Alternatively, an empirical approach has been employed that utilizes load test results and normalizes the test load-displacement curves to obtain a single representative design curve (Phoon *et al.* 1995; Phoon *et al.* 2003b). For drilled shafts under undrained compression, the load-displacement curves can be represented reasonably well by the following hyperbolic model (Phoon *et al.* 1995; Phoon *et al.* 2003b):

$$\frac{Q}{Q_{uls}} = \frac{y/B}{a + b (y/B)} \quad (5)$$

in which  $Q$  = compression load,  $Q_{uls}$  = ULS capacity,  $y$  = axial butt displacement,  $B$  = drilled shaft diameter, and  $a$  and  $b$  = hyperbolic model parameters.

Phoon *et al.* (1995) compiled a database that includes load tests of 27 drilled shafts with pile diameter  $B$  varying from 0.18 m to 1.3 m. The drilled shafts were installed in clay and tested under axial compression. Figure 1 shows a scatter plot of the hyperbolic model parameters  $a$  and  $b$ . It is found that  $a$  and  $b$  are virtually uncorrelated and have the following statistics: Mean ( $m_a = 0.0040$  and  $m_b = 0.7798$ ), standard deviation ( $s_a = 0.0024$  and  $s_b = 0.1492$ ), and coefficient of correlation ( $\rho_{a,b} = -0.05$ ). For simplification herein, the value of  $\rho_{a,b}$  is taken as 0.

## 5. LIMITING TOLERABLE SETTLEMENTS FOR FOUNDATIONS

The limiting tolerable foundation settlement ( $y_{lt}$ ) is the maximum settlement that a foundation can sustain before causing any serviceability failure, and it corresponds to the SLS capacity  $Q_{sls}$ , expressed as:

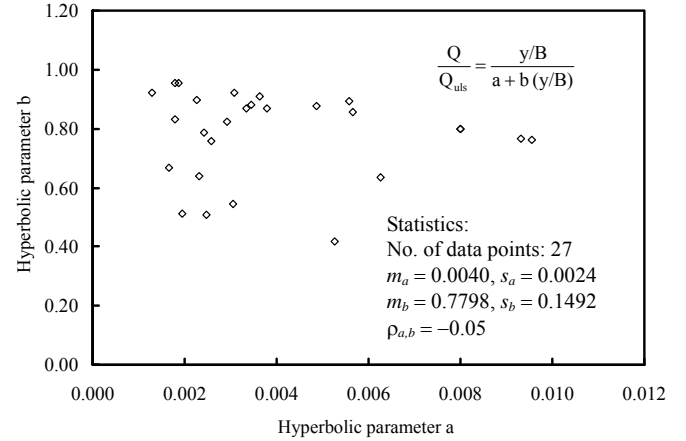
$$R = \frac{Q_{sls}}{Q_{uls}} = \frac{y_{lt}/B}{a + b (y_{lt}/B)} \quad (6)$$

The  $y_{lt}$  for foundations has been examined by many researchers (*e.g.*, Skempton and MacDonald 1956; Lumb 1964; Grant *et al.* 1974; Wahls 1981, 1994; Zhang and Ng 2005), and deterministic  $y_{lt}$  values have been proposed and adopted in design codes around the world. Zhang and Ng (2005) synthesized the  $y_{lt}$  values for pile foundations supporting buildings or bridges and used fragility curves to represent the cumulative probability distribution of  $y_{lt}$ . The  $y_{lt}$  statistics, including the mean ( $m_{y_{lt}}$ ), standard deviation ( $s_{y_{lt}}$ ) and coefficient of variation ( $COV_{y_{lt}}$ ), were then obtained from the cumulative probability distribution of  $y_{lt}$ . Detailed development of these statistics is referred to Zhang and Ng (2005).

Table 4 summarizes the  $y_{lt}$  statistics, including the mean ( $m_{y_{lt}}$ ), standard deviation ( $s_{y_{lt}}$ ) and coefficient of variation ( $COV_{y_{lt}}$ ). For pile foundations supporting buildings,  $m_{y_{lt}} = 96$  mm,  $s_{y_{lt}} = 56$  mm, and  $COV_{y_{lt}} = 0.583$ . In contrast, for pile foundations supporting bridges,  $m_{y_{lt}} = 135$  mm,  $s_{y_{lt}} = 89$  mm, and  $COV_{y_{lt}} = 0.659$ . Note that, as the structure types are different for buildings and bridges, their respective  $m_{y_{lt}}$  are also different considerably. Nonetheless, these  $y_{lt}$  statistics are significantly larger than the

**Table 4 Statistics of limiting tolerable settlement ( $y_{lt}$ ) for pile foundations (after Zhang and Ng 2005)**

Statistics	Supporting Buildings	Supporting Bridges
Mean, $m_{y_{lt}}$ (mm)	96	135
Standard Deviation, $s_{y_{lt}}$ (mm)	56	89
Coefficient of Variation, $COV_{y_{lt}}$	0.583	0.659



**Fig. 1 Scatter plot of hyperbolic parameters  $a$  and  $b$**

allowable settlement limit of 25 mm that is used frequently in deterministic SLS designs of most foundation types (*e.g.*, Peck *et al.* 1974; Wahls 1994). The  $y_{lt}$  statistics summarized in Table 4 are therefore used as a starting point in this work. The effect of  $y_{lt}$  on  $\beta_{sls}$  is discussed later.

## 6. CLOSED-FORM APPROXIMATIONS OF $m_R$ AND $COV_R$

With the probabilistic load-displacement model for drilled shafts and mean and standard deviation of  $y_{lt}$ , the mean ( $m_R$ ) and standard deviation ( $s_R$ ) of  $R$  can be approximated by a Taylor series expansion as follows:

$$m_R \approx \frac{m_{y_{lt}}/B}{m_a + m_b (m_{y_{lt}}/B)} \quad (7)$$

$$s_R^2 \approx \left( \frac{\partial R}{\partial a} \right)^2 s_a^2 + \left( \frac{\partial R}{\partial b} \right)^2 s_b^2 + \left( \frac{\partial R}{\partial (y_{lt}/B)} \right)^2 s_{(y_{lt}/B)}^2 \\ = \frac{m_{(y_{lt}/B)}^2 s_a^2 + m_{(y_{lt}/B)}^4 s_b^2 + m_a^2 s_{(y_{lt}/B)}^2}{(m_a + m_b m_{(y_{lt}/B)})^4} \quad (8)$$

Table 5 summarizes statistics of  $R$  for drilled shafts supporting both buildings and bridges. The  $R$  statistics are calculated

**Table 5** Statistics of  $R = Q_{sls}/Q_{uls}$

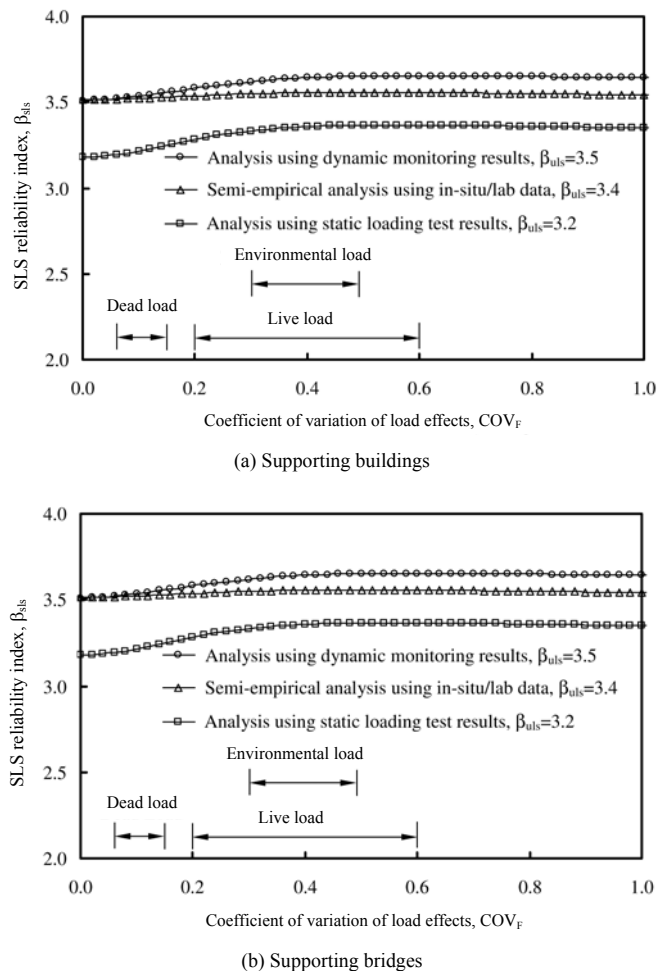
Type of Supporting structure		Mean, $m_R$	Standard deviation, $s_R$	Coefficient of variation, $COV_R$
Buildings	B = 0.18 m	1.27	0.24	0.19
	B = 1.30 m	1.20	0.22	0.19
	Average	<b>1.24</b>	<b>0.23</b>	<b>0.19</b>
Bridges	B = 0.18 m	1.27	0.24	0.19
	B = 1.30 m	1.22	0.23	0.19
	Average	<b>1.25</b>	<b>0.24</b>	<b>0.19</b>

for the values of shaft diameter  $B$  varying from 0.18 m to 1.30 m which are the respective minimum and maximum shaft diameters in the database compiled by Phoon *et al.* (1995) for the development of the hyperbolic model defined in the previous section. The  $R$  statistics vary slightly for different types of supporting structure and different  $B$  values. For drilled shafts supporting buildings, the value of  $m_R$  changes from 1.27 for  $B = 0.18$  m to 1.20 for  $B = 1.30$  m. The value of  $COV_R$  remains virtually constant at 0.19. For drilled shafts supporting bridges, the value of  $m_R$  changes from 1.27 to 1.22 with a  $COV_R$  value of 0.19, which equals to the  $COV_R$  for drilled shafts supporting buildings. The values of  $m_R$  and  $COV_R$  are insensitive to the variation of shaft diameters and types of supporting structures. Consequently, the average of  $m_R$  (*i.e.*, 1.24 and 1.25) and  $COV_R$  (*i.e.*, 0.19 and 0.19), as highlighted by the bold fonts in Table 5, are used in this work to estimate the values of  $\beta_{sls}$  for drilled shafts supporting buildings and bridges, respectively.

**7. SLS RELIABILITY INDEX ESTIMATED FOR NBCC**

Becker (1996) described the development of RBD methodologies for the NBCC and summarized the calibration process, corresponding ULS reliability index ( $\beta_{uls}$ ), and proposed resistance factors ( $\psi$ ), as shown in Table 2. The values of  $\psi$  are calibrated for three different design methods (*i.e.*, semi-empirical analysis using in situ and laboratory test data, analysis using static loading test results, and analysis using dynamic monitoring results) and their associated values of  $\beta_{uls}$  and  $COV_{Q_{uls}}$ . For example, the proposed resistance factor  $\psi = 0.6$  for the axial compression capacity of a pile foundation, when interpreted from static loading tests. The corresponding  $\beta_{uls} = 3.2$ , and the coefficient of variation of  $Q_{uls}$  ( $COV_{Q_{uls}} = 0.25$ ).

Using Eqs. (2) ~ (4), the SLS reliability index ( $\beta_{sls}$ ) can be estimated directly for NBCC. Figure 2 shows the estimated  $\beta_{sls}$  as a function of the coefficient of variation for load effects ( $COV_F$ ) for drilled shafts under undrained compression and supporting either buildings (Fig. 2a) or bridges (Fig. 2a). As  $COV_F$  increases from 0 to 1.0,  $\beta_{sls}$  varies slightly and it is generally larger than 3,



**Fig. 2** SLS reliability index  $\beta_{sls}$  estimated for NBCC

corresponding to an expected performance level of “above average” (see Table 3). As summarized in Table 1 and illustrated in Fig. 2, the  $COV_F$  of various load effects for foundations lies in the range of 0.05 to 0.6 (Meyerhof 1993, 1995).

For three different design methods (*i.e.*, semi-empirical analysis using in situ and laboratory test data, analysis using static loading test results, and analysis using dynamic monitoring results), the estimated  $\beta_{sls}$  is larger than their respective  $\beta_{uls}$  values of 3.4, 3.2, and 3.5 that are specified in the NBCC. This result indicates that, for the undrained compression capacity of drilled shafts designed in accordance with the NBCC, the designs automatically satisfy the SLS design requirements and have a  $\beta_{sls}$  larger than the  $\beta_{uls}$ . This result can be attributed to the probabilistic characterization of  $R$ , which shows that  $R$  is larger than 1 (see Table 5) and therefore the SLS capacity  $Q_{sls}$  is larger than  $Q_{uls}$ . Since  $Q_{sls}$  is larger than  $Q_{uls}$  and the probability of  $Q_{uls} < F$  is  $\Phi(-\beta_{uls})$ , the  $\Phi(-\beta_{sls})$ , or the probability of  $Q_{sls} < F$ , is smaller than  $\Phi(-\beta_{uls})$ . Therefore,  $\beta_{sls}$  is larger than  $\beta_{uls}$ .

It should be pointed out that the values of  $\beta_{sls}$  reported in this paper are calculated using typical values of  $COV_{Q_{uls}}$ ,  $COV_F$ ,  $COV_R$  and  $m_R$ . They therefore are general estimates of the typical range of  $\beta_{sls}$ , which might not be necessarily valid for the  $\beta_{sls}$  value of a specific foundation design, particularly when the values of  $COV_{Q_{uls}}$ ,  $COV_F$ ,  $COV_R$  or  $m_R$  of the specific foundation design deviate significantly from their typical values used herein.

### 8. EFFECT OF LIMITING TOLERABLE SETTLEMENTS

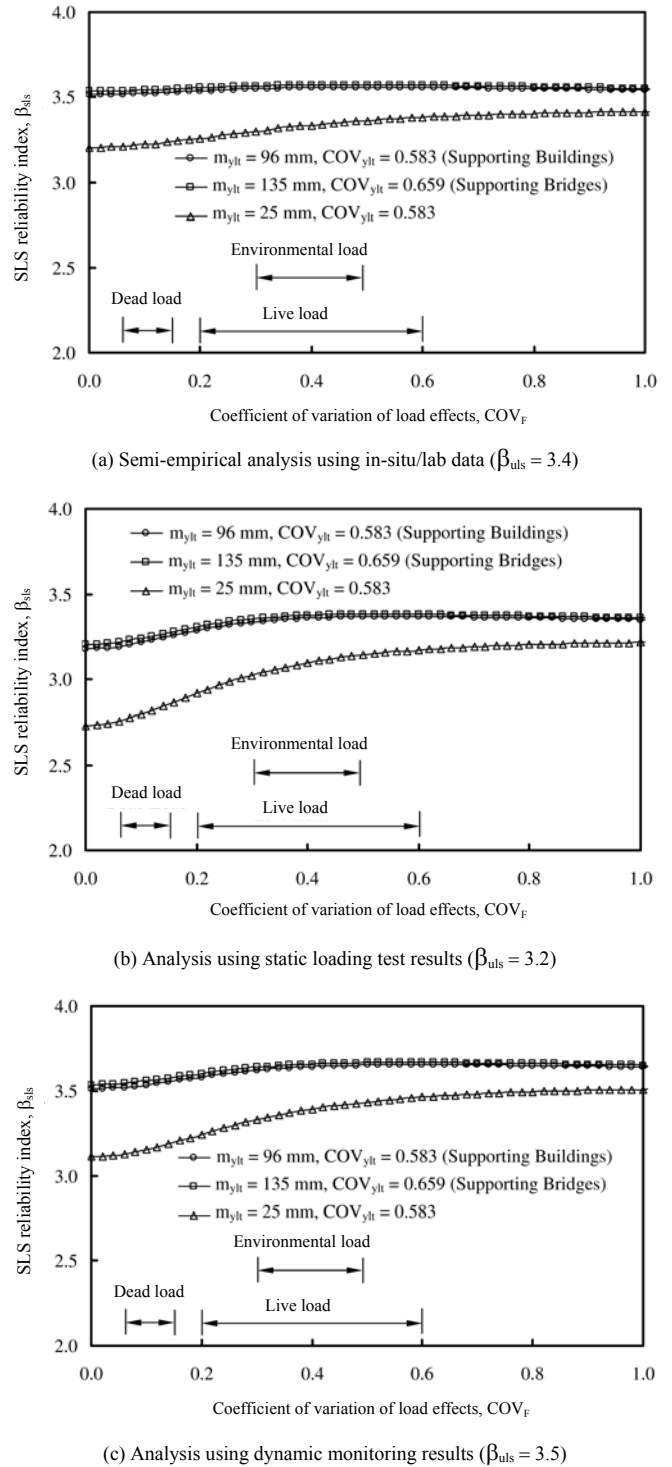
The  $y_{lt}$  statistics (*i.e.*,  $m_{y_{lt}} = 96$  mm and  $s_{y_{lt}} = 56$  mm for piles supporting buildings and  $m_{y_{lt}} = 135$  mm and  $s_{y_{lt}} = 89$  mm for piles supporting bridges) reported by Zhang and Ng (2005) and used herein are significantly larger than the displacement limit of 25 mm that is used frequently in deterministic SLS designs (*e.g.*, Peck *et al.* 1974; Wahls 1994). To explore the effect of  $y_{lt}$ , the values of  $\beta_{sls}$  are estimated using a  $m_{y_{lt}} = 25$  mm in combination with the  $COV_{y_{lt}} = 0.583$  reported by Zhang and Ng (2005) for piles supporting buildings. For  $m_{y_{lt}} = 25$  mm,  $s_{y_{lt}} = m_{y_{lt}} (COV_{y_{lt}}) = 25 (0.583) = 15$  mm. Using Eqs. (7) and (8), the values of  $m_R$  and  $COV_R$  are taken as 1.124 and 0.210, respectively. Then,  $\beta_{sls}$  is estimated using these  $m_R$  and  $COV_R$  values.

Figure 3 shows the estimated  $\beta_{sls}$  as a function of the  $COV_F$  for  $m_{y_{lt}} = 25$  mm for three different design methods. As  $COV_F$  increases from 0 to 1.0, the values of  $\beta_{sls}$  increase gradually for three different design methods as shown in Figs. 3a, b, and c, respectively. For example, when the drilled shafts are designed by semi-empirical analysis using in-situ/lab data, the value of  $\beta_{sls}$  increases from 3.20 to 3.41 (see Fig. 3a). Although these values of  $\beta_{sls}$  are slightly smaller than  $\beta_{uls} = 3.4$ , they correspond to an expected performance level that is among “above average” (see Table 3). In addition, they are larger than the target  $\beta_{sls} = 2.6$  that is given in the EPRI study (Phoon *et al.* 1995; Phoon *et al.* 2003a, 2003b). The values of  $\beta_{sls}$  similar to those for designs by semi-empirical analysis are shown in Figs. 3b and c for designs based on static or dynamic test results. Therefore, even with  $m_{y_{lt}} = 25$  mm and  $s_{y_{lt}} = 15$  mm, the NBCC designs of drilled shafts under undrained compression still are acceptable for the SLS design requirements.

Figure 3 also includes the  $\beta_{sls}$  versus  $COV_F$  relationships for drilled shafts supporting buildings ( $m_{y_{lt}} = 96$  mm) and bridges ( $m_{y_{lt}} = 135$  mm) from Fig. 2. The  $\beta_{sls}$  versus  $COV_F$  relationships for  $m_{y_{lt}} = 96$  mm and  $m_{y_{lt}} = 135$  mm are virtually identical. The hyperbolic load-displacement model defined by Eq. (6) shows that, when  $y_{lt}/B$  is relatively large, the value of  $R$  approaches the asymptotic maximum of  $1/b = 1.28$ . For  $m_{y_{lt}} = 96$  mm and  $m_{y_{lt}} = 135$  mm, the respective means of  $R$  value are estimated as 1.24 and 1.25, and they approach the maximum value of 1.28. Therefore, even the  $m_{y_{lt}} = 135$  mm for piles supporting bridges is significantly larger than the  $m_{y_{lt}} = 96$  mm for piles supporting buildings, only inconsiderable difference (*i.e.*,  $1.25 - 1.24 = 0.01$ ) is observed for the values of  $m_R$ . On the other hand, when  $m_{y_{lt}}$  decreases from 96 mm to 25 mm,  $\beta_{sls}$  decreases significantly, particularly for relatively small  $COV_F$ . The value of  $\beta_{sls}$  may decrease to 2.7. This figure shows clearly that the limiting tolerable foundation settlement ( $y_{lt}$ ) must be defined rationally and thoughtfully.

### 9. SUMMARY AND CONCLUSIONS

This paper dealt with the reliability index of serviceability limit state of drilled shafts under undrained compression. It made use of a relationship between  $\beta_{sls}$  and  $\beta_{uls}$  to infer the  $\beta_{sls}$  of drilled shafts under undrained compression from the  $\beta_{uls}$  that is specified already in the design codes. The probabilistic distribution of  $y_{lt}$  and the uncertainties associated with the hyperbolic



**Fig. 3** Estimated SLS Reliability Index  $\beta_{sls}$  for Varying  $m_{y_{lt}}$  and  $COV_{y_{lt}}$

load-displacement model of drilled shafts were accounted for explicitly in this paper. Key variables in this relationship were explored, including the ratio ( $R$ ) of the SLS capacity ( $Q_{sls}$ ) to the ULS capacity ( $Q_{uls}$ ), coefficient of variation of the ULS capacity ( $COV_{Q_{uls}}$ ), and coefficient of variation of the load effect  $F$  ( $COV_F$ ).

The values of  $\beta_{sls}$  were estimated for drilled shafts designed in accordance with the NBCC. For assumed input variables consistent with recent data summaries for piles supporting buildings

( $m_{y_{lt}} = 96$  mm and  $COV_{y_{lt}} = 0.583$ ) or bridges ( $m_{y_{lt}} = 135$  mm and  $COV_{y_{lt}} = 0.659$ ), it was found that, for  $COV_F$  ranging from 0 to 1.0,  $\beta_{s_{ls}}$  varies slightly and it is generally larger than 3, corresponding to an expected performance level of “above average”. For three different design methods (*i.e.*, semi-empirical analysis using in situ and laboratory test data, analysis using static loading test results, and analysis using dynamic monitoring results) in NBCC, the estimated  $\beta_{s_{ls}}$  is larger than their respective  $\beta_{u_{ls}}$  values of 3.4, 3.2, and 3.5 that are specified in the NBCC. This result indicates that, for the undrained compression capacity of drilled shafts designed in accordance with the NBCC, the designs automatically satisfy the SLS design requirements and have a  $\beta_{s_{ls}}$  larger than the  $\beta_{u_{ls}}$ . This result can be attributed to the input variables and the probabilistic characterization of  $R$ , which shows that  $R$  is larger than 1, and therefore  $Q_{s_{ls}}$  is larger than  $Q_{u_{ls}}$  and  $\beta_{s_{ls}}$  is larger than  $\beta_{u_{ls}}$ .

The effect of  $y_{lt}$  on the estimated  $\beta_{s_{ls}}$  also was examined. For  $m_{y_{lt}} = 25$  mm,  $\beta_{s_{ls}}$  increases gradually as  $COV_F$  increases. Although these values of  $\beta_{s_{ls}}$  are slightly smaller than their respective  $\beta_{u_{ls}}$  values, they are larger than the target  $\beta_{s_{ls}} = 2.6$  that is given in the EPRI study. Therefore, even with  $m_{y_{lt}} = 25$  mm and  $COV_{y_{lt}} = 0.583$ , the NBCC designs of drilled shafts under undrained compression still are acceptable for the SLS design requirements. It was also found that, the  $\beta_{s_{ls}}$  versus  $COV_F$  relationships for  $m_{y_{lt}} = 96$  mm and  $m_{y_{lt}} = 135$  mm are virtually identical because the values of  $R$  for these relatively large  $y_{lt}/B$  values already approach the asymptotic maximum of  $1/b = 1.28$  of the hyperbolic load-displacement model. On the other hand, when  $m_{y_{lt}}$  decreases from 96 mm to 25 mm,  $\beta_{s_{ls}}$  decreases significantly, particularly for relatively small  $COV_F$ . The value of  $\beta_{s_{ls}}$  may decrease to 2.7. Clearly, the limiting tolerable foundation settlement must be defined rationally and thoughtfully.

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