CALIBRATION OF INFORMATION-SENSITIVE PARTIAL FACTORS FOR ASSESSING EARTH SLOPES

Gwangha Roh[1] and H. P. Hong[2]

ABSTRACT

Use of the limit state design with the calibrated load and resistance factors (or partial factors of safety) has a long tradition, especially for structural design codes. The load and resistance factors are calibrated using statistics, reliability, probabilistic analyses and selected target safety levels. To take advantage of the reliability-based design approach, to achieve a greater consistency in the safety level for designed or assessed earth slopes, and to cope with the degree of uncertainty in soil properties, in this study, calibration of the information-sensitive partial factors is carried out. The calibration is based on the first-order reliability method, and considers that the critical slip surface for a given set of soil properties and geometric variables of slope can be estimated based on the generalized method of slices. The calibrated factors depend on the degree of uncertainty in the soil properties (*i.e.*, coefficients of variation of cohesion and friction angle), and on the selected target reliability levels. Results of calibration are used to develop empirical equations for estimating the partial factors that are to be used for slope stability analysis and to assess the adequacy of slope for a selected target safety level. It is hoped that the developed relations could be used to aid the development of reliability-consistent design and checking of earth slopes, and to promote the practical application of the limit state design in geotechnical engineering.

Key words: Calibration, earth slope, partial factor, reliability, uncertainty.

1. INTRODUCTION

Concepts of statistics, reliability and probabilistic analysis are employed to calibrate the load and resistance factors used in the limit state design adopted in the structural design codes. Use of the limit state design instead of the allowable stress design is aimed at achieving a greater consistency in the reliability of designed structures and engineered systems. Commonly considered limit states are the ultimate limit state (ULS) and the serviceability limit state (SLS). The former is focused on safety considering strengths of materials, while the latter is concentrated on service conditions. Use of the limit state design with the calibrated load and resistance factors (or partial factors of safety) has a long tradition, especially for structural design codes (*e.g.*, CSA S408 1981, NRC 2005). The calibration of the load and resistance factors (or partial safety factors) is often carried out for selected target reliability levels (Madsen *et al*. 1986); the selection of target reliability levels takes into account the observed performance of engineered systems as well as engineering judgement and optimum use of limited available resources.

However, by comparison, the use of the limit state design is less well received in geotechnical practice, even though Taylor (1948) showed the possible usefulness of the partial factor of safety in evaluating the factored shear strength capacity (Christian 2003), and Hansen (1965) suggested the use of partial factors for load and soil parameters. More recent implementations of the use of partial factors of safety and limit state design can be found in Canadian Foundation Engineering Manual (CGS 1992) and Eurocode 7 (European Committee for Standardization 1998); calibration of partial factors for foundation design can be found in Becker (1996), Orr and Farrell (1999) and Phoon *et al*. (2003). This lack of enthusiasm of using the limit state design perhaps is partly due to unavailability of simple and agreeable framework, and that natural material such as soil and rock is less statistically homogeneous than the man-made construction material. The level of uncertainty in the properties of man-made materials such as the coefficient of variation is relatively consistent and, that the same probabilistic model could be adopted for the same material type provided by different manufactures. These result in the required probability levels for evaluating the quantiles or fractiles of the material property to achieve specified target reliability levels to be fairly consistent. Consequently, it facilitates the implementation of the calibrated material resistance factors in design codes and ensures their use leading to reliability consistent designs.

To take advantage of the reliability-based design approach, to achieve a greater consistency in the safety level for designed or assessed earth slopes, and to cope with the degree of uncertainty in soil properties, one could calibrate a set of (statistical) information-sensitive partial factors for specified target reliability levels. The calibration of the information-sensitive set of partial factors and the development of simple to use partial factors to be used to evaluate the earth slope form the main objective of this study. For the calibration of information-sensitive partial factors, only simple slopes are considered, and possible effect of spatial variability (Roh 2007) is ignored. The latter is a conservative assumption. The application of the calibrated information-sensitive factors is illustrated through numerical examples.

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[1] Project Manager, Coffey Geotechnics Inc., 20 Meteor Drive Etobicoke Ontario M9W 1A4 Canada, Formerly Graduate Student in the Department of Civil and Environmental Engineering, University of Western Ontario, London, Ontario, Canada N6A 5B9.

[2] Professor (corresponding author), Department of Civil and Environmental Engineering, University of Western Ontario, London, Ontario, Canada N6A 5B9 (e-mail: hongh@eng.uwo.ca).

2. FORMATS FOR DESIGN CHECKING AND CALIBRATION PROCEDURE

The traditional allowable stress design or the working stress design (ASD/WSD) uses a single factor of safety to judge whether a design or assessed earth slopes is satisfactory. The factor of safety, F, is a function of a set of random or deterministic variables such as the cohesion, friction angle, soil density and the slope geometry. Some of these variables could be treated as random variables $\mathbf{X}$ such as the cohesion, friction angle and unit weight of soil, and others as deterministic parameters $\mathbf{X}_D$. Let,

$$\psi_0(F,\ \mathbf{x}) = 0 \tag{1}$$

denote the function relating the factor of safety F and the values of $\mathbf{X}$, $\mathbf{x}$, where $\mathbf{x}$ represents the values of $\mathbf{X}$, and the deterministic parameters $\mathbf{X}_D$ are excluded from the equation to simplify the notation. The calculation of the factor of safety can be carried out by using the limit equilibrium methods or the finite element methods (Duncan 1996). If F is greater than a selected target value, the designed or assessed earth slopes is considered to be adequate. The well-known problem with this checking or design format, especially if $\mathbf{x}$ is taken equal to the mean of $\mathbf{X}$, is that a consistent F does not imply a consistent reliability since the degrees of variability or uncertainty in $\mathbf{X}$ differ and vary from design to design.

To improve the reliability consistency in designing and checking, the reliability-based limit state design (also known as the load and resistance factor design) considers that Eq. (1) is replaced by,

$$\psi(\mathbf{x}_f) \geq 0 \tag{2}$$

where $\psi(\mathbf{x}_f)$ denotes the limit state function, $\mathbf{x}_f$ represents the quantile or fractile of $\mathbf{X}$. The fractile of the i-th random variable X_i in $\mathbf{X}$, x_{fi}, is often expressed as a multiplication of a partial factor (*i.e.*, load or resistance factor) γ_i and its corresponding nominal value x_{Ni}, or mean value m_i (*i.e.*, $x_{fi} = \gamma_i x_{Ni}$, or $x_{fi} = \gamma_i m_i$). Note that the probability levels used to calculate the fractiles of each random variable are likely to differ. Note also that Eq. (2) can be stated as,

$$\text{factored resistance} - \text{sum of effects of factored loads} \geq 0 \tag{3}$$

which simply indicates that the capacity and demand (or resistance and load effect) are balanced at the considered limit and performance level if equality occurs.

The implied safety level by satisfying Eqs. (2) or (3) depends on the target reliability index or the tolerable failure probability level, P_{fT}, employed in calibrating the fractiles of $\mathbf{X}$, $\mathbf{x}_f$, or the partial factors since calibration is aimed at finding $\mathbf{x}_f$ such that if the design engineered system satisfies,

$$\psi(\mathbf{x}_f) = 0 \tag{4}$$

and its associated probability is given by,

$$\text{Prob}\left(\psi(\boldsymbol{X}) \leq 0\right) = P_{fT} \tag{5}$$

The selection of target reliability index (or tolerable failure probability level), the definition of limit state function for assessing the stability of earth slope, and the procedure used to calibrate the required fractiles of $\mathbf{X}$, $\mathbf{x}_f$, such that Eqs. (4) and (5) hold, are presented in the following. For the analysis, it is considered that $\mathbf{X}$ only contains three random variables: effective cohesion, effective friction angle and unit weight of soil.

3. LIMIT STATE FUNCTION, RELIABILITY ANALYSIS, AND TARGET RELIABILITY LEVEL

To evaluate the reliability of earth slopes, a method for assessing the stability of the slope with given soil parameters must be selected, and reliability analysis needs to be carried out. In this study, the generalized method of slices for slope stability analysis given by Chen and Morgenstern (1983), which is an extension of the well-accepted Morgenstern-Price method (Morgenstern and Price 1965), is adopted; the reliability analysis of slopes formulated based on the first-order reliability method (FORM) and summarized below (Hong and Roh 2008), is considered.

Given the soil properties and geometric variables of a slope, this method can be used to find the critical slip surface and its corresponding factor of safety F (Chen and Shao 1988). The problem of search F expressed as a constrained optimization problem is:

$$\begin{cases} \text{Minimize} & F \\ \text{Subject to} & G(F,\ \lambda,\ \mathbf{P}) = 0 \\ & M(F,\ \lambda,\ \mathbf{P}) = 0 \\ & \mathbf{P} \text{ is in the allowable domain} \end{cases} \tag{6}$$

In the above equation, λ is a parameter used in defining the inclination of interslice force (see Fig. 1) defined by,

$$\tan\beta(x) = f_0(x) + \lambda\, f(x) \tag{7}$$

where $\beta(x)$ is the inclination of the interslice force, and $f_0(x)$ and $f(x)$ are assumed functions. $\mathbf{P}$ is a set of parameters defining the slip surface in the allowable domain, and $G(F, \lambda, \mathbf{P})$ and $M(F, \lambda, \mathbf{P})$ are functions established based on force and moment equilibrium and are detailed in Chen and Morgenstern (1983).

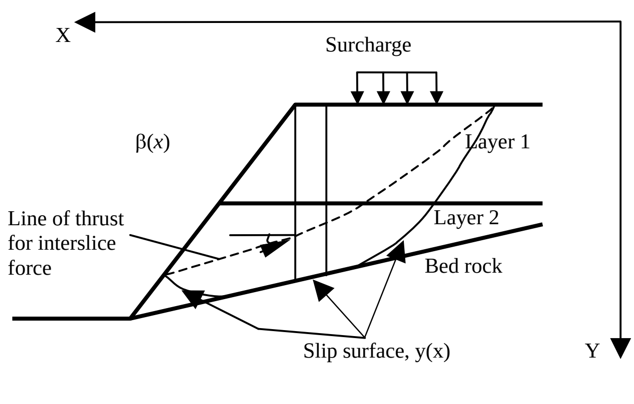


Fig. 1 Schematic of earth slope

It is considered that the mobilized shear strength τ, is related to F by the following equation,

$$\tau = (c' + \sigma_n' \tan\phi')/F = c_e' + \sigma_n' \tan\phi_e' \tag{8}$$

where c' is the effective cohesion, ϕ' is the effective friction angle, $c_e' = c'/F$, and $\tan\phi_e' = \tan\phi'/F$. Chen and Shao (1988) indicated that the Davidson-Fletcher-Power algorithm is adequate to solve the constrained optimization problem shown in Eq. (6), while Hong and Roh (2008) successfully analyzed a few slopes using the nonlinear sequential quadratic programming (SQP) method (Schittkowski 1985), which is adopted in the present study for the numerical analysis.

For a single potential slip surface, denoted by $\mathcal{S}$, and given the values of the **X**, (which contains three random variables: c', ϕ' and unit weight of soil γ), **x**, the limit state function $g_e(\mathbf{x}|\mathcal{S})$ can be defined in terms of the factor of safety F (*i.e.*, ratio of the shear strength to the mobilized shear strength) and expressed as,

$$g_e(\mathbf{x}|\mathcal{S}) = F - 1 \tag{9}$$

where F is obtained by solving Eq. (6) and depends on **x**, and the slope is in the failure domain and safe domain, and on the limit state surface if $g_e(\mathbf{x}|\mathcal{S})$ is less than zero, greater than zero and equal to zero, respectively.

Since the slope with its many potential slip surfaces can be considered as a series system, any $g_e(\mathbf{x}|\mathcal{S}) < 0$ implies the failure and consequently implies the failure of the system. By considering all potential slip surfaces, denoted as (all $\mathcal{S}$), the limit state function of the system $\psi(\mathbf{x})$ is given by (Hong and Lo 2001, Hong and Roh 2008),

$$\psi(\mathbf{x}) = \min_{\text{all } \mathcal{S}} \left(g_e(\mathbf{x}|\mathcal{S}) \right) \tag{10}$$

and the probability of failure of the slope, P_f, can be evaluated by solving,

$$P_f = \int_{\psi(\mathbf{X})\le 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} \tag{11}$$

where $f_x(\mathbf{x})$ is the joint probability density function of the random variables **X**. This integral equation can be evaluated efficiently using first-order reliability method (FORM) (Rackwitz and Fiessler 1978, Madsen *et al*. 1986), which gives the reliability index β_R and the failure probability P_f is then approximated by,

$$P_f = \Phi(-\beta_R) \tag{12}$$

where $\Phi(\bullet)$ is the standard normal distribution function. Also, the vector of sensitivity factors (sensitivity of β_R to each random variable) as well as the design point (*i.e.*, the point representing the most likely combination of the values of the random variables **X** that satisfies $\psi(\mathbf{x}) = 0$) are obtained by using the FORM. For easy reference, the critical slip surface for which the soil parameters equal their corresponding values at the design point, $\mathbf{x}_d$, will be referred to as the "design critical" slip surface.

Based on the above, the objective of calibration for a selected P_{fT} is to identify $\mathbf{x}_d$ through the reliability analysis such that,

$$\text{Prob}\left(\psi(\mathbf{X}) \le 0\right) = \int_{\psi(\mathbf{X})\le 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} = P_{fT} \tag{13}$$

At the design point the limit state function satisfies,

$$\psi(\mathbf{x}_d) = \min_{\text{all } \mathcal{S}} \left(g_e(\mathbf{x}_d|\mathcal{S}) \right) = 0 \tag{14}$$

In other words, the task to calibrate the partial factors of interest is to solve Eq. (13) for selected P_{fT}, and to use the design point $\mathbf{x}_d$, the means and standard deviations of the random variables to assess the partial factors.

In particular, if a considered random variable X_i is normally distributed and its value at the design point is x_{di}, one could define a partial factor γ_i that relates x_{di} to the mean of X_i, m_{xi}, by,

$$\gamma_i = x_{di}/m_{Xi} = 1 + k_i\, v_{Xi} \tag{15}$$

where

$$k_i = (x_{di} - m_{Xi})/(m_{Xi}\, v_{Xi}) \tag{16}$$

and v_{xi} is the coefficient of variation of X_i. Similarly, if X_i is lognormally distributed, the partial factor γ_i can be expressed as,

$$\gamma_i = x_{di}/m_{Xi} = \exp\left(k_i \sqrt{\ln\ (1+v_i^2)}\right) / \sqrt{1+v_i^2} \tag{17}$$

and, k_i, is given by

$$k_i = \left(\ln\ (x_{di}) - \ln\left(m_{Xi}/\sqrt{1+v_i^2}\right)\right) / \sqrt{\ln\ (1+v_i^2)} \tag{18}$$

The above indicates that k_i or γ_i define a set of information-sensitive partial factors that can be used together with available statistical information on **X** and Eq. (14) to formulate the limit state design or checking for earth slopes.

For the calibration, it is noted that the suggested tolerable failure probability levels for developing the limit state design depend on the considered failure mode and importance of the engineering system. The suggested tolerable failure probabilities by CSA S408 (1981) range from 3.40×10^{-6} to 6.21×10^{-3} (*i.e.*, reliability index ranging from 2.5 to 4.5) for buildings based on 30-year reference period, depending on the type of failure and the safety class. Comparison of reliability levels for different constructions and activities has been summarized by Whitman (1984), while Meyerhof (1982) recommended the failure probability of 1.0×10^{-2} for earth work and of 1.0×10^{-3} for earth retaining structure and foundation. These values correspond to reliability indices of 2.33, and 3.09, respectively. Baecher and Christian (2003) recommended that a target reliability index of 2.0 to 3.0 should be adequate for assessing geotechnical problems. Based on these considerations, the target reliability indices ranging from 2.0 to 3.0 (*i.e.*, P_{fT} of 2.28×10^{-2} to 1.35×10^{-3}) are employed to calibrate the information-sensitive partial factors.

4. CALIBRATION RESULTS

To evaluate the information-sensitive partial factors, earth slopes of simple geometry with statistically homogeneous soil and four different slope ratios (*i.e.*, 1:1, 1:1.5, 1:2, and 1:3 slopes) are considered. The considered random variables for the slopes are the cohesion, friction angle, and the unit weight. These random variables are commonly considered to be normal or lognormally distributed. Statistics of these random variables have been discussed in the literature (Lacasse and Nadim 1996; Phoon and Kulhawy 1999; Orr and Farrell 1999; Phoon 2008).

Lacasse and Nadim (1996) suggested that the cohesion, c', can be modeled as a lognormal variate, the friction angle, ϕ', can be modeled as a normal variate, and the unit weight, γ, can be modeled as a normal variate. Orr and Farrell (1999) recommended that the typical coefficients of variation (cov) range from 0.2 to 0.4 for cohesion, from 0.05 to 0.15 for the friction angle, and from 0.01 to 0.10 for the unit weight. The suggested ranges of cov values and the probabilistic models that are summarized in Table 1 are considered in the following for the numerical analysis.

4.1 Effect of Mean and Coefficient of Variation on the Estimated Failure Probability

The impact of the uncertainty in c', ϕ', and γ on the estimated failure probability must be assessed for calibrating the partial factors. First consider a statistically homogenous earth slope with a face angle of 45° (*i.e.*, 1:1 slope) as illustrated in Fig. 2. The slope consists of highly cohesive soil. This implies that the safety of the slope depends mostly on the cohesion. The means of c', ϕ' and γ, denoted by $m_{c'}$, $m_{\phi'}$, and m_γ, are considered to be equal to 30 kN/m^2, 5° and 16 kN/m^3, respectively. The cov values of c', ϕ' and γ, denoted by $\nu_{c'}$, $\nu_{\phi'}$, and ν_γ, are considered to be equal to 0.3, 0.1 and 0.05, respectively. As mentioned previously, c', ϕ' and γ are modeled as lognormal, normal and normal variates, respectively. By using these probabilistic models and the analysis procedure presented earlier, the obtained reliability index β_R equals 1.19 and the corresponding failure probability, P_f, equals 1.17×10^{-1}. Note that if c', ϕ' and γ are taken equal to their corresponding mean values, the calculated factor of safety of the slope equals 1.37. For easy reference, this considered case will be referred to as Case 1.

By maintaining the values of the parameters to be the same as those for Case 1, except that m_γ = 14 and 18 kN/m^3 instead of 16 kN/m^3, the obtained failure probabilities are shown in Figure 3a and compared with that of Case 1. The figure shows that as m_γ increases P_f increases. For comparison purpose, the calculated factor of safety using the mean values of the random variables is also indicated in the figure.

By repeating the above analysis, but considering ν_γ equal to 0.01 and 0.1, the obtained P_f is shown in Fig. 3(a) as well. The figure suggests that the impact of ν_γ on P_f is not very significant.

Similarly, a sensitivity analysis of P_f to $m_{c'}$ and $\nu_{c'}$ is carried out, and the obtained values of P_f are shown in Fig. 3(b). The figure suggests that the increase in $m_{c'}$ leads to the decrease of P_f, and that P_f could be increased by an order of magnitude as $\nu_{c'}$ increases.

Now, consider a slope, denoted as Case 2, whose shear strength depends highly on the friction capacity of soil. The geometry of the slope and the probabilistic distribution types for the random variables for Case 2 are considered to be the same as for Case 1. $m_{c'}$, $m_{\phi'}$ and m_γ for the slope are considered to be 5 kN/m^2, 40° and 16 kN/m^3, respectively; $\nu_{c'}$, $\nu_{\phi'}$, and ν_γ are considered to be equal to 0.3, 0.1 and 0.05, respectively. The obtained failure probability, P_f, equals 7.50×10^{-3}. Note that the factor of safety for Case 2 is equal to 1.40. By varying the values of m_γ and ν_γ and, the values of $m_{\phi'}$ and $\nu_{\phi'}$ for Case 2, the calculated P_f are depicted in Fig. 4(a). By maintaining the values of the variables to be the same as those for Case 2, except that m_γ varies from 14 to 18 kN/m^3 instead of m_γ equal to 16 kN/m^3, the obtained failure

Table 1 Probability distribution types and coefficients of variations (cov) of soil parameters suggested in the literature

Parameters	Range of cov (typical cov–*)	Probability distribution type
Unit weight (kN/m^3)	0.01 ~ 0.10 (0.05)	Normal
Cohesion (kN/m^2)	0.2 ~ 0.4 (0.3)	Lognormal
Friction angle (°)	0.05 ~ 0.15 (0.10)	Normal

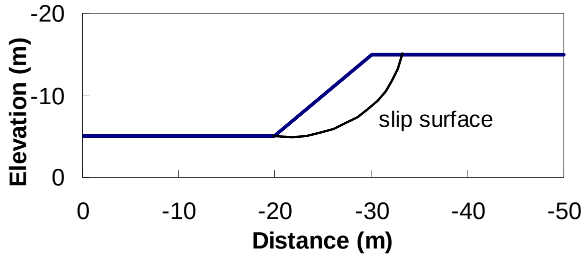
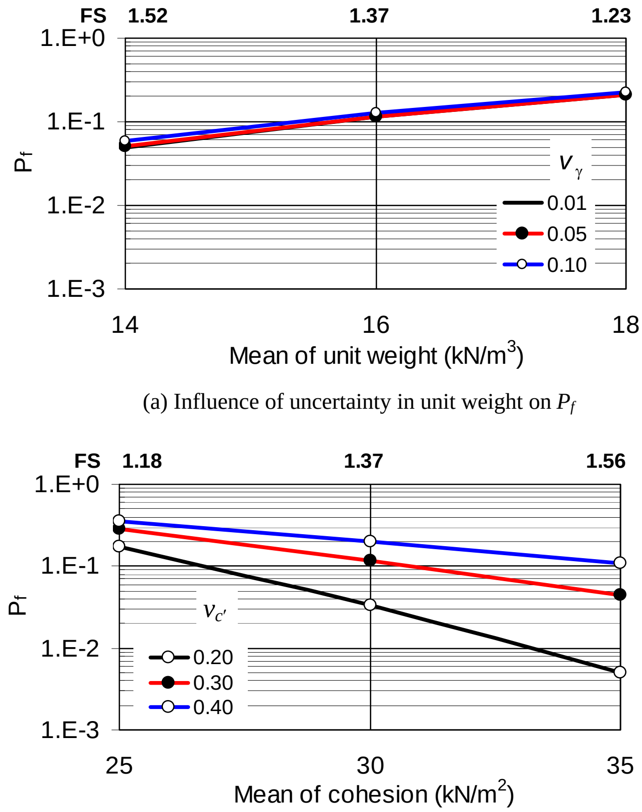


Fig. 2 Soil slope geometry and critical slip surface



(a) Influence of uncertainty in unit weight on P_f

(b) Influence of uncertainty in cohesion on P_f

Fig. 3 Effect of varying statistics of unit weight and cohesion on P_f (1 to 1 slope and cohesive soil)

probabilities are shown in Fig 4(a) and compared with that of Case 2. The figure shows that P_f increases as m_γ increases which is expected. For comparison, the calculated factor of safety based on the mean values of the random variables is also shown in the figure.

By repeating the above analysis, but considering ν_γ equal to 0.01 and 0.1, the obtained P_f is shown in Fig. 4(a), indicating that the impact of ν_γ on P_f is negligible.

Similarly, a sensitivity analysis of P_f to $m_{\phi'}$ and $\nu_{\phi'}$ is carried out, and the obtained values of P_f are shown in Fig. 4(b), indicating that an increase in $m_{\phi'}$ leads to the decrease of P_f, and that P_f could be increased by orders of magnitude as $\nu_{\phi'}$ increases.

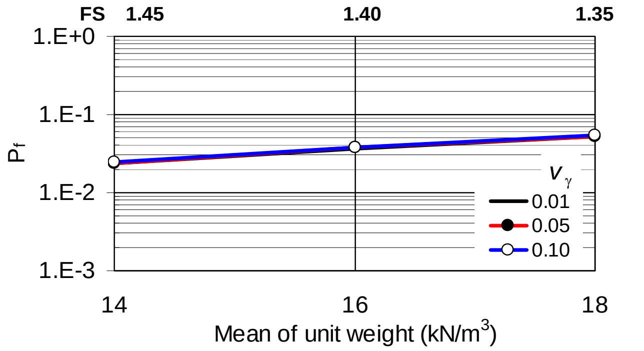
In short, P_f is sensitive to the uncertainty in the cohesion for slopes of highly cohesive soil, and is sensitive to the uncertainty in friction angle if the shear strength of the slope is mainly controlled by friction. The results also suggest that P_f is relatively insensitive to the considered range of the cov of the unit weight. Therefore, in the following analysis, only a set of typical values of m_γ and ν_γ is considered.

4.2 Calibration of the Information-Sensitive Partial Factors

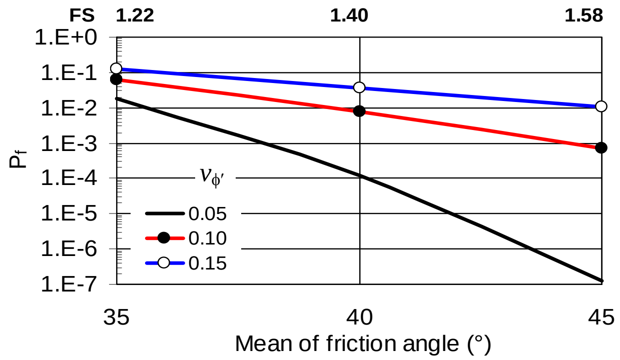
For the calibration, consider the slope shown in Fig. 2 but with a slope ratio of 1:1, 1:1.5, 1:2, or 1:3. The unit weight of the soil is considered as a normal variate with m_γ of 16 kN/m^3 and ν_γ of 0.05.

First, consider that the slope consists of cohesive soil (without friction). The cohesion is modeled as a lognormal variate with $m_{c'}$ varying within 15 to 70 kN/m^2, and $\nu_{c'}$ equal to 0.2, 0.3 and 0.4. For each value of $\nu_{c'}$, the calculated P_f as a function of $m_{c'}$ is shown in Fig. 5(a). During the calculation, values of c' and γ at the design point for the considered cases are also recorded. The value of c' at the design point is then used in Eqs. (17) and (18) to evaluate the factor k_i and γ_i (*i.e.*, $k_{c'}$ and $\gamma_{c'}$) for c'. The obtained values of $k_{c'}$ are presented in Fig. 5(b). The results shown in the figure suggest that $k_{c'}$ decreases and P_f decreases as $m_{c'}$ increases. To meet the tolerable failure probability, P_{fT}, of 2.28×10^{-2} (*i.e.*, target reliability index of 2.0), the required $m_{c'}$ values for $\nu_{c'}$ equal to 0.2, 0.3, and 0.4, as shown in Fig. 5(a), are 40.31, 50.24, and 62.99 kN/m^2. The corresponding $k_{c'}$ values shown in Fig. 5(b) are –1.85, –1.95, and –2.00. Similarly, for $\nu_{c'}$ equal to 0.2, 0.3, and 0.4, the obtained $k_{c'}$ values are –2.35, –2.45, and –2.50 for P_{fT} equal to 6.21×10^{-3} (*i.e.*, target reliability index of 2.5); and are –2.90, –3.00, and –3.00 for P_{fT} equal to 1.35×10^{-3} (*i.e.*, target reliability index of 3.0). These suggest that for a given P_{fT}, $k_{c'}$ is relatively insensitive to $\nu_{c'}$. This observation is valuable since it indicates that for a given P_{fT} one could adopt a constant $k_{c'}$ for $\nu_{c'}$ ranging from 0.2 to 0.4 and using Eq. (17) to evaluate the partial factor for cohesion, $\gamma_{c'}$.

The above analysis is repeated for the earth slope shown in Fig. 2 but with a slope ratio of 1:1.5, 1:2, and 1:3. The obtained results are then employed to estimate $k_{c'}$ values for different tolerable failure probability levels, which are summarized in Table 2. The table suggests that $k_{c'}$ for a given P_{fT} and a range of $\nu_{c'}$ is relatively insensitive to the slope angle. One could consider, therefore, that $k_{c'}$ for a given P_{fT} equals the average of the values of $k_{c'}$ obtained for the considered $\nu_{c'}$ values and slope angles. This leads to $k_{c'}$ equal to –1.93, –2.44, and –2.95 for P_{fT} equal to 2.28 $\times 10^{-2}$, 6.21×10^{-3} and 1.35×10^{-3}, respectively, and $\gamma_{c'}$ is evaluated using (see Eq. (17)),

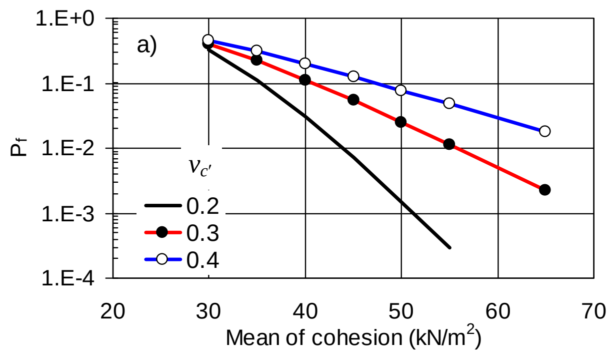


(a) Influence of uncertainty in unit weight on P_f

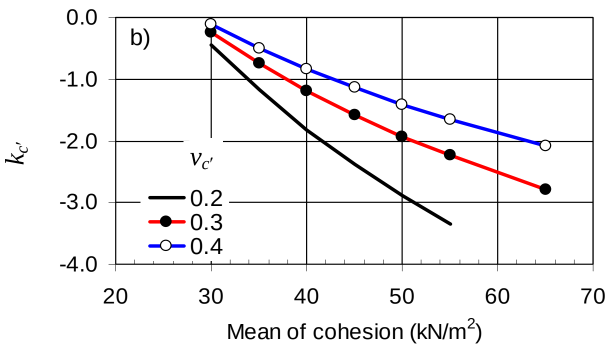


(b) Influence of uncertainty in friction angle on P_f

Fig. 4 Effect of varying statistics of unit weight and friction angle on P_f (1 to 1 slope and cohesive soil)



(a)



(b)

Fig. 5 Failure probability and the corresponding $k_{c'}$ value (1:1 slope with cohesive soil only)

Table 2 Estimated $k_{c'}$ and $k_{\phi'}$ values for different target reliability indices and slope ratios

Slope ratio	Target reliability index, β_R	Cohesive soil			Friction soil		
		$k_{c'}$			$k_{\phi'}$		
		$\nu_{c'}=0.2$	$\nu_{c'}=0.3$	$\nu_{c'}=0.4$	$\nu_{\phi'}=0.05$	$\nu_{\phi'}=0.10$	$\nu_{\phi'}=0.15$
1:1	2.0	–1.85	–1.95	–2.00	–1.70	–1.90	–2.00
	2.5	–2.35	–2.45	–2.50	–2.35	–2.40	–2.45
	3.0	–2.90	–3.00	–3.00	–2.80	–2.95	–3.00
1:1.5	2.0	–1.90	–1.90	–1.95	–1.80	–1.95	–2.00
	2.5	–2.35	–2.50	–2.50	–2.40	–2.45	–2.50
	3.0	–2.90	–2.90	–2.95	–2.85	–2.95	–3.00
1:2	2.0	–1.85	–1.95	–2.00	–1.80	–1.95	–2.00
	2.5	–2.35	–2.45	–2.50	–2.40	–2.45	–2.50
	3.0	–2.90	–2.95	–3.00	–2.85	–2.95	–3.00
1:3	2.0	–1.90	–1.95	–2.00	–1.85	–1.95	–2.00
	2.5	–2.40	–2.45	–2.50	–2.40	–2.45	–2.50
	3.0	–2.90	–2.95	–3.00	–2.85	–2.95	–3.00

$$\gamma_{c'} = \exp\left(k_{c'}\sqrt{\ln\ (1+\nu_{c'}^2)}\right) / \sqrt{1+\nu_{c'}^2} \tag{19}$$

which could further simplified to

$$\gamma_{c'} = (1-\nu_{c'}^2/2)\exp\ (k_{c'}\ \nu_{c'}) \tag{20}$$

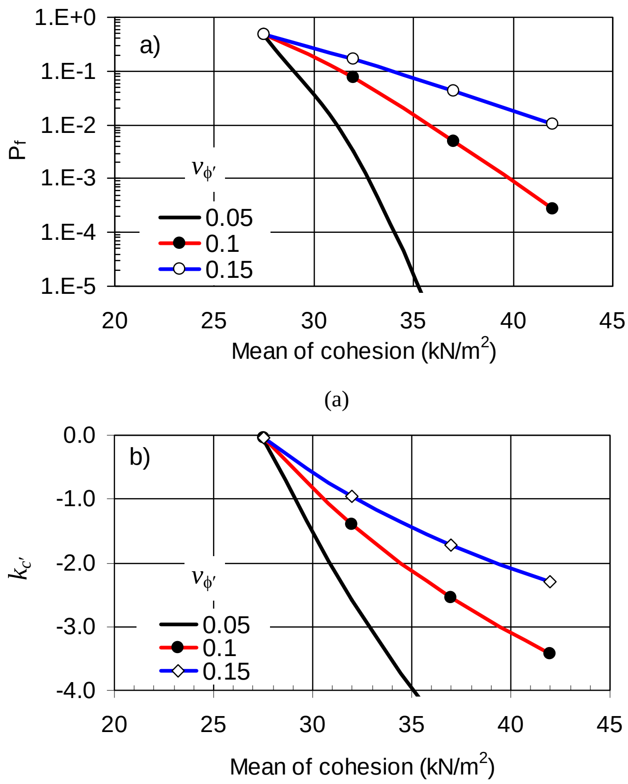
Note that a simple analysis shows that the average value of $k_{c'}$ for the considered P_{fT} is approximately equal to $0.98\Phi^{-1}(P_{fT})$. Therefore, for a tolerable failure probability within the considered range of values of P_{fT}, one could use $k_{c'}$ equal to $0.98\Phi^{-1}(P_{fT})$ into Eqs. (19) or (20) to estimate the partial factor $\gamma_{c'}$. The suggested two equations (*i.e.*, Eqs. (19) and (20)) to estimate partial factor $\gamma_{c'}$, which is denoted by $\gamma_{c'0}$, are summarized in Table 3.

It is noteworthy that the results obtained through the reliability analyses indicate that the value of γ at the design point is slightly higher than m_γ, resulting in the partial factor for the unit weight, γ_γ (see Eq. (15)) ranges from 1.00 to 1.02 for typical ν_γ of 0.05. The higher value of γ_γ is associated with smaller values of P_{fT}, and $\nu_{c'}$. Since the variation of γ_γ is relatively small, as an approximation, γ_γ could be considered to be equal to 1.02.

Now, consider that the shear strength of the earth slope is controlled by the soil friction capacity. At the extreme, the soil would be considered as the friction only soil. However, this would lead to the surficial slumps, and is outside of the scope of this study. Consequently, a certain fixed amount of cohesion, say, 5 kN/m^2 is included for the considered earth slopes. The friction angle for the slope is modeled as a normal variate with $m_{\phi'}$ varying from about 10° to 45°, and $\nu_{\phi'}$ equal to 0.05, 0.10, and 0.15. For each given $\nu_{\phi'}$ value, the calculated P_f as a function of $m_{\phi'}$ is shown in Fig. 6(a). During the calculation, ϕ' and γ values at the design point for the considered cases are also recorded. The value of ϕ' at the design point is then used in Eqs. (15) and (16) to evaluate the factor k_i and γ_i (*i.e.*, $k_{\phi'}$ and $\gamma_{\phi'}$) for ϕ'. The obtained values of $k_{\phi'}$ are presented in Fig. 6(b). Note that to meet P_{fT} of 2.28×10^{-2} (*i.e.*, target reliability index of 2), the required $k_{\phi'}$ values are –1.70, –1.90, and –2.00 for $\nu_{\phi'}$ equal to 0.05, 0.10, and 0.15, respectively. Similarly, for $\nu_{\phi'}$ equal to 0.05, 0.10, and 0.15,

Table 3 Recommended information-sensitive partial factors (and $\gamma_\gamma = 1.02$) for a given P_{fT} value and extreme cases

Condition	$\gamma_{c'0}$ (for cohesive soil)	$\gamma_{\phi'0}$ (for ϕ' – soil)
For typical cov values of c' and ϕ'	$\dfrac{\exp\left(0.98\Phi^{-1}(P_{fT})\sqrt{\ln\ (1+\nu_{c'}^2)}\right)}{\sqrt{1+\nu_{c'}^2}}$ or $(1-\nu_{c'}^2/2)\exp\ \left(0.98\Phi^{-1}(P_{fT})\ \nu_{c'}\right)$	$1+0.97\Phi^{-1}(P_{fT})\ \nu_{\phi'}$
For typical cov values ($\nu_{c'}=0.3$ and $\nu_{\phi'}=0.1$)	$0.96\exp\left(0.29\Phi^{-1}(P_{fT})\right)$	$1+0.097\Phi^{-1}(P_{fT})$
Relation between $\gamma_{c'}$ and $\gamma_{\phi'}$	$\gamma_{\phi'} = (1.00-\gamma_{\phi'0})\exp\ \left(-7(\gamma_{c'}\ \gamma_{c'0})^{0.7}\right)+\gamma_{\phi'0}$	



(b)

Fig. 6 Failure probability and the corresponding $k_{\phi'}$ value for (1:1 slope with friction dominate soil)

$k_{\phi'}$ values are –2.35, –2.45, and –2.45 for P_{fT} equal to 6.21×10^{-3}; and are –2.80, –2.95, and –3.00 for P_{fT} equal to 1.35×10^{-3}. These suggest that for a given value of P_{fT} the value of $k_{\phi'}$ is somewhat sensitive to the value of $\nu_{\phi'}$, and $k_{\phi'}$ increase with increasing value of $\nu_{\phi'}$.

By repeating the above analysis for the same slope but with slope ratios of 1:1.5, 1:2, and 1:3, the obtained $k_{\phi'}$ values are presented in Table 2 for different tolerable failure probability levels. The table suggests that $k_{\phi'}$ is relatively insensitive to the slope angle, and that in general $k_{\phi'}$ is an increasing function of $\nu_{\phi'}$ for a given value of P_{fT}. Based on the results shown in Table 2, it is suggested that $k_{\phi'}$ could be approximated by,

$$k_{\phi'} = 0.97\Phi^{-1}(P_{fT}) \tag{21}$$

Note that this approximation, which is obtained from an average value, did not consider possible slightly increasing trend of $k_{\phi'}$ due to the increased $\nu_{\phi'}$ value or increased slope ratio. By adopting this suggested approximation and Eq. (15), $\gamma_{\phi'}$ for a given value of P_{fT} can be evaluated using,

$$\gamma_{\phi'} = 1 + k_{\phi'}\ \nu_{\phi'} = 1 + 0.97\Phi^{-1}(P_{fT})\ \nu_{\phi'} \tag{22}$$

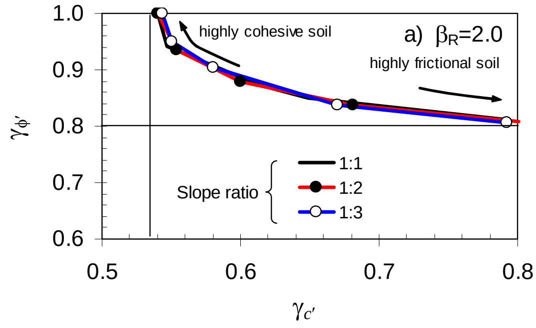
Note also that again in all the calculations the value of unit weight γ at the design point is slightly higher than m_γ resulting in γ_γ range from 1.01 to 1.04 for ν_γ equal to 0.05. The higher value of γ_γ is associated with smaller value of P_{fT}, and $\nu_{\phi'}$. Again, since this variation of γ_γ is relatively small, as an approximation, γ_γ equal to 1.02 is considered.

Based on the above analysis results, the recommended $\gamma_{\phi'}$, denoted by $\gamma_{\phi'0}$, is also summarized in Table 3. It must be emphasized that the factors shown in the table are to be applied to the mean values of the cohesion, friction angle, and unit weight of soil.

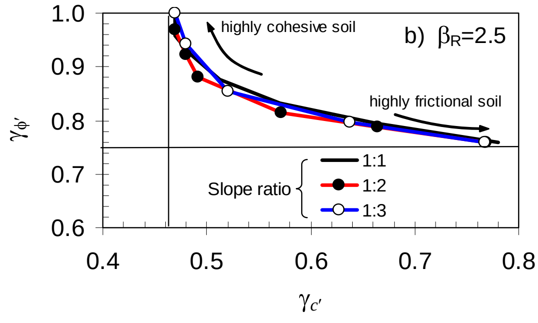
To assessing the required partial factors for the statistically homogenous earth slopes with c'-ϕ' soils, we again consider the slope shown in Fig. 2 with slope ratios of 1:1, 1:2, and 1:3. Typical cov values of cohesion, friction angle, and unit weights of 0.3, 0.1 and 0.05 are considered, and the mean of the unity weight is assumed to be equal to 16 kN/m^3. The required partial factors are evaluated by varying the mean of c' and/or ϕ' such that the obtained reliability index equals a selected target reliability index.

For example, consider in particular the earth slope with the 1:1 slope ratio, and a target reliability index of 2.0. By considering the mean of cohesion c' equal to 5 kN/m^3 and varying the mean of the friction angle ϕ' within 0 to 40°, we found that the mean of ϕ' must be equal to 36.5° for the target reliability index equal to 2.0. The values of c' and ϕ' corresponding to the design point obtained by using the FORM are 3.25 kN/m^3 and 31.03°. Consequently, the partial factors $\gamma_{c'}$ and $\gamma_{\phi'}$ calculated based on the design point and using the procedure similar to the previous section, are 0.65 and 0.85, respectively. These factors are presented in Fig. 7(a). By carrying out this analysis but considering different $m_{c'}$ values, a set of values of $\gamma_{c'}$ and $\gamma_{\phi'}$ is obtained and is also included in Fig. 7(a). By repeating this analysis but considering slope ratios of 1:2 and 1:3, the obtained values of $\gamma_{c'}$ and $\gamma_{\phi'}$ are shown in Fig. 7(a) as well. The figure indicates that the relation between $\gamma_{c'}$ and $\gamma_{\phi'}$ is insensitive to the slope ratio. This is advantageous as it implies that there is no need to develop different partial factors for different slope ratios. Furthermore, Fig. 7(a) indicates that $\gamma_{c'}$ and $\gamma_{\phi'}$ approach to the recommended values shown in Table 3 as the mean capacity due to cohesion increases (*i.e.*, cohesive soil), or as the mean capacity due to friction increases (*i.e.*, friction soil), respectively.

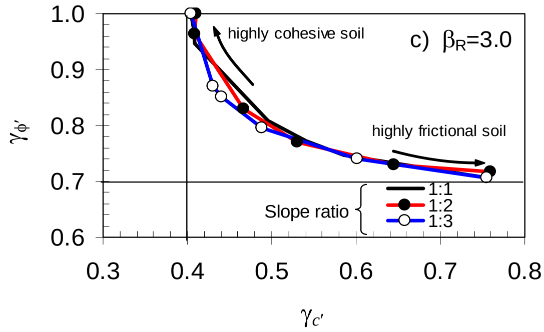
The above analysis is repeated for the selected target reliability index equal to 2.5 and 3.0. The results are presented in Figs. 7(b) and 7(c). In general, the observations drawn from Fig. 7(a) are applicable to the results shown in Figs. 7(b) and 7(c).



(a)



(b)



(c)

Fig. 7 Partial factors for cohesion and friction angle considering $\nu_{c'}$ = 0.30, $\nu_{\phi'}$ = 0.10 and ν_γ = 0.05 (the limit values of $\gamma_{\phi'}$ and $\gamma_{c'}$ represented by horizontal and vertical lines are those given in the Table 3)

Comparison of the results presented in the figures suggests that $\gamma_{c'}$ and $\gamma_{\phi'}$ decreases as the target reliability index increases, which is expected.

To suggest a single curve relating $\gamma_{c'}$ and $\gamma_{\phi'}$ for slopes with different slope ratios, a simple curve fitting is carried out for the results presented in Fig. 7. It was found that the following empirical approximation could be considered,

$$\gamma_{\phi'} = (1.00 - \gamma_{\phi'0})\ \exp\left(-A(\gamma_{c'} - \gamma_{c'0})^B\right) + \gamma_{\phi'0} \tag{23}$$

where A = 7.0 and B = 0.7 are the model parameters, and $\gamma_{\phi'0}$ and $\gamma_{c'0}$ are shown in Table 3. The fitted curve shown by Eq. (23) is shown in Fig. 8, and included in Table 3.

It is suggested that the curves presented in Fig. 8 and Eq. (23) can be used to select the partial factors. Since these calibrated partial factors depends on the mean values of c' and ϕ', we considered a slope with the slope ratio 1:1 and tested a few slopes

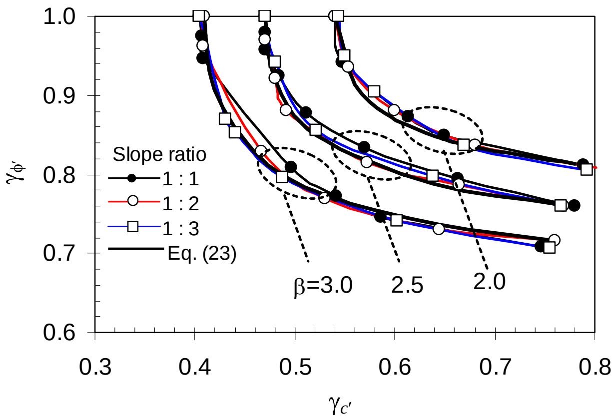


Fig. 8 Fitted empirical relation between $\gamma_{c'}$ and $\gamma_{\phi'}$ (for $\nu_{c'} = 0.30$, $\nu_{\phi'} = 0.10$ and $\nu_{\gamma} = 0.05$)

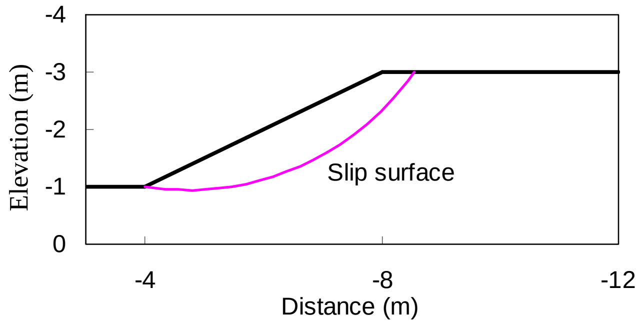


Fig. 9 A simple slope used to illustrate the application of the information-sensitive partial factors

Table 4 Soil properties for the slope shown in Fig. 9

	Cohesion, c' (kN/m^2)	Friction angle, ϕ'(°)	Unit weight, γ (kN/m^3)
Mean	1.80	20.00	18.00
Coefficient of variation	0.30	0.10	0.05
Partial factor for $P_{fT} = 6.21 \times 10^{-3}$	0.65	0.79	1.02
Factored value for $P_{fT} = 6.21 \times 10^{-3}$	1.17	15.80	18.36

with different $m_{c'}$ and $m_{\phi'}$, and concluded that most probable ranges of partial factors for c' and ϕ' can be separated into three groups. The first group is for low cohesion and high friction angle soil, the third group is for high cohesion and low friction angle soil, and the second group is for the soil with the properties lying between the first group and third group. However, we did not find more precise rules that can be used to separate these three groups. The use of these curves is illustrated in the next section.

5. ILLUSTRATIVE APPLICATIONS

To illustrate the application of the information-sensitive partial factors, two example slopes are considered. The first example is a simple slope, considered by Griffiths and Lane (1999) and shown in Fig. 9 with soil properties presented in Table 4. The specified tolerable failure probability level P_{fT} is considered to be equal to 6.21×10^{-3} (target reliability index $\beta_R = 2.5$). Our task is to assess whether the slope has acceptable safety level or it needs to be strengthened to achieve the considered target reliability index. Since the slope is a c'-ϕ' soil slope, one should use the values of $\gamma_{c'}$ and $\gamma_{\phi'}$ corresponding to second group that are between the cohesive soil and friction soil. For example, one could select $\gamma_{c'}$ equal to 0.65. By using this value, Eq. (23) and the results shown in Table 3, one finds that the required value of $\gamma_{\phi'}$ equals 0.79. Using these partial factors and the suggested value of γ_{γ} which equals 1.02 and the means of the variables shown in Table 4, the factored values of c', ϕ' and γ are calculated (*i.e.*, cohesion $0.65 \times 1.8 = 1.17$ kN/m^2, friction angle $0.79 \times 20 = 15.8°$ and unit weight $1.02 \times 18 = 18.36$ kN/m^3) and are shown in Table 4 as well. By employing these factored values in the slope stability analysis, the obtained factor of safety equals 1.01. This indicates that the slope with the adopted means and cov values of c', ϕ' and γ is acceptable for the considered target reliability level. It is noteworthy that the partial factors for c' and ϕ' suggested in the Canadian Foundation Engineering Manual (CGS. 1992) are equal to 0.65 and 0.8. These values are consistent with the ones used for the analysis.

Instead of considering $\gamma_{c'}$ equal to 0.65, one could take another reasonable value of $\gamma_{c'}$, say, 0.55, and repeat the above analysis. This results in $\gamma_{\phi'}$ equal to 0.83, and the factor of safety obtained by using the factored values of the material properties equals 1.01. This implies that the obtained factor of safety by using the factored values of the material properties is not very sensitive to the adopted values of $\gamma_{c'}$ and $\gamma_{\phi'}$ as long as they satisfy the suggested empirical equation (*i.e.*, Eq. (23)), and the selected value of $\gamma_{c'}$ is between the cohesive soil and friction soil. Furthermore, if one carries out a reliability assessment of the slope with the statistics shown in Table 4 and considering that c', ϕ', and γ are independent lognormal, normal and normal variates, respectively, one concludes that the reliability index for the slope equals 2.55, which is slightly higher than the target reliability level of 2.5. This confirms the adequacy of using the proposed information-sensitive partial factors for assessing the safety of this earth slope.

Now consider a second slope with 3 layers as shown in Fig. 10 (Donald and Giam 1992). The statistics of the material properties of the soil are presented in Table 5, and the considered tolerable failure probability level P_{fT} is 6.21×10^{-3} (*i.e.*, target reliability index of 2.5). Following the same procedure as was done for the first example, one first adopts a set of partial factor for each layer. Since the first layer consists of cohesionless soil, $\gamma_{\phi'}$ is taken equal to the value suggested in Table 3. For the second and third layers, if $\gamma_{c'}$ equal to 0.65 is considered, the required $\gamma_{\phi'}$ calculated according to Eq. (23) is 0.79. These partial factors as well as the factored material properties are also presented in Table 5. By employing these factored material property values in slope stability analysis, the obtained factor of safety equals 1.04. We repeated the analysis by using $\gamma_{c'} = 0.55$ and 0.6, the obtained factor of safety equals to 1.04 and 1.035, respectively. This again indicates that the analysis results are not very sensitive to the adopted values of $\gamma_{c'}$ and $\gamma_{\phi'}$ as long as they satisfy Eq. (23) and $\gamma_{c'}$ is within the extreme cases.

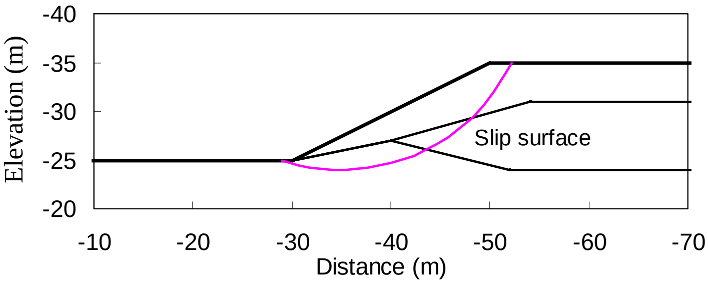


Fig. 10 A three-layer slope used to illustrate the application of the information-sensitive partial factors

Table 5 Soil properties for the slope shown in Fig. 10

Soil properties		Cohesion, c' (kN/m^2)	Friction angle, ϕ' (°)	Unit weight, γ(kN/m^3)
Layer 1	Mean	0.00	38.00	19.50
	Coefficient of varaition	–	0.10	0.05
	Partial factor $P_{fT} = 6.21 \times 10^{-3}$	–	0.76	1.02
	Factored value for $P_{fT} = 6.21 \times 10^{-3}$	–	28.88	19.38
Layer 2	Mean value	5.30	23.00	19.50
	cov	0.30	0.10	0.05
	Partial factor $P_{fT} = 6.21 \times 10^{-3}$	0.65	0.79	1.02
	Factored value for $P_{fT} = 6.21 \times 10^{-3}$	3.44	18.17	19.38
Layer 3	Mean value	7.20	20.00	19.50
	cov	0.30	0.10	0.05
	Partial factor $P_{fT} = 6.21 \times 10^{-3}$	0.65	0.79	1.02
	Factored value for $P_{fT} = 6.21 \times 10^{-3}$	4.68	15.80	19.38

6. SUMMARY AND CONCLUSIONS

In this study, calibration of the information-sensitive partial factors is carried out using the first-order reliability method, and considering that the critical slip surface for given set of soil properties and geometric variables of slope can be estimated based on the generalized method of slices. The calibrated factors depend on the degree of uncertainty in the soil properties (*i.e.*, coefficient of variation of the random variable), and on the selected target reliability levels.

The results of calibration analyses are used to develop approximate empirical equations for evaluating the partial factors for cohesion and friction angle. In particular, equations are given for estimate the required partial factors for soil strength that is controlled by the cohesion or by the friction angle. For c'-ϕ' soil, an empirical equation that relates the partial factors for cohesion and friction angle is also developed. It is hoped that the developed relations could be used to aid the development of reliability-consistent design and checking of earth slopes, and to promote the application of the limit state design in geotechnical engineering.

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