

SHEAR WAVE VELOCITY AND SUCTION OF UNSATURATED SOIL USING BENDER ELEMENT AND FILTER PAPER METHOD

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ABSTRACT

A study was conducted to determine the dynamic properties of compacted clayey soil with matric suction measurement. The effects of matric suction of unsaturated residual lateritic soil on small-strain shear wave velocity and small-strain shear modulus were investigated. Soil specimens were compacted at optimum moisture content, wetted to various moisture contents, then, tested for small-strain shear wave velocity and matric suction using the bender element and the filter paper method, respectively. Test results demonstrated that the small-strain shear wave velocity and small-strain shear modulus decrease with increasing moisture content and decreasing matric suction. Test results also indicated that when the degree of saturation changes slightly, although matric suction changes greatly, soil can be regarded as to have a nearly consistent soil skeleton. Hence, reductions of shear wave velocity and shear modulus are limited.

Key words: Shear wave velocity, unsaturated soil, matric suction, bender element, filter paper method.

1. INTRODUCTION

Compacted soils are commonly used in geotechnical engineering, such as buffer materials in radioactive waste repositories, embankments, dams, dikes, levees, liners, *etc.* Dynamic soil properties, such as shear wave velocity V_s , shear modulus G , and material damping ratio D , of compacted soils need to be known to determine the engineering behavior under dynamic loading of different origin including vibrations from machine foundations, pile driving, traffic, earthquakes, conventional blasting operations, wind and wave loads, *etc.* Shear modulus usually varies with shear strain level. However, as shear strain level is very low, shear strain has slight effect on shear modulus. Consequently, under this condition shear modulus can be defined as small-strain shear modulus, G_{\max} . Hence, small-strain shear modulus can be treated as a special case of shear modulus. The value of G_{\max} can be calculated as:

$$G_{\max} = \rho \times V_s^2 \quad (1)$$

where ρ is the total density and V_s is the shear wave velocity of the soil mass.

From Eq. (1), shear wave velocity governs shear modulus. Sheeran, *et al.*, (1967) studied the shear wave velocity of com-

pacted unsaturated soils. They correlated shear wave velocity of compacted clayey soils to compaction water content. At constant water content, the shear wave velocity increased with dry density until a maximum was reached and then the velocity decreased sharply when dry density continuously increased. Besides, compacted soils are usually unsaturated during the entire life of the facility, and thus matric suction is an important property of compacted soils. Several studies proved that mechanical and hydraulic behaviors of compacted soils are influenced by matric suction (Alonso, *et al.*, 1999; Yang, *et al.*, 2005; Kung, *et al.*, 2006; Yang, *et al.*, 2006). However, does matric suction have influence on dynamic properties of compacted soils?

Qian, *et al.*, (1991) studied thirteen sands for the resonant column tests to investigate dynamic properties under unsaturated conditions. They indicated that capillary stresses significantly increased the shear modulus of unsaturated sands. According to Inci, *et al.*, (2003), the shear wave velocity increases from 300 m/s to 800 m/s as matric suction increases from 300 kPa to 15,000 kPa for soils compacted at optimum moisture content and then experienced drying path. Besides, the shear modulus could be correlated to soil suction. In their study, it was observed that if the soil is cohesive or well-graded, the relationship between shear modulus and the suction is proportional.

The main purpose of this study is to comprehend the effects of matric suction on the small-strain shear wave velocity and small-strain shear modulus for unsaturated residual lateritic soil. Facilities to measure the shear wave velocity and the matric suction of unsaturated lateritic soils were set up. Soil specimens were compacted at optimum moisture content, wetted to various moisture contents, and then tested for small-strain shear wave velocity. The small-strain shear wave velocity was measured using bender elements. The results were then used to deduce the small-strain shear modulus using Eq. (1). Soil specimens were allowed to wet subsequent to compaction to investigate the effects of water content and degree of saturation on small-strain shear wave velocity and small-strain shear modulus. Matric suc-

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tion was measured by the filter paper method after the small-strain shear wave velocity test. The effects of matric suction on the dynamic properties, small-strain shear wave velocity and small-strain shear modulus, of the soil were also assessed.

2. SOIL AND SAMPLE PREPARATION

2.1 Soil Basic Properties

The soil, residual lateritic soil, used in this study was taken from Linkuo located at about 20 km south of Taipei city in Taiwan. This soil is cohesive, dark brown in color, with natural moisture content 31% ~ 37%, Sand ($> 74 \mu\text{m}$) 10%, silt ($74 \mu\text{m} \sim 2 \mu\text{m}$) 30%, clay ($< 2 \mu\text{m}$) 60%, Modified Proctor optimum moisture content (OMC) 19.5%, maximum dry density (MDD) 1.69 Mg/m^3 , specific gravity (G_s) 2.68, liquid limit (LL) 49%, and plasticity index (PI) 23%, activity (A_c) 0.38. The soil is classified as A-7-6 and CL according to AASHTO and USCS, respectively.

2.2 Soil-Water Characteristic Curves

The soil-water characteristic curve (SWCC) defines the relationship between water content, volumetric water content or degree of saturation versus suction. Many methods can be used to obtain the SWCC, including pressure plate method, salt solution method, and filter paper method. In this study, SWCC measurement was performed using pressure-plate apparatus, and the soil specimen was subjected to an increasing value of matric suction in accordance with ASTM D 2325-68.

During the testing process, the air pressure in pressure-plate apparatus increased, expelled water out of the soil specimen, and consequently increased matric suction. The weight of soil specimen was measured every day to determine the water content in soil at equilibrium. In other words, the matric suction maintained as the equilibrium condition was reached. Then the matric suction was increased to another predetermined level and similar procedure was performed until matric suction reached equilibrium again. In this study, matric suction was staged up to the limit of 1,400 kPa of the pressure-plate apparatus used. At the end of the test, the soil specimen was oven-dried to determine its water content.

Figure 1 plots the SWCC for the tested soil compacted at optimum moisture content. It represents the relationship between the matric suction and the water content along the drying path. Figure 1 shows that the test results exhibit a typical SWCC behavior of clayey soil. As the water content decreases, the matric suction increases. At high water content the concave-down curve indicates small changes in moisture content at around saturated conditions and at low matric suction value smaller than 10 kPa. At low volumetric water content (matric suction value greater than 10 kPa) the more rapidly decreasing water content with a nearly constant rate occurs. Besides, from Fig. 1 it can be known that the air entry value is about 40 kPa.

2.3 Sample Preparation

Modified Proctor compaction tests were performed on the tested soil. Soil was compacted at optimum moisture content. The amount of water, by weight of dry soil, was calculated from the pre-selected moisture content. Air-dried soil was then thoroughly mixed with the required amount of water to ensure homogeneity. The fully mixed soil was then compacted in five equal layers into a 10 cm diameter and 12 cm height split miter box. Each layer was compacted using a 44.5 N, 45.7 cm drop, U.S. Army Corps hammer with 25 uniformly distributed blows, corresponding to a compactive effort of $2,695 \text{ kJ/m}^3$. After compaction (Fig. 2(a)), the specimen was cut off the side, top, and bottom portions of the sample to a 5 cm diameter and 10 cm height (Fig. 2(b) and (c)), and then dug a striated notch for setting bender element prior to shear modulus test (Fig. 2(d)).

2.4 Simulation of the In-Service Moisture Content

To simulate the in-service moisture content, this study developed a sample wetting procedure and an environment simulation room to enhance the sample wetting process. Specimens were compacted at optimum moisture content (simulation of construction phase), and then wetted to different moisture contents (simulation of in-service phase). To prevent soil specimen from lateral expanding during wetting, the specimen was constrained by a mold, and a surcharge weight of 2.5 kg was placed on top of the upper spacer to simulate the overburden (Fig. 2(e)). Then the entire set-up (with the surcharge weight on top) was placed in the environment simulation room (Fig. 2(f)) and conditioned in moist air with 100 percent relative humidity.

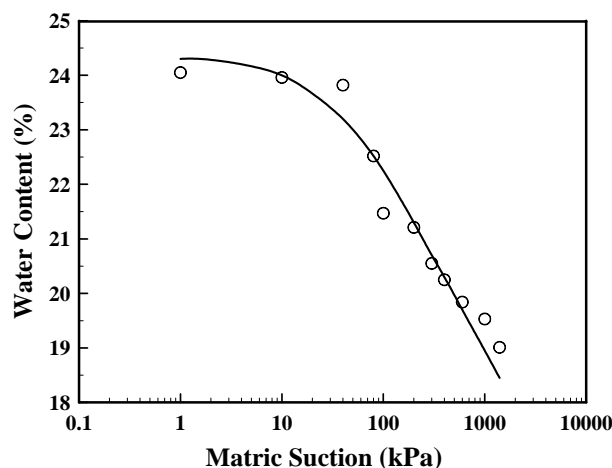


Fig. 1 SWCC of the tested soil compacted at optimum moisture content

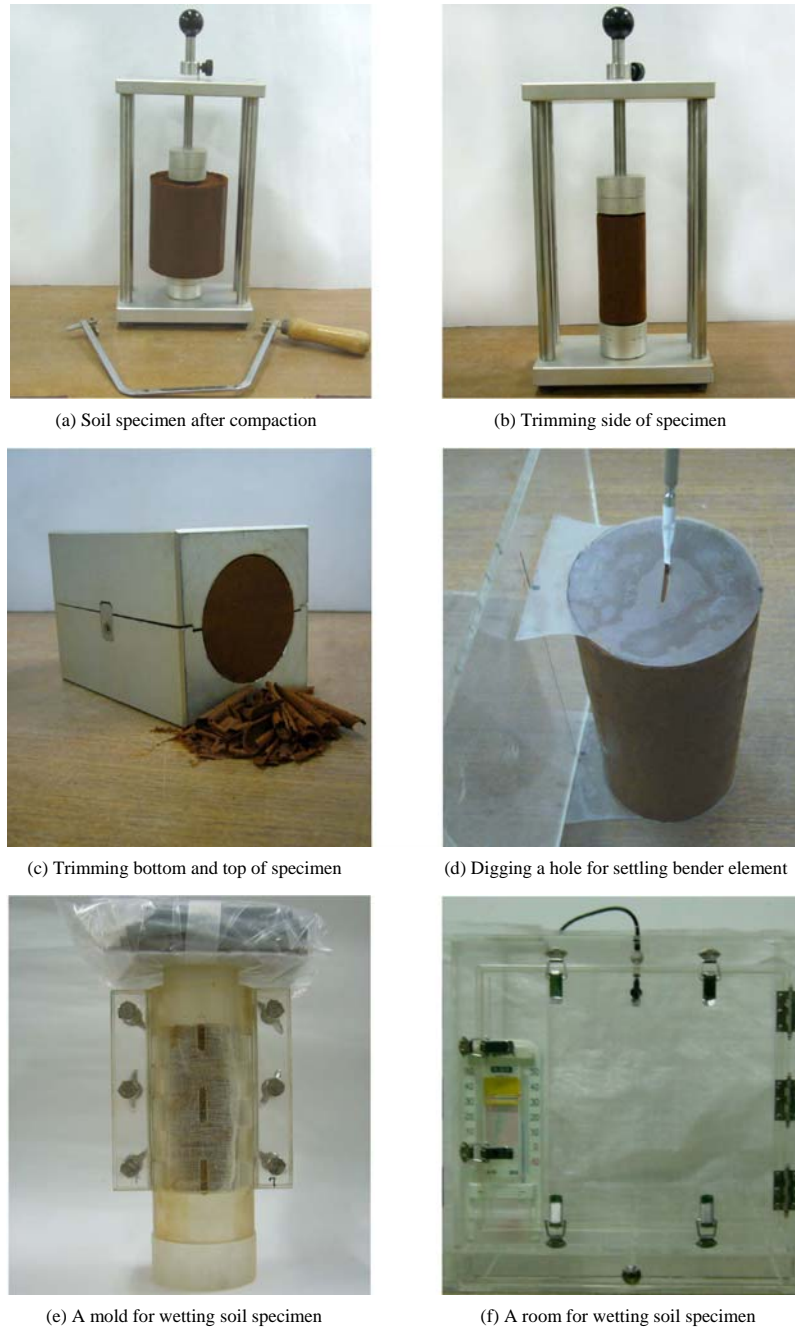


Fig. 2 Photographic view showing sample preparation

3. TESTING PROGRAM

To comprehend the influence of matric suction on small-strain shear modulus, the apparatus of the small-strain shear wave velocity test and the filter paper method were integrated to develop a systematic testing procedure. The representative soil specimens were compacted at optimum moisture content. The specimens were wetted in the environment simulation room subsequent to compaction to different moisture contents under different wetting time. After wetting the specimens to a pre-assigned state, the small-strain shear wave velocity test was first conducted using the bender element method, and then, the filter paper method was used to measure the matric suction. The small-strain shear wave velocity test and the filter paper method are briefly described below.

3.1 Small-Strain Shear Wave Velocity Measurement

The bender element tests were performed after wetting soil specimens. The measured small-strain shear wave velocity (V_s) was used to calculate the G_{max} . The bender element test setup includes three components: bender element, function generator, and oscilloscope. In this study, the bender element composed by two piezoelectric ceramics has thickness of 1 mm, width of 12 mm, and height of 10 mm. It was coated with epoxy glue for waterproofing as shown in Fig. 3. Besides, the use of the bender elements requires a high definition oscilloscope and an electronic function generator. In this study, the high definition oscilloscope is a Nicolet Type Inergra-10 with a maximum sampling rate of 1 MS/s in each channel and a definition of 12 bit. A sine pulse

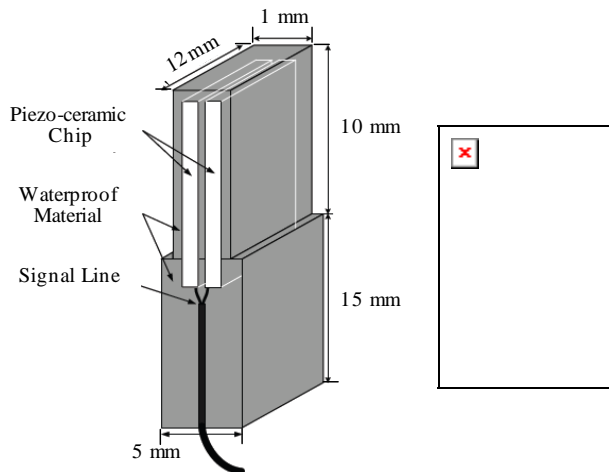


Fig. 3 Bender element sketch for shear wave velocity measurement

generated by a TECH FG-506 with 6 MHz pulse and ± 10 voltage function generator is used to activate excitation voltage (transmitters). The data recorded by the digital oscilloscope can be transferred to a computer for further signal processing.

Prior to the small-strain shear wave velocity test, the specimens were dug a striated notch using a razor blade having the same thickness like that of the piezoelectric ceramics. The bender elements were then set into the striated notches on top and bottom of the specimens, and the interval is filled with soil so that intimate contact between the bender element and the surrounding soil can be assured.

The small-strain shear wave velocity of a soil specimen can be measured by applying a voltage pulse to one of the bender elements and recording the elapsed time until the other element registers the arrival signal. The shear wave arrival time was obtained from oscilloscope readings. There are several methods of interpreting the travel time (V_s) of the waves in the soil, based on the transmitted and received signals. The travel time can be determined by the time between corresponding peaks or troughs of the transmitter and receiver signals and used to compute the shear wave velocity by using Eq. (2).

$$V_s = \frac{L_t}{t} \quad (2)$$

where L_t is the distance between the transmitter and receiver bender elements (tip-to-tip) and t is the travel time of the signal from the transmitter to the receiver.

Time histories of the transmitted signal and received signal for one specimen compacted at optimum moisture content are shown in Fig. 4. The small-strain shear modulus is related to the small-strain shear wave velocity and the density of the soil through Eq. (1).

3.2 Soil Suction Measurement

Filter paper method is specified in ASTM D 5298-94 (2000), describing calibration and test procedures for measuring matric suction (contact method) and total suction (noncontact method). Both the contact and noncontact methods determine suction of

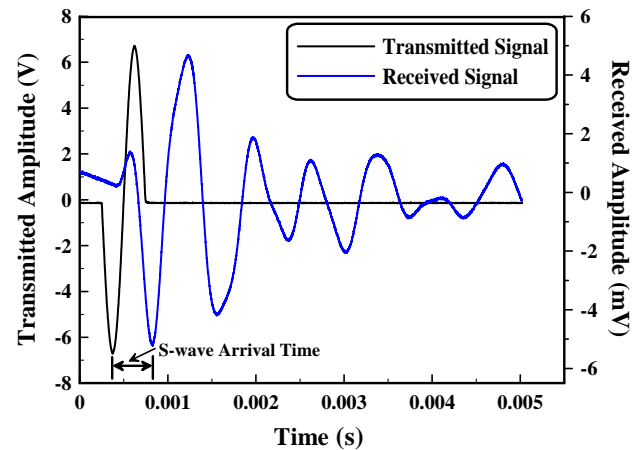
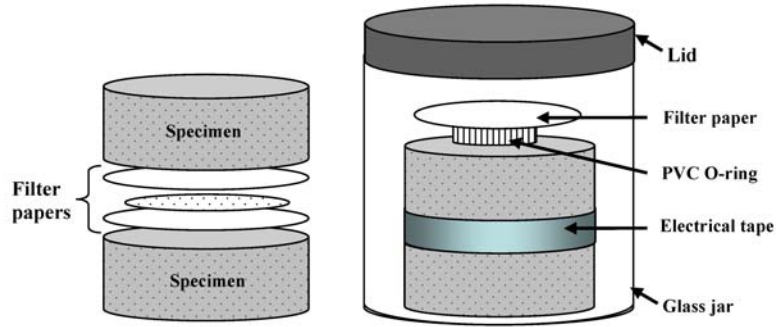


Fig. 4 Shear wave arrival time for a sample at optimum moisture content

unsaturated soils indirectly by measuring the amount of moisture transferred from soil mass to dry filter papers. Figure 5 illustrates the testing setups for total and matric suction measurements. In this study because only matric suction was measured after the shear wave velocity test, the contact method will be described below.

Filter papers, Whatman No. 42, ash-free quantitative Type II with a diameter of 5.5 cm, were used in this study as indirect sensors to measure the matric suction of the soil specimens. One filter paper is sandwiched between two sacrificial filter papers used to protect the central filter paper from soil fouling or contamination, as shown in Fig. 5(a). To measure matric suction of compacted unsaturated soil, the soil specimens were compacted and cut into two layers. Each specimen has 80 ~ 100 g of moist soil mass nearly filled the containers. Next, the stack of three filter papers (composed of a filter paper sandwiched between two protective filter papers) were placed at the center of the compacted two specimens and then put into a specimen container in 150 mL. The moisture content of filter papers was equilibrated with the compacted soil specimens. Besides, because a good contact between the filter papers and the compacted specimens needs to be satisfied, the flat surface of specimens is required.

During cutting soil specimens, the remaining soil was weighted to calculate the moisture content of specimens. The container was immediately closed with an airtight lid and was put into an airtight box, and the latter, in turn, was kept in a temperature-controlled cabin. Then, the filter papers and the soil specimen were allowed to equilibrate. ASTM specification requires a 7-day equilibration period. In this study a 10-day period is sufficient for allowing the filter paper-soil to reach equilibrium. After 10 days of equilibration, the moist central filter paper was removed from the specimen container and put into a pre-weighed metal container capable of an airtight seal. The metal container was then weighted by electronic balance with 0.0001 g accuracy. After measuring the wet mass of the central filter paper, the moist central filter paper was oven-dried at 110°C for 24 hours. The difference between the wet and dry mass of the filter paper gave the equilibrium filter paper water content (ω_{fp}). The equilibrium filter paper water content of a compacted specimen



(a) Filter paper method for measuring total and matric suction



(b) A well-insulated container for suction equilibrium



(c) Monitoring temperature of container during suction equilibrium



(d) Using a balance at least to the nearest 0.1 mg accuracy



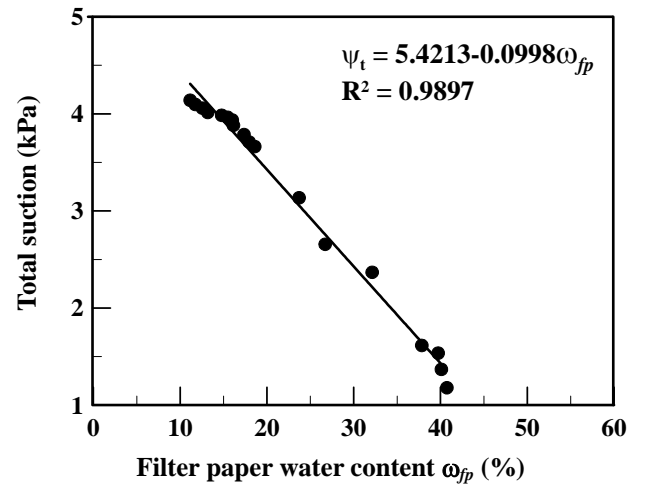
(e) Soil suction measurement set up

Fig. 5 Filter paper method configuration for soil suction measurement

was converted to matric suction values by calibration curve. The filter papers are calibrated by determining the relationship between equilibrium filter paper water content (ω_{fp}) and vapor-phase relative humidity, as shown in Fig. 6. Salt solutions of known concentrations were used to control relative humidity. In this study, NaCl solution was used for calibration. More details on the calibration of filter papers can refer to ASTM D 5298-94. The following calibration equation, based on the calibration data, was used:

$$\psi_t = 5.467 - 0.1094 \omega_{fp} \quad R^2 = 0.982 \quad (3)$$

where ψ_t is total suction (log kPa) and ω_{fp} is the filter paper water content (%).

**Fig. 6 Calibration curve of Whatman No. 42 filter paper**

4. INFLUENCES OF MATRIC SUCTION ON SHEAR MODULUS

4.1 Moisture Content of Specimens after Wetting Process

Table 1 presents the moisture contents and matric suctions of soil specimens before and after wetting. Most soil specimens were compacted at near optimum moisture content and then wetted to different moisture content under different wetting time except the specimens of No. 1 and No. 2. These two specimens were tested without wetting for comparison. From Table 1, the initial moisture contents of soil specimens are between 18.80% and 19.32%; the moisture contents of soil specimens after wetting are between 19.60% and 20.53%. For specimens (No. 2 and No. 5) having the same initial moisture content of 19.32%, when the specimen is wetted to moisture content of 20.12%, the matric suction measured by the filter paper method decreases significantly from 610 kPa to 330 kPa. If the soil is compacted at dry of optimum and then wetted, the matric suction will decrease more obviously.

Test results show that the wetting progress has influence on the matric suction even though the variations in moisture content are limited. Compared with the matric suctions deduced from the SWCC performed using pressure-plate apparatus under drying progress, the matric suctions are lower than those at the same moisture content. It is expected that the matric suctions obtained under drying progress are higher than those obtained under wetting progress. This is because hysteresis in the relationship between the water content and matric suction occurs between the wetting and drying processes. Therefore, the measured results are qualitatively in good agreement with the SWCC.

Table 1 Moisture contents and matric suctions of soil specimens after wetting

Specimen No.	1	2	3	4	5	6	7
Moisture content ¹ (%)	19.12	19.32	18.93	18.80	19.32	18.81	19.02
Moisture content ² (%)	—	—	19.60	20.11	20.12	20.31	20.53
Matric suction ³ (kPa)	790	610	439	343	330	97	194
Matric suction ⁴ (kPa)	1315	1160	910	470	470	380	300

¹ Moisture content after compaction

² Moisture content after wetting

³ Matric suction measured by filter paper method after wetting

⁴ Matric suction corresponding to drying part of SWCC

4.2 Variation of Shear Wave Velocity under Different Wetting Moisture Content

Figure 7 plots the relationship between the small-strain shear wave velocity (S-wave velocity) and the moisture content. Figure 8 plots the relationship between the small-strain S-wave velocity and the degree of saturation. Figures 7 and 8 show that the small-strain S-wave velocity decreases with increasing moisture content and degree of saturation. The reason probably can be attributed to the pore water in soil and soil skeleton. It has been pointed out that wave propagation through saturated soils involves the soil skeleton and water in the void spaces. For S-wave propagation, the pore water has no rigidity to shear. The S-wave

in soils is dependent only on the properties of the soil skeleton (Das, 1983). Similar principle can be applied to unsaturated soils, the abilities to resist shearing decrease when moisture content in soils increases, because pore water that have no rigidity to shear increases. If soils experience significant change in soil skeleton, usually accompanied by significant change in moisture content, the S-wave velocity is expected to exhibit larger changes. On the other hand, when the degree of saturation changes slightly, soils maintain nearly consistent soil skeleton. Hence, the reduction of small-strain S-wave velocity is limited.

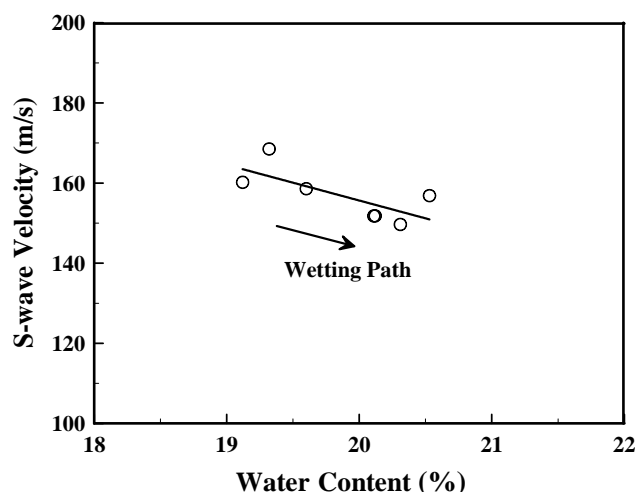


Fig. 7 Variation of S-wave velocity with water content

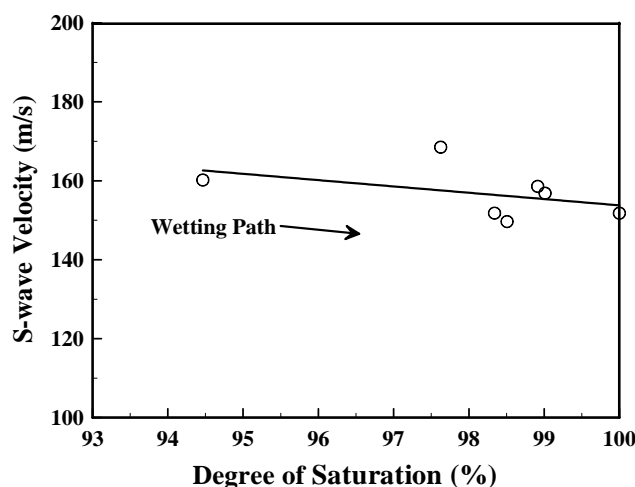


Fig. 8 Variation of S-wave velocity with degree of saturation

Figure 9 presents the relationship between the small-strain S-wave velocity and the matric suction. From Fig. 9 it is observed that the small-strain S-wave velocity increases, when the matric suction increases from 100 kPa to 800 kPa. The fact that small-strain S-wave velocity increases with increasing matric suction can be attributed to the reason that high matric suction produces a stiffening effect on the soil specimens due to increased rigidity of the soil skeleton. For most soils, the range of this suction change has great influences on soil's mechanical behavior. However, in this case, the variation of the small-strain S-wave velocity is only about 10%, when the matric suction ranges from 100 kPa to 800 kPa. This is somewhat unexpected and may be explained by the slight change in the degree of satu-

ration. From Fig. 8 it shows that the degree of saturation of the soil compacted at optimum moisture content has a very high value of 94%. After wetting to the moisture content of 20.64%, the degree of saturation is about 100%; consequently, the soil can be regarded as to have similar soil skeleton before and after wetting. Under such conditions the soil skeleton change may not be significantly enough to induce large change in the small-strain S-wave velocity. However, the behavior may be totally different for soil specimens compacted at dry of optimum. It can be inferred that if the soil is compacted at dry of optimum and then wetted, the small-strain S-wave velocity might change obviously. This is because soil skeleton changes from flocculated structure to dispersed structure (Das, 2006). On the contrary, if the soil is compacted at wet of optimum, the small-strain S-wave velocity might change even more unobvious than that compacted at optimum moisture content. Clearly, further studies are needed to verify the above speculation.

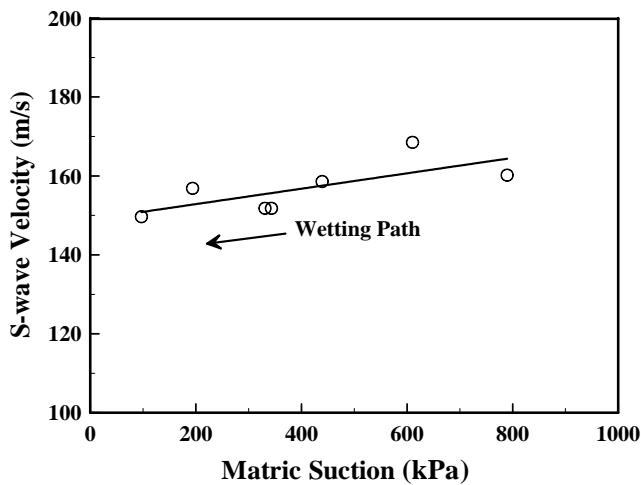


Fig. 9 Variation of S-wave velocity with matric suction

4.3 Relationship between Matric Suction and Shear Modulus

To evaluate the effect of increased moisture content on the small-strain shear modulus of the soil, the soil specimens were prepared with different moisture contents by the aforementioned wetting procedures and were subjected to the small-strain S-wave velocity test using the bender element. The small-strain S-wave velocity test results were then used to deduce the small-strain shear modulus using Eq. (1). Figure 10 presents the relationship between the small-strain shear modulus and the moisture content. Figure 11 presents the relationship between the small-strain shear modulus and the degree of saturation. As expected, the small-strain shear modulus decreases with increasing moisture content and degree of saturation, since the small-strain S-wave velocity decreases with increasing moisture content and degree of saturation.

To confirm the validity of the test method, the test results were compared with others in the literatures. According to Das (1983), the small-strain shear wave velocity of the silty clay is 135 m/sec. In this study, the small-strain shear wave velocity measured using the bender element is between 150 and 170 m/sec. Compared with the small-strain shear wave velocity for the clayey soil in the literature (Das, 1983), the small-strain shear wave velocity measured in this study seems reasonable. On the

other hand, the degree of saturation of the specimens is all above 94%. Inci, *et al.* (2003) investigated the small-strain shear modulus of compacted clayey soils. Their test results show that the small-strain shear modulus is below 100 MPa as the degree of saturation is above 90%. From above discussions, the test results are reasonable against experimental data conducted by the theoretical equation as well as by other data available in the literature.

Figure 12 shows the relationship between the small-strain shear modulus and the matric suction. Tested results indicate that the small-strain shear modulus decreases with decreasing matric suction. Inci, *et al.* (2003) also found the same behavior. Figure 12 also shows that the small-strain shear modulus and matric suction have high linear relationship. Besides, test results indicate that the small-strain shear modulus decreases more notably than the small-strain S-wave velocity. Reduction in small-strain shear modulus is about 20%, when matric suction increases from 100 kPa to 800 kPa. It is twice as much as the reduction in small-strain S-wave velocity due to same amount of matric suction change. From Eq. (1), the small-strain S-wave velocity has a power relation with the shear modulus. Hence, the matric suction has more influence on small-strain shear modulus than small-strain S-wave velocity, as would be expected.

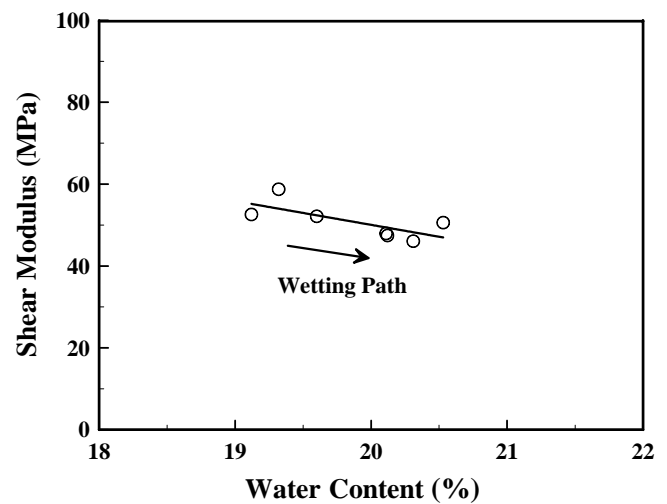


Fig. 10 Variation of shear modulus with water content

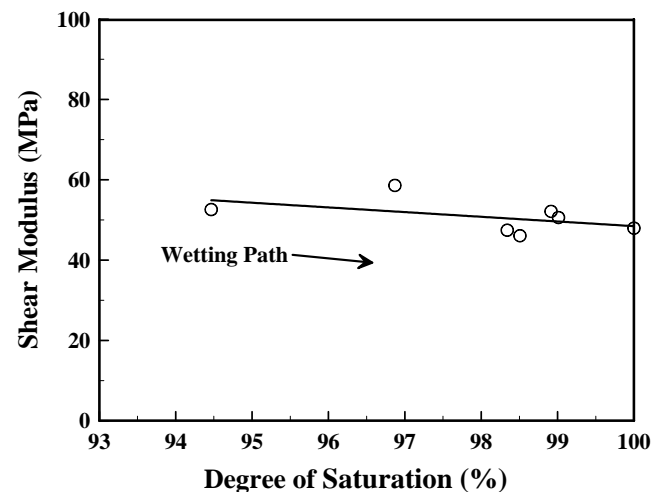


Fig. 11 Variation of shear modulus with degree of saturation

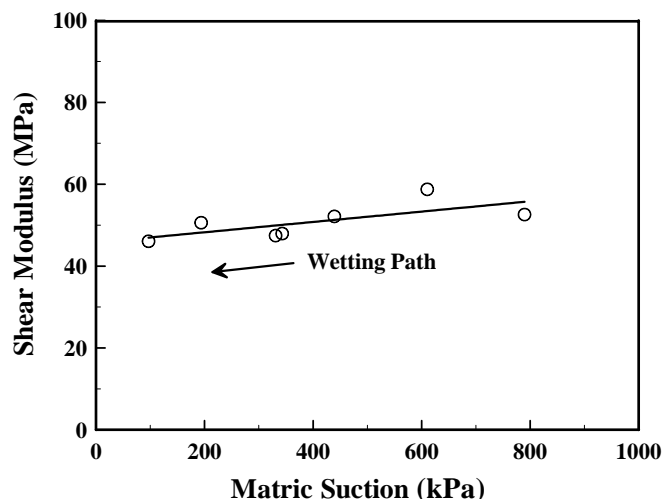


Fig. 12 Variation of shear modulus with matric suction

5. CONCLUSIONS

In this study, experimental test results of a residual lateritic soil were investigated to study the small-strain S-wave velocity and small-strain shear modulus behaviors considering matric suction influences. The soil were compacted at optimum moisture content to simulate the construction moisture content, and then wetted to simulate the in-service moisture content. A bender element testing system and a filter paper method were successfully integrated for the determination of the shear modulus of unsaturated soil.

Test results indicated that increase in moisture content is limited after wetting for the soil compacted at optimum moisture content. However, under this circumstance matric suction has large decrease from 800 kPa to 100 kPa. Laboratory data also indicate that decrease in matric suction results in decreases in small-strain S-wave velocity and small-strain shear modulus. However, the amount of decrease is not proportional to the amount of change in soil suction. This is because under the studied test conditions the degree of saturation changes only slightly even though the change in suction is more pronounced; consequently, the soil can be regarded as to have a nearly consistent soil skeleton. Hence, the reductions of small-strain shear wave velocity and small-strain shear modulus are limited. Besides, reduction in shear modulus is about twice as much as the reduction in shear wave velocity at the same change range in matric suction. The behavior of suction-induced change in small-strain shear wave velocity may be different from what have been observed herewith if the soil are compacted at different suction and moisture states. Further studies are warranted for clarification.

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