

RELIABILITY BASED DESIGN OF PILE FOUNDATIONS CONSIDERING BOTH PARAMETER AND MODEL UNCERTAINTIES

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ABSTRACT

There are two types of uncertainties in the design of large diameter bored piles, *i.e.*, parameter uncertainty and model uncertainty. This paper illustrates how model uncertainty associated with a pile capacity prediction model can be characterized, and describes a procedure to develop resistance factors for the design of large-diameter bored piles with explicit consideration of both types of uncertainties. The deterministic pile capacity model is first described. Then, the parameter uncertainty associated with the model is characterized. It is shown that the parameter uncertainty alone cannot explain the disparity between predicted and measured pile capacities. Thereafter, the model uncertainty associated with the prediction model is characterized using pile load test data. The effect of pile load tests not conducted to failure on model uncertainty characterization is discussed. Finally, based on the characterized parameter and model uncertainties, resistances factors are developed for practical reliability based design using the design point method. To comply with load factors specified for structural design, the resistance factors determined directly based on the design point method are scaled such that the safety of the pile foundation is not lower than the target reliability level.

Key words: Pile foundation, reliability based design, parameter uncertainty, model uncertainty, resistance factors.

1. INTRODUCTION

Accurate prediction of the capacity of large diameter bored piles is generally not easy, since both soil properties and construction procedures could affect the capacity of such piles. The capacity of a large diameter bored pile generally consists of two parts, *i.e.*, the shaft resistance and the toe resistance. As summarized in Rollins *et al.* (2005), commonly used methods for predicting shaft and toe resistances of bored piles include standard penetration test (SPT) methods (*e.g.*, Meyerhof 1976), effective stress methods (*e.g.*, Reese and O'Neill 1988; O'Neill 1994), and soil mechanics based approaches (*e.g.*, Kulhawy 1991).

In general, the uncertainty involved in a design can be divided into two types, *i.e.*, the uncertainty associated with model input parameters and the uncertainty associated with the model itself. Traditionally, a global factor of safety is used to accommodate all sources of uncertainty in the design of bored piles. As the global factor of safety method cannot explicitly consider the level of uncertainty involved in a design, piles with the same global factor of safety may in fact correspond to different levels of risk. To achieve a consistent level of safety, probabilistic theory can be used to develop resistance factors for reliability based design of pile foundations (*e.g.*, Barker *et al.* 1991; AASHTO 2004; Paikowsky *et al.* 2004).

At present, parameter and model uncertainties are often not considered separately when calibrating resistance factors with reliability theory. One possible reason for this might be the lack of methods for separating these two types of uncertainties. Recently, Ching *et al.* (2008) suggested a method to develop resistance factors considering both parameter and model uncertainties for reliability based design of soil anchors based on Markov Chain Monte Carlo simulation. A general method for separating parameter and model uncertainties has been developed based on Bayes' theorem (*e.g.*, Zhang *et al.* 2009; Zhang 2009). The objectives of this paper are then to illustrate how to separate the two types of uncertainties involved in a pile design, and to describe a procedure for developing resistance factors for the design of large diameter bored piles with explicit consideration of both types of uncertainties. The structure of this paper is as follows. First, a deterministic model for predicting the capacity of a large diameter bored pile is introduced. Then, the parameter uncertainty associated with the model is characterized, and the necessity to characterize the model uncertainty is highlighted. Thereafter, the model uncertainty associated with the pile capacity model is determined, and the effect of piles not loaded to failure on the model uncertainty characterization is studied. Finally, based on characterized parameter and model uncertainties, resistances factors for practical reliability based design are developed. How to adjust the resistance factors to comply with the load factors specified in the structural codes is also described.

2. PILE CAPACITY MODEL AND ASSOCIATED UNCERTAINTIES

2.1 Deterministic Pile Capacity Model

It is common to predict the capacity of a bored pile based on empirical correlations with uncorrected SPT blow count N , which is an indicator of the state of the soil before the pile is installed into the ground. Based on local experience in Hong Kong (*e.g.*,

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Malone *et al.* 1992; GEO 2006), the deterministic model used to predict the total capacity of a large-diameter bored pile used in this study is as follows

$$g(\theta) = \sum_{i=1}^n \pi D_i l_i \bar{N}_i + 9.5\pi D_n^2 N_n / 4 \tag{1}$$

where $g(\theta)$ = pile capacity model; θ = uncertain input parameters; n = number of soil layers; N_n = SPT blow count N of soil layer n ; and D_i, l_i and \bar{N}_i = diameter, length of the pile in soil layer i and average N of the soil layer, respectively. In Eq. (1), the values of \bar{N}_i and N_n are often hard to determine accurately, so uncertain model input parameters can be denoted as $\theta = \{ \bar{N}_1, \bar{N}_2, \dots, \bar{N}_n, N_n \}$.

2.2 Characterization of Parameter Uncertainty

To get some feeling about the magnitude of the parameter uncertainty, data of 17 pile-load tests are collected (Chu 2007), as summarized in Table 1. All these piles are founded in soils. The diameters of these piles are in the range of 1.0 m to 1.5 m. The

lengths of these piles are in the range of 17.3 to 72.7 m. The information available for each pile test includes: Soil stratum profile, SPT N value profile, pile material, pile geometry and load-settlement curve of the pile-load test. For the 17 piles, the encountered soils included fill, marine deposit, alluvium, residual soil, completely decomposed granite, and completely decomposed volcanic. The uncertainties in N and \bar{N} of the soil layers were evaluated based on in-situ SPT N profiles assisted with engineering judgment. It is found that the coefficients of variation (COV) of N of different soil layers are typically in the range of 0.10-0.70. For comparison, Phoon and Kulhawy (1999) reported that the COV values of N in sand layers are in the range of 0.19-0.62. Note that \bar{N} denotes the mean of N in a soil layer, so it has a smaller COV value due to the averaging effect. The difference between the COV values of \bar{N} and N of a soil layer mainly depends on the thickness of the soil layer. For the soil layers involved in the 17 piles, the COV values of \bar{N} are typically in the range of 0.05 ~ 0.20.

Table 1 Information of 17 test piles for calibrating model uncertainty factor (adapted from Chu 2007)

Pile Number	Site ¹	Length (m)	Diameter (m)	Construction details	Stratum
					Soil type (thickness in meter)
1	NC	48.8	1.5	Grabs, RCD (casing, water)	Fill (15.7), Marine deposit (1), Alluvium (9.5), CDG ² (22.7)
2	NC	54.4	1.3	Grabs (casing, water)	Fill (21), Marine deposit (0.9), Alluvium (1.1), CDG (31.4)
3	TC	38.5	1.2	Grabs (casing, water)	Fill (0.6), Alluvium (10.5), CDV ² (27.8)
4	TC	25.5	1.2	Grabs (casing, water)	Fill (5.4), Alluvium (12.7), CDV (7.5)
5	TC	39.3	1.2	Grabs (casing, water)	Fill (2.2), Alluvium (9.6), CDV (27.5)
6	TSW	30.2	1.5	Grabs (casing, water)	Fill (4.5), Alluvium (3.0), Metasiltstone (22.7)
7	YL	31.8	1.2	Grabs (casing, water)	Fill (3.1), Alluvium (10.8), CDV (18.0)
8	MOS	72.7	1.5	Grabs (bentonite)	Fill (12.5), Marine deposit (0.1), Alluvium (15.9), CDG (44.2)
9	WKC	51.6	1.5	Grabs, RCD (casing, water)	Fill (10.0), Marine deposit (3.5), Alluvium (5.5), Residual soil (10.0), CDG (22.7)
10	WKC	43.5	1.5	Grabs (casing, water)	Fill (29.0), CDG (14.5)
11	WHC	45.0	1.8	Grabs (casing, water)	Fill (10.0), Marine deposit (3.0), Alluvium (2.0), CDG (30.0)
12	TG	36.9	1.0	Grabs (casing, water)	Fill (14.0), Alluvium and Marine deposit (11.0), CDG (12.0)
13	TG	16.8	1.5	Grabs (casing, water)	Fill (7.3), Alluvium and Marine deposit (7.3), CDG (2.2)
14	TG	17.3	1.5	Grabs (casing, water)	Fill (4.4), Alluvium and Marine deposit (4.5), CDG (8.4)
15	TG	25.1	1.5	Grabs (casing, water)	Fill (13.3), Alluvium and Marine deposit (7.0), CDG (4.8)
16	TG	42.0	1.0	Grabs (casing, water)	Fill (14.0), Alluvium and Marine deposit (8.0), CDG (20.0)
17	TG	21.2	1.2	Grabs (casing, water)	Fill (6.1), Alluvium and Marine deposit (6.1), CDG (9.0)

1. NC = Nam Cheong Station; TSW = Tsuen Wan West Station; TC = Tung Chung Station; YL = Yuen Long Station; WHC = Western Harbour Corridor; TG = Telford Garden; WKC = West Kowloon Corridor; IEC = Island Eastern Corridor; MOS = Ma On Shan Rail; FPMTR = First Phase of Mass Transit Railway; 2. CDG = completely decomposed granite; HDG = highly decomposed granite; CDV = completely decomposed volcanic.

Based on the failure threshold suggested by Hinary and Kulhawy (1989), 6 out of the above 17 piles did not reach failure during the load testing. To study the effect of parameter uncertainty on pile capacity prediction, Fig. 1 compares the measured pile capacity, or the maximum load applied in a load test if the pile was not loaded to failure, with the lower and upper bounds of the predicted pile capacities corresponding to a confidence level of 97.5%. The bounds of the predicted pile capacities are calculated using Monte Carlo simulation considering only the uncertainty in θ . As can be seen in Fig. 1, out of the 17 piles, the measured capacities of four piles are outside the 97.5% confidence bounds of the predicted capacity; the maximum loads applied to three piles are above the upper bound of the predicted pile capacity. Such a large discrepancy suggests that the disparity between measured and predicted pile capacities cannot be fully explained by parameter uncertainties alone (*i.e.*, uncertainty in θ).

The reason for the above phenomenon is that the model uncertainty associated with Eq. (1) for pile capacity prediction is not considered in the probabilistic analysis. The model uncertainty may arise from several sources, such as:

- (1) SPT N is only an approximate index for representing soil property but it cannot reflect the effects of construction procedure on pile capacity, which is crucial for the capacity of a bored pile;
- (2) The mobilized shaft friction and toe resistance depend on the stress-strain relationship of the pile-soil interface, which is not explicitly considered in Eq. (1); and
- (3) The relationships used to predict shaft and toe resistances based on SPT N are not accurate. For a more realistic prediction of pile capacity, the model uncertainty associated with Eq. (1) must be assessed.

2.3 Characterization of Model Uncertainty

To model the effect of model uncertainty on pile capacity prediction, a model correction factor α is applied to the prediction model as follows

$$y = \alpha g(\theta) \tag{2}$$

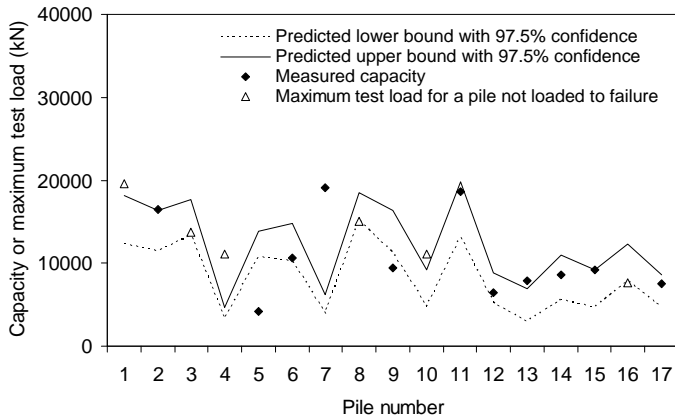


Fig. 1 Comparison of measured capacity or maximum load applied in a load test with bounds of predicted pile capacity (without considering model uncertainty)

where y = actual pile capacity. Let $E(\alpha)$ and $Std(\alpha)$ denote the mean and standard deviation of α , respectively. These two parameters quantify the characteristics of the model uncertainty. More specifically, while $E(\alpha)$ denotes how on average the model prediction is close to the actual pile capacity, $Std(\alpha)$ denotes the scatter of the model error around its mean value. Therefore, the model would be more biased when $E(\alpha)$ is far away from 1, and the magnitude of model uncertainty would increase when $Std(\alpha)$ increases.

Let $\varepsilon = \ln(\alpha)$ and assume α is lognormally distributed. As such, ε is a normal random variable. Let μ_ε and σ_ε be the mean and standard deviation of ε . Based on the above assumptions, characterizing the model uncertainty is equivalent to determining the distributions of μ_ε and σ_ε . In this study, the model uncertainty is characterized based on the Bayesian method suggested in Zhang et al. (2009), where the effects of parameter uncertainty on model uncertainty characterization can be explicitly considered, as described below.

Let P_i be the measured capacity of the i th pile or the maximum load on the i th pile if it did not fail during the loading test. Based on Zhang et al. (2009), the posterior distribution of $\{\mu_\varepsilon, \sigma_\varepsilon\}$ can be calculated as

$$f(\mu_\varepsilon, \sigma_\varepsilon | P_1, P_2, \dots, P_{17}) \approx k f(\mu_\varepsilon, \sigma_\varepsilon) \prod_{i=1}^{11} \phi \left[\frac{\ln P_i - \mu_{\ln g(\theta_i)} - \mu_\varepsilon}{\sqrt{(\sigma_{\ln g(\theta_i)}^2 + \sigma_\varepsilon^2)}} \right] \cdot \prod_{i=12}^{17} \left\{ 1 - \Phi \left[\frac{\ln P_i - \mu_{\ln g(\theta_i)} - \mu_\varepsilon}{\sqrt{(\sigma_{\ln g(\theta_i)}^2 + \sigma_\varepsilon^2)}} \right] \right\} \tag{3}$$

where k = a normalization constant to make the posterior probability density function valid; $f(\mu_\varepsilon, \sigma_\varepsilon)$ = prior distribution of μ_ε and σ_ε ; $\theta_i = \theta$ of the i th pile; $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$ = mean and standard deviation of $\ln g(\theta_i)$; ϕ = probability density function of a standard normal distribution; and Φ = cumulative distribution function of a standard normal variable. In Eq. (3), $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$, which denote the effect of uncertainty in θ_i on model uncertainty characterization, are obtained by lumping the uncertainties in θ_i into $\ln g(\theta_i)$. $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$ can be calculated using methods like Monte Carlo simulation or Taylor's expansion (*e.g.*, Ang and Tang 2007).

The prior information we used for $\{\mu_\varepsilon, \sigma_\varepsilon\}$ is

$$f(\mu_\varepsilon, \sigma_\varepsilon) \propto 1, \quad -\infty < \mu_\varepsilon < +\infty, \quad 0 < \sigma_\varepsilon < S \tag{4}$$

in which S is a sufficiently large number so that the value of the likelihood function is negligible. With this prior distribution, $f(\mu_\varepsilon, \sigma_\varepsilon | P_1, P_2, \dots, P_{17})$ is proportional to the likelihood function, so the model uncertainty is solely determined by the observed data.

As it is very costly to conduct load tests on large diameter piles, such test data are rare. In this study, all the 17 pile tests in Table 1 are used for model uncertainty characterization. To obtain $f(\mu_\varepsilon, \sigma_\varepsilon | P_1, P_2, \dots, P_{17})$, the values of $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$ of the 17 piles are first calculated using Monte Carlo simulation. Then, these values are substituted into Eq. (3), and the posterior density of μ_ε and σ_ε is calculated using a grid calculation method

suggested in Zhang (2009). With this grid calculation method, the obtained posterior distributions of μ_ϵ and σ_ϵ are shown in Fig. 2. The posterior mean and standard deviation of α , *i.e.*, $E(\alpha)$ and $\text{Std}(\alpha)$, are 2.135 and 2.147, respectively. The value of $E(\alpha)$ is larger than 1, implying that the model described above is on average biased towards the conservative side. The value of $\text{Std}(\alpha)$ is also quite large, indicating that the magnitude of model uncertainty is also large. The model correction factor, α , has a COV of 1.005, which is significantly larger than the COV values of \bar{N} and N as mentioned previously. This highlights the importance to characterize model uncertainty: If the model uncertainty is not characterized, the major uncertainty involved in a design may be ignored. Also, large model uncertainty implies that the model studied here may be an inefficient design method. As the major purpose of this paper is to illustrate how to separate parameter and model uncertainties in the pile design and how to develop resistance factors considering both two types of uncertainties, the following study is still based on the model as described in Eq. (1). In practice, it is possible that one may use a different model for design of large diameter bored piles; in such a case, the procedure presented in this paper can still be used to derive resistance factors when a different model is adopted.

One possible reason for the large model uncertainty may be due to the fact that some of the piles in the database for model uncertainty characterization were not loaded to failure. For a pile that was not loaded to failure, we only know its capacity is larger than the maximum load applied in the load test. To see the effect

of piles not loaded to failure, the model uncertainty is characterized again, assuming that the maximum load is the capacity of a pile if it was not loaded to failure. In such a case, the Bayesian formulation for model uncertainty characterization is (Zhang *et al.* 2009)

$$f(\mu_\epsilon, \sigma_\epsilon | P_1, P_2, \dots, P_{17}) \approx kf(\mu_\epsilon, \sigma_\epsilon) \prod_{i=1}^{17} \phi \left[\frac{\ln P_i - \mu_{\ln g(\theta_i)} - \mu_\epsilon}{\sqrt{(\sigma_{\ln g(\theta_i)}^2 + \sigma_\epsilon^2)}} \right] \quad (5)$$

Substituting the values of $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$ of the 17 piles into Eq. (5), the posterior distributions of μ_ϵ and σ_ϵ are calculated based on Eq. (5) using the grid calculation method, and the results are also shown in Fig. 2. Note whether a pile was loaded to failure or not does not affect the calculation of $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$, so the values of $\mu_{\ln g(\theta_i)}$ and $\sigma_{\ln g(\theta_i)}$ are the same as those used in Eq. (3). Figure 2 shows that both the distributions of μ_ϵ and σ_ϵ become less spread when Eq. (5) is used for model uncertainty characterization. As a result, the values of $E(\alpha)$ and $\text{Std}(\alpha)$ in this case are 1.410 and 0.963, respectively, which are significantly smaller than those obtained previously. The value of $E(\alpha)$ is reduced because by regarding the maximum load as the capacity of a pile if the pile did not fail, one actually takes the lowest possible capacity as the capacity of the pile, thus underestimating the actual pile capacity and hence the mean of model correction factor. The value of $\text{Std}(\alpha)$ also decreases because failed piles contain more accurate information than piles that were not loaded to fail, and therefore are more effective in characterizing the model uncertainty. Nevertheless, assuming the maximum load as the capacity of a pile does not reflect the real state of information, hence model uncertainty characterization based on this convenient assumption is only notional. The following study is based on the model uncertainty characterized based on Eq. (3), which can utilize both piles loaded to failure and piles not loaded to failure for model uncertainty characterization.

3. CALIBRATING RESISTANCE FACTORS CONSIDERING BOTH TYPES OF UNCERTAINTIES

To improve practical reliability based design, the parameter and model uncertainties should be considered in developing resistance factors. In this study, the resistance factors are determined using the design point method (*e.g.*, Ang and Tang 1984), as briefly described below.

3.1 Design Point Method

Let $u(\boldsymbol{\phi}) = 0$ denote a limit state function that can be used to discriminate whether a system is safe or not, where $\boldsymbol{\phi} = \{\phi_1, \phi_2, \dots, \phi_n\}$ are uncertain variables in the limit state function. If $\boldsymbol{\phi}$ is multivariate normal, the failure probability can be calculated approximately as follows

$$p_f \approx 1 - \Phi(\beta) \quad (6)$$

$$\beta = \min_{u(\boldsymbol{\phi})=0} \sqrt{(\boldsymbol{\phi} - \boldsymbol{\mu}_\boldsymbol{\phi})^t \mathbf{C}_\boldsymbol{\phi}^{-1} (\boldsymbol{\phi} - \boldsymbol{\mu}_\boldsymbol{\phi})} \quad (7)$$

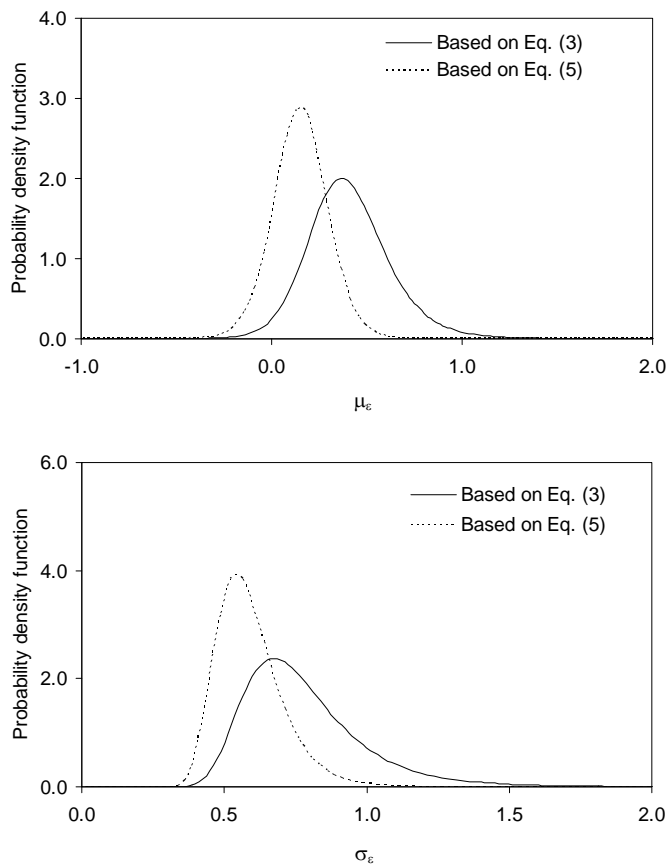


Fig. 2 Effects of piles not loaded to failure on model uncertainty characterization

where β = reliability index; $\mu_{\boldsymbol{\varphi}}$ = mean of $\boldsymbol{\varphi}$; and $\mathbf{C}_{\boldsymbol{\varphi}}$ = covariance matrix of $\boldsymbol{\varphi}$. The point minimizing the expression in Eq. (7), which is denoted as $\boldsymbol{\varphi}^*$ here, is usually called design point. When the target reliability index is met, the partial factor for φ_i can be determined based on the design point as follows

$$\gamma_{\varphi_i} = \frac{\varphi_i^*}{\varphi_{ni}} \quad (8)$$

where γ_{φ_i} = partial factor for φ_i ; and φ_{ni} = nominal value of φ_i , which can be related to the mean of φ_i , *i.e.*, μ_{φ_i} , through a bias factor λ_{φ_i} as follows

$$\varphi_{ni} = \frac{\mu_{\varphi_i}}{\lambda_{\varphi_i}} \quad (9)$$

In Eq. (7), it is assumed that $\boldsymbol{\varphi}$ follows the multivariate normal distribution. If $\boldsymbol{\varphi}$ is not multivariate normal, the design point can be found with the assistance of “equivalent normal” variables (*e.g.*, Ang and Tang 1984). After the design point is found, Eq. (8) can be used to find partial factors for the uncertain variable in the limit state function. Calibrating partial factors with the design point method can be implemented conveniently in a spreadsheet (*e.g.*, Low and Tang 1997, Low and Tang 2007).

3.2 Application to the Pile Capacity Model

For the corrected pile capacity model as described by Eq. (2), its limit state function can be written as:

$$\alpha(R_S + R_T) - S_L - S_D = 0 \quad (10)$$

where R_S = shaft resistance; R_T = toe resistance; S_L = live load; and S_D = dead load. The load resistance factor design (LRFD) format of this limit state function can be written as

$$\gamma_{\alpha} \alpha_n (\gamma_{R_S} R_{nS} + \gamma_{R_T} R_{nT}) = \gamma_{S_L} S_{nL} + \gamma_{S_D} S_{nD} \quad (11)$$

where γ_{α} , γ_{R_S} , γ_{R_T} , γ_{S_L} , γ_{S_D} = partial factor for α , R_S , R_T , S_L ; and S_D , respectively; and α_n , R_{nS} , R_{nT} , S_{nL} , S_{nD} = nominal values for α ; R_S , R_T , S_L , and S_D , respectively.

For illustrative purpose, in this study S_L is assumed to follow the lognormal distribution with a coefficient of variation (COV) of 0.18; S_D is assumed to be a lognormal variable with a COV of 0.1. The bias factors for S_L and S_D , *i.e.*, λ_{S_L} and λ_{S_D} , are taken as 1.05 and 1.0, respectively. The above load statistics and bias factors are taken from Ellingwood *et al.* (1980). Let μ_{S_L} and μ_{S_D} denote the means of S_L and S_D , respectively. The ratio of μ_{S_L} to μ_{S_D} is structure specific, and assumed to be 0.33 here. Barker *et al.* (1991), McVay *et al.* (2000), and Zhang (2004) showed that β is relatively insensitive to the ratio of μ_{S_L} to μ_{S_D} . The bias factors for resistance factors, *i.e.*, λ_{R_S} and λ_{R_T} , are taken as 1.0. Note that the bias factors of λ_{R_S} and λ_{R_T} here are defined according to Eq. (9), and are not relevant to the model correction factor α . The nominal values of R_S and R_T are calculated using the mean values of $\boldsymbol{\theta}$. The nominal value for α , *i.e.* α_n is taken as 1.0.

In practice, the statistics of R_S and R_T vary from one pile to another. As a result, the obtained partial factors may also vary from pile to pile, even when the target reliability index is the same. To see how the partial factors may vary from pile to pile,

the following procedure is used to determine the partial factors of each of the 17 piles to achieve a certain target reliability index:

- (1) As the partial factors do not depend on the absolute values of load and resistance variables, μ_{S_L} is set as 1.05 for simplicity. Using $\mu_{S_L}/\mu_{S_D} = 0.33$, $\mu_{S_D} = 0.35$. The COV values of S_L and S_D are 0.18 and 0.1, respectively. The mean values and standard deviations of S_L and S_D can be calculated based on their mean and COV values.
- (2) Calculate the COV values of the shaft and tip resistances, respectively. Calculate the ratio of the mean shaft resistance to the mean tip resistance.
- (3) Adjust the mean shaft resistance while maintaining the ratio of the mean shaft resistance to the mean toe resistance until the target reliability index is achieved. During this process, the COV values of shaft and toe resistances are fixed. The standard deviations of shaft and toe resistances are calculated based on their mean and COV values.
- (4) Calculate the partial factors based on Eq. (8).

Figures 3(a)-3(e) show the values of γ_{α} , γ_{R_S} , γ_{R_T} , γ_{S_L} , and γ_{S_D} for the 17 piles when the target reliability index is 2.5, respectively. There is relatively larger scatter in the values of γ_{R_S} and γ_{R_T} of the 17 piles; for comparison, the values of γ_{α} , γ_{S_L} , and γ_{S_D} of the 17 piles are almost identical. This is probably because while the distributions of R_S and R_T vary from pile to pile, the distributions of α , S_L and S_D are the same for all the 17 piles.

4. ADJUSTING RESISTANCE FACTORS TO COMPLY WITH STRUCTURAL CODES

One feature for limit state design in geotechnical engineering is that the factors for loads shall be consistent with those specified in structural codes. However, load factors obtained directly from the design point method may not be the same as those specified in structural codes. For instance, according to BD (2004), the partial factors for dead load and live load for buildings are 1.4 and 1.6, respectively. In the above analysis, the factors for dead load and live load are all lower than 1.4 [See Figs. 3(d) and 3(e)]. Thus, to adopt the load factors specified in BD (2004), the resistance factors determined directly based on the design point method should be increased. Based on this idea, the following design equation can be adopted to determine the required pile resistance, if the partial factors for loads specified in BD (2004) are to be used

$$\eta \gamma_{\alpha} \alpha_n (\gamma_{R_S} R_{nS} + \gamma_{R_T} R_{nT}) = 1.6 S_{nL} + 1.4 S_{nD} \quad (12)$$

where η is defined as

$$\eta = \min \left\{ \frac{1.4}{\gamma_{S_L}}, \frac{1.6}{\gamma_{S_D}} \right\} \quad (13)$$

It can be shown that if Eq. (12) is used for design, the reliability level of the pile foundation will at least meet the target reliability level (see Appendix). This idea was also used previously in Foye *et al.* (2006) to determine partial factors for shallow foundations.

Assuming the target reliability index is 2.5, the η values of 17 piles are shown in Fig. 3(f), which are almost identical. This is because the values of γ_{S_L} and γ_{S_D} are quite similar for all the 17

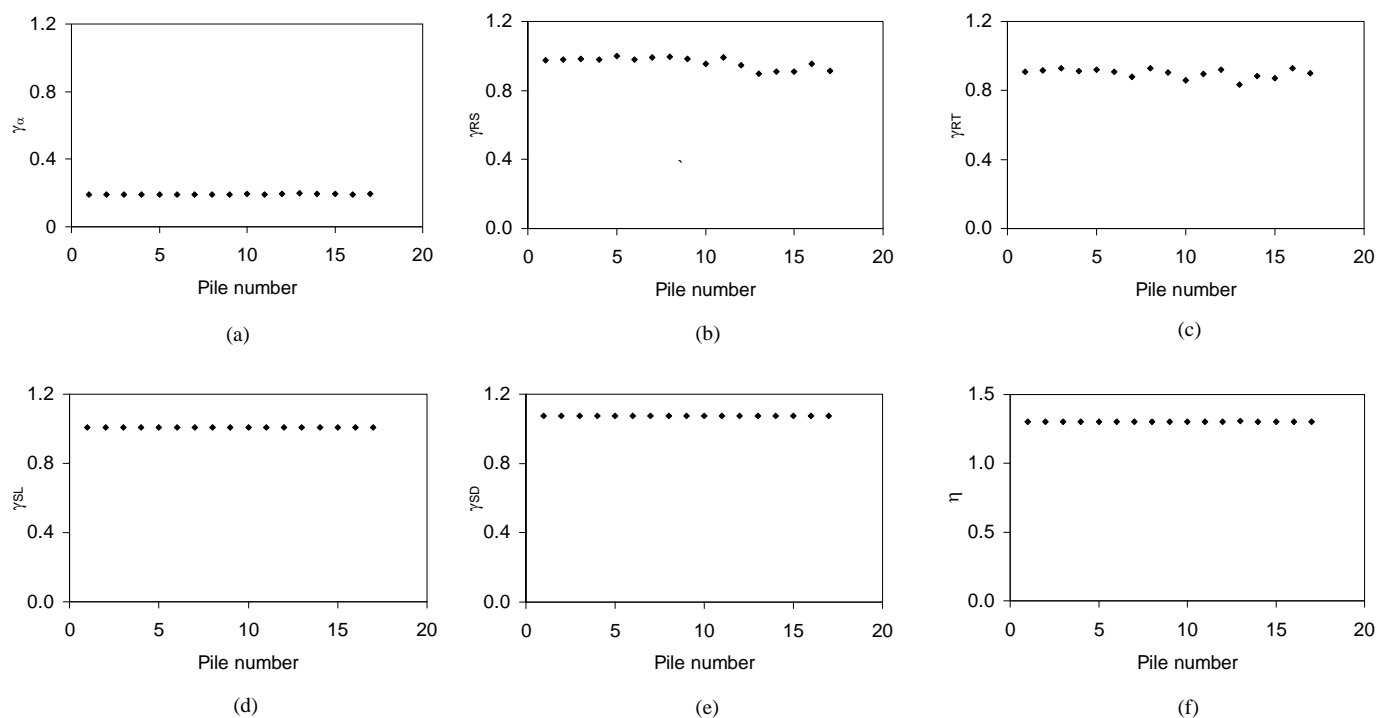


Fig. 3 Calibrated partial factors and η values for the 17 piles (target reliability index = 2.5)

piles, as shown in Figs. 3(d) and 3(e). The η values of the 17 piles are all about 1.29, indicating that the resistance factors should be increased by about 30% to be consistent with structural codes. Such an amount of change is appreciable. If the resistance factors determined from the design point method is directly used for design without adjustment, the design would be unduly conservative.

As the values of γα and η for different piles are similar [See Figs. 3(a) and 3(f)], these two quantities may be lumped with the partial factors for shaft and toe resistances to reduce the number of partial factors involved in the design as follows

$$\gamma'_{RS} = \eta \gamma_{\alpha} \alpha_n \gamma_{RS} \tag{14}$$

$$\gamma'_{RT} = \eta \gamma_{\alpha} \alpha_n \gamma_{RT} \tag{15}$$

where γ'RS and γ'RT are lumped shaft and toe resistance factors, respectively. With Eqs. (14) and (15), the design equation, Eq. (12), becomes

$$\gamma'_{R_s} R_{nS} + \gamma'_{R_t} R_{nT} = 1.6S_{nL} + 1.4S_{nD} \tag{16}$$

Based on Eqs. (14) and (15), the lumped resistance factors are calculated and shown in Fig. 4(a), for the case where the target reliability index is 2.5. The values of γ'RS of the 17 piles have a mean of 0.241 and a standard deviation of 0.005; the values of γ'RT of the 17 piles have a mean of 0.225 and a standard deviation of 0.004.

5. SUGGESTED RESISTANCE FACTORS FOR DESIGN

In the above, the resistance factors for the 17 piles are calculated. One practical problem one may raise is: What are the

resistance factors one should use in a future design? Figure 4(a) shows that the variation of lumped resistance factors from pile to pile is small when the target reliability index is 2.5. Thus, for the design of a future pile, the values of γ'RS and γ'RT can take 0.241 and 0.225, respectively, if the target reliability index is 2.5.

In practice, the target reliability index for a single pile is often in the range of 2.0 ~ 3.5. The lumped resistance factors have also been determined for the 17 piles assuming target reliability index values of 2.0, 3.0, and 3.5, respectively, as shown in Figs. 4(b) ~ 4(d). As in the case where the target reliability index is 2.5, it is also found that the variation in the values of a certain lumped resistance factor of the 17 piles is small at a given target reliability index. Therefore, the mean values of resistance factors of the 17 piles are suggested as the resistance factors for a future design, as in the case of a target reliability index of 2.5.

Figure 5 shows the suggested lumped resistance factors when the target reliability index varies from 2.0 to 3.5. The resistance factors decrease as the target reliability index increases, implying lower resistance factors should be adopted if a higher level of reliability is required, which is reasonable. In particular, Fig. 5 shows that the resistance factors highly depend on the specified target reliability index. For instance, the resistance factors in the case where the target reliability index is 2.0 are about two times of those in the case where the target reliability index is 3.0. The reliability theory provides a quantitative way to determine resistance factors considering the target reliability level, as well as the amount of uncertainty involved in the design. One may observe that the resistance factors shown in Fig. 5 are slightly lower than those reported in the literature (e.g., AASHTO 2004). As noticed previously, the pile prediction model on average underestimates the actual pile capacity, which implies that larger resistance factors can be adopted. On the other hand, the COV of the model correction factor is also large,

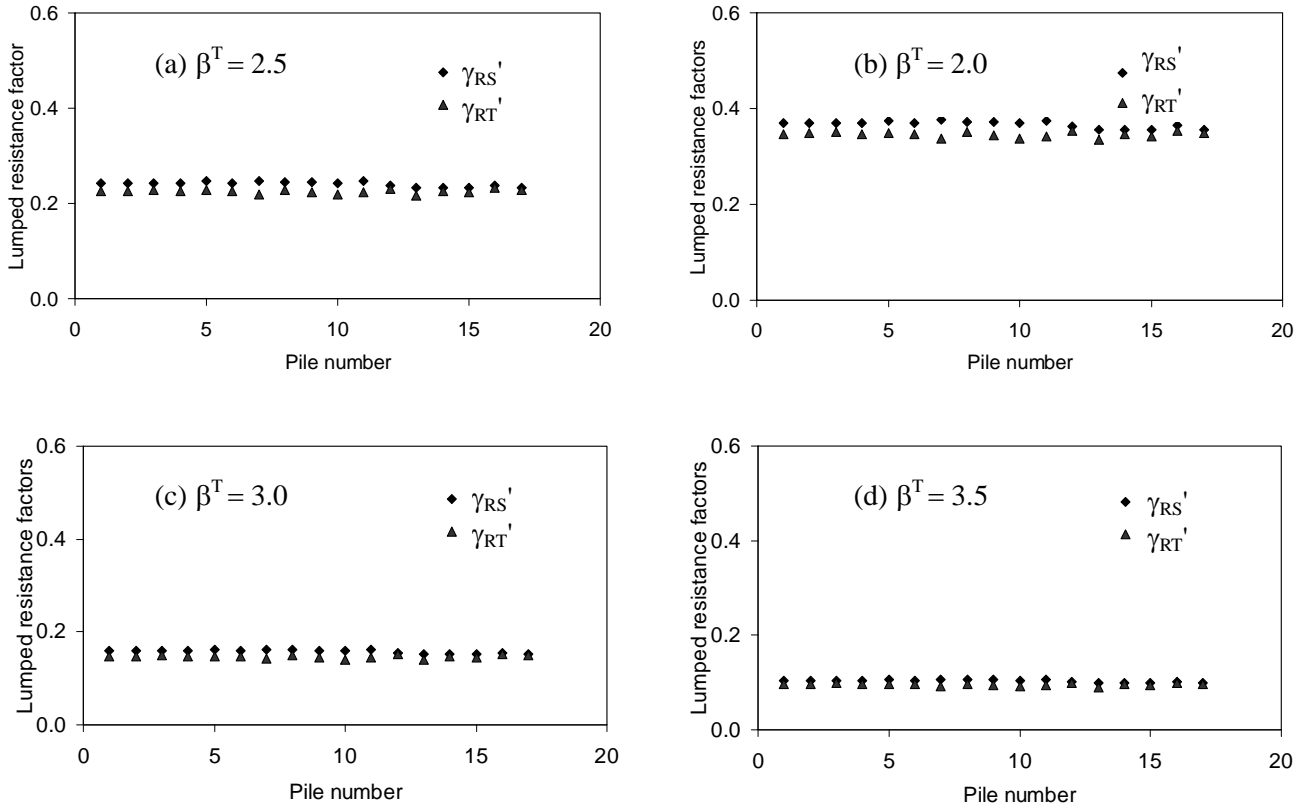


Fig. 4 Lumped resistance factors of the 17 piles (β^T : target reliability index)

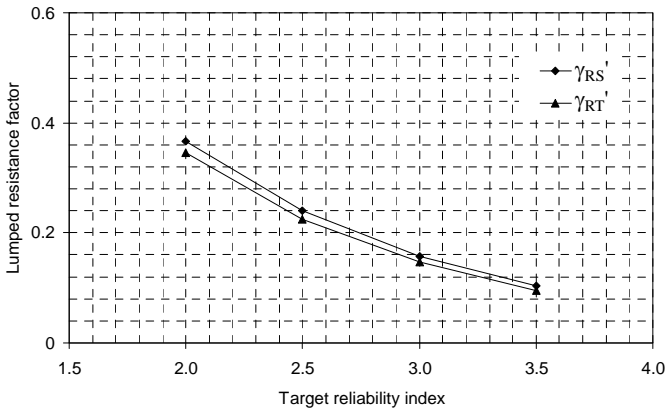


Fig. 5 Relationship between lumped resistance factors and target reliability index

implying that smaller resistance factors should be employed. Since the resistances factors in Fig. 5 are relatively low, it means that the effect of COV of the model correction factor is more obvious in the present study.

Note the resistance factors shown in Fig. 5 are developed based on the 17 piles in Table 1. These resistance factors are thus ideally suitable for a future design that has similar amount of parameter and model uncertainties as those of the 17 piles. If a pile is significantly different from those piles in the calibration database, one should use methods like FORM for site-specific reliability based design.

6. SUMMARY AND CONCLUSIONS

The research and results reported in this paper can be summarized as follows:

- (1) Two types of uncertainty exist in the design of large diameter bored piles, *i.e.*, uncertainty in model input parameters and model uncertainty. The parameter uncertainty cannot solely explain the disparity between predicted pile capacity and measured pile capacity. For a more realistic pile capacity prediction, the model uncertainty must be considered.
- (2) The model uncertainty can be characterized using pile load test data through a Bayesian method. Piles not failed during load tests have an important effect on model uncertainty characterization. Taking the maximum load as the capacity of a pile not loaded to failure may be a convenient assumption, but it underestimates both the mean and standard deviation of the model correction factor.
- (3) The design point method can be used to determine the resistance factors for pile design. To use load factors specified in the structural codes, the resistance factors determined from the design point method can be scaled such that the reliability level in a geotechnical design will be equal to or higher than the target reliability level. In this study, to use load factors specified in a structural code, the partial factors for resistance determined from the design point method are increased by about 30%, when the target reliability index is 2.5.

(4) As the partial factor for the model correction factor and the adjustment factor for resistance factors to comply with structural codes are similar for different piles, these factors can be lumped with resistance factors to reduce the number of partial factors. At a given target reliability level, the values of a lumped resistance factor vary little from one pile to another. For practical reliability based design, the mean value of the lumped resistance factor determined for various piles is suggested as the resistance factor for a future design. The lumped resistance factors depend heavily on the specified target reliability index. For instance, the lumped resistance factors are reduced by about one half when the target reliability index increases from 2.0 to 3.0. The lumped resistance factors suggested in this study is ideally suited for the design of a pile similar to those of the piles in the calibration database.

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APPENDIX

Consider the case

$$\frac{1.4}{\gamma_{SD}} \geq \frac{1.6}{\gamma_{SL}} \quad (\text{A1})$$

According to Eq. (13),

$$\eta = \frac{1.6}{\gamma_{SL}} \quad (\text{A2})$$

Then the following equation holds

$$\eta \gamma_{SL} S_{nL} + \eta \gamma_{SD} S_{nD} = \frac{1.6}{\gamma_{SL}} \gamma_{SL} S_{nL} + \frac{1.6}{\gamma_{SL}} \gamma_{SD} S_{nD} \quad (\text{A3})$$

According to Eq. (A1), the right hand side of Eq. (A3) can be written as

$$1.6 S_{nL} + \frac{1.6}{\gamma_{SL}} \gamma_{SD} S_{nD} \leq 1.6 S_{nL} + \frac{1.4}{\gamma_{SD}} \gamma_{SD} S_{nD} \quad (\text{A4})$$

Note the right hand side of Eq. (A6) in fact is

$$1.6 S_{nL} + \frac{1.4}{\gamma_{SD}} \gamma_{SD} S_{nD} = 1.6 S_{nL} + 1.4 S_{nD} \quad (\text{A5})$$

Based on Eqs. (A3) ~ (A5), the following inequality exists

$$\eta \gamma_{SL} S_{nL} + \eta \gamma_{SD} S_{nD} \leq 1.6 S_{nL} + 1.4 S_{nD} \quad (\text{A6})$$

Note both the terms at the left and right hand sides of Eq. (A6) may be used to calculate the desired amount of resistance. If the left hand side term is used, the design equation is the same as Eq. (11). In such a case, the designed reliability index of the system is the same as the target reliability index. On the other hand,

if the right hand side term is used, which is the case of Eq. (13), the calculated resistance would not be less than the case where the left hand side term in Eq. (A6) is used for calculation of resistance. This means the resistance in the design would not be less than what is required to achieve the target reliability index. Therefore, if Eq. (A1) is used for design, the reliability index of the design would be equal to or higher than the target reliability index.

Following the reasoning shown above, it can be shown that when η defined in Eq. (13) is used, the reliability level in a design will also not be lower than the target reliability level in the following case

$$\frac{1.4}{\gamma_{SD}} < \frac{1.6}{\gamma_{SL}} \quad (\text{A7})$$

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