

THE EFFECTS OF A SETBACK ON THE BEARING CAPACITY OF A SURFACE FOOTING NEAR A SLOPE

Ching-Chuan Huang¹ and Wen-Wei Kang²

ABSTRACT

The effects of setback on the ultimate bearing capacity of rigid surface footings placed on the crest of the slope are evaluated using a limit-equilibrium-based method. It is shown that the values of the bearing capacity coefficient or correction factor for the ultimate bearing capacity of a footing placed on the crest of a slope can be expressed as linear functions of the setback-to-footing-width ratio up to a certain threshold value namely, $(b/B)_t$. Beyond this threshold value, the ultimate bearing capacity is the same as those obtained for a similar footing placed on semi-infinite level ground. The value of $(b/B)_t$ can be uniquely related to the internal friction angle of the foundation soil, ϕ , regardless of the change in the slope angle, α . Good agreements are obtained between the value of correction factors obtained herein and those reported in the literature. It is also shown that a conventional indirect approach which deals with the effects of a setback using an equivalent surcharge on the slope surface also works well in the sense that the indirect approach generate small errors in the correction factors when compared to those obtained using the direct approach such as the one proposed in the present study.

Key words: Bearing capacity, surface footing, slope, limit equilibrium method.

1. INTRODUCTION

A simple and straightforward way of mitigating possible disasters induced by the bearing capacity failure of a shallow footing located near a slope is to increase the setback of the footing (b) as schematically shown in Fig. 1. Regardless of the importance of setback to practical applications, research into this aspect has been very limited (*e.g.*, Meyerhof, 1957; Graham, 1988). One of the reasons that analytical solutions for the ultimate bearing capacity taking into account the setback of footing (expressed by a setback distance ' b ') has rarely been provided is schematically shown in Fig. 2. That is, the effect of setback has conventionally only been indirectly considered by using the effect of surcharge, namely, $\gamma \cdot D_f \cdot N_q$ (γ : unit weight of soil, D_f : depth of overburden soil, N_q : Terzaghi's bearing capacity coefficient for surcharge). In such cases, the ultimate bearing capacity of footing placed on the crest of a slope with a slope angle ' α ' namely, $q_{u(\alpha>0, b>0)}$ can be expressed using Terzaghi's bearing capacity formula (Terzaghi, 1943):

$$q_{u(\alpha>0, b>0)} = \frac{1}{2} \cdot B \cdot \gamma \cdot N_{\gamma(\alpha>0, b=0)} + \gamma \cdot D_f \cdot N_{q(\alpha>0, b=0)} \quad (1)$$

in which

B : width of footing

$N_{\gamma(\alpha>0, b=0)}$: bearing capacity coefficient due to self-weight of soils for a footing adjacent to the slope with a slope angle α .

$N_{q(\alpha>0, b=0)}$: bearing capacity coefficient due to surcharge for a footing adjacent to the slope with a slope angle α .

D_f : virtual depth of footing comparable to the footing with a setback. ($D_f = b \cdot \tan \alpha$ is used in the following).

However, this indirect approach has some shortcomings. First, the surcharge is non-uniform adjacent to the toe of the footing as illustrated by the shaded area in Fig. 2, and the effect

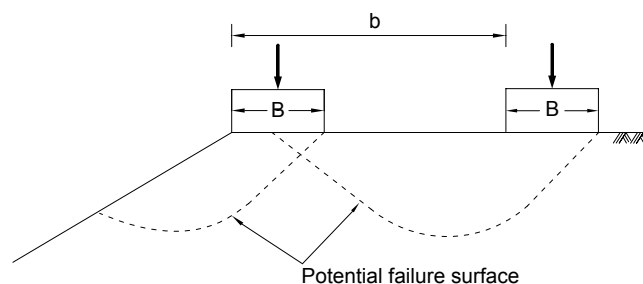


Fig. 1 Schematic figure showing the effect of setback on the failure mechanism of footings placed on the crest of the slope

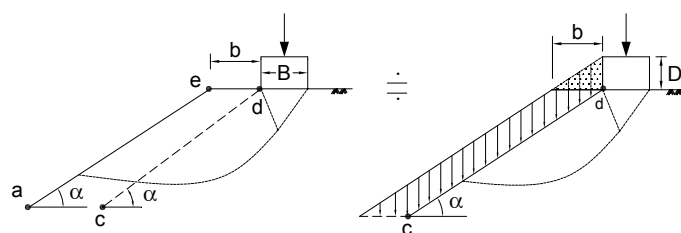


Fig. 2 Schematic figure showing the approximation of a footing with a setback using a footing with a uniform surcharge

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¹ Professor (corresponding author), Department of Civil Engineering, National Cheng Kung University, Tainan, Taiwan 70101, R.O.C. (e-mail: samhcc@mail.ncku.edu.tw).

² Graduate student, Department of Civil Engineering, National Cheng Kung University, Tainan, Taiwan 70101, R.O.C.

of non-uniformity of surcharge increases when the distance of setback 'b' increases. Second, the crust of the soil that provides overburden pressure to the slope surface is subjected to possible slope instability similar to that of an infinite slope, *i.e.*, a driving force is acting on the interface c-d (see Fig. 2) which is not accounted for in Eq. (1). A possible way to eliminate the above drawbacks is to use a direct approach to evaluate the effect of a setback of the footing on the bearing capacity coefficient, N_γ using a correction factor, η_b , as discussed below. Various correction factors have been successfully used in foundation engineering practices in evaluating the effects of slope inclinations, load inclinations and load eccentricities (Meyerhof, 1963; Hansen, 1970; Vesic, 1973). To the best knowledge of the authors, a simplified equation expressing the correction factor η_b is yet to be developed.

2. ANALYTICAL METHODS AND VERIFICATION

Figure 3 schematically shows the failure mechanism assumed in the present study, which consists of a triangular active wedge under the footing with a width 'B', a transitional zone bounded by a logarithmic failure line (defined by the following equation) and a passive zone.

$$r = r_o \cdot e^{\mu \cdot \tan \phi} \tag{2}$$

where r : radius from the toe of the footing
 r_o : radius at the interface between the active wedge and the transitional zone
 μ : angle between r_o and r .

In the case of gentle slopes, a passive zone may be fully or partially included in the failure mechanism, depending on the relative position of the slope and footings. An example is shown by the dotted lines in Fig. 3. The failure mechanism shown in Fig. 3 has been verified by Huang and Tatsuoka (1994) based on the model test results reported by Huang, *et al.* (1994). The zone bounded by the failure mechanism illustrated in Fig. 3 is divided vertically into slices with a width of 10 mm. It was found that slices with this width consistently provide accurate results in the sense that further reducing the slice width gives no improvement for the calculated values of q_u . Janbu's rigorous slice method (Janbu, 1973) which satisfies force and moment equilibria is used in the present study. It has been shown that this method is as accurate as other rigorous methods, such as Spencer's and Morgenstern and Price's slice methods (Fredlund and Krahn, 1977) and

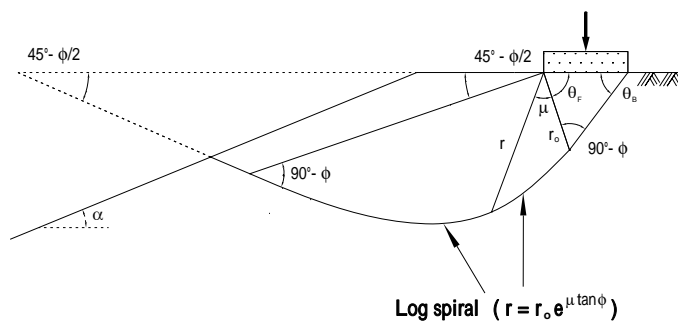


Fig. 3 Failure mechanism used in the present study

the generalized variational method (Leshchinsky and Huang, 1992). The equilibrium formulation of the slice method and procedure for deriving ultimate bearing capacity (coefficients) of footings are described in Appendix A. Formulations and verifications of this method have been described in-detail by Huang, *et al.* (1994). In the present study, and also in the study of Huang, *et al.* (1994), a minimum value of vertical footing load is searched via an optimization of θ_B and μ , *i.e.*, via a systematic search of critical failure mechanisms. The ultimate bearing capacity of footing, q_u is then calculated based on Eqs. (A1) through (A3). The bearing capacity coefficients for a footing adjacent to a slope ($\alpha > 0$) with or without a set back, namely, $N_{\gamma(\alpha>0, b=0)}$ or $N_{\gamma(\alpha>0, b>0)}$, can be calculated using Eq. (A4).

Note that in Huang's analyses, the exponential function in Eq. (2) is replaced by ' $e^{\mu \cdot \tan \eta}$ ' and the value of ' η ' is determined via a trial-and-error procedure searching for an optimized value of ' η ' that generates a minimum value of N_γ . Huang and Tatsuoka (1994) found that the optimized value of η is only slightly different from the internal friction angle of soil ' ϕ '. Therefore, the difference in the value of N_γ when using ' $e^{\mu \cdot \tan \phi}$ ' or ' $e^{\mu \cdot \tan \eta}$ ' in Eq. (2) is small, as shown in Fig. 4. This figure also shows that the value of N_γ obtained using the present method is close to the lower bound of various theoretical solutions compiled by Graham, *et al.* (1988). As has been examined by Huang and Tatsuoka (1994) that the assumption of the thrust height of interslice force at 1/3 of the side-face of the slice generates acceptable interslice and slice base forces in the sense that positive interslice and slice base forces are obtained along the entire critical failure surface.

Figure 5 compares the analytical values of N_γ for a rigid footing placed on the slope with various slope angles ($\alpha = 20^\circ, 30^\circ$, and 40°) and setbacks ($b/B \geq 0$). Comparable results on the N_γ vs. b/B relationship showing the transition of N_γ from $b/B = 0$ into $b/B = \infty$ (a case of footing placed on a level ground) can be obtained. All these curves plateau when the value of b/B is beyond the threshold value of b/B , namely $(b/B)_t$, indicating the diminishing of the influence of footing setback on the ultimate bearing capacity of footing when placed with a setback-to-footing width ratio ' b/B ' larger than $(b/B)_t$. Figure 5 also shows that in the cases of $\phi = 40^\circ, \alpha = 20^\circ$ and $\phi = 40^\circ, \alpha = 40^\circ$, Meyerhof (1957) provided $(b/B)_t = 4.5$ which is close to that provided in the present study ($(b/B)_t = 5.0$). It is also seen that except in the case of $b/B = 0$ for $\phi = \alpha = 30^\circ$ and $\phi = \alpha = 40^\circ$ conditions, which are unlikely to exist in practice, the difference in the value of N_γ between these two analytical solutions is generally less than 19% ~ 32%. Comparing this variation with those found in other theoretical approaches, as typically shown by the shaded area in the logarithmic scale in Fig. 4, a variation in theoretical values of N_γ of only 19% ~ 32% can be considered very small.

3. THE EFFECT OF THE SETBACK OF A FOOTING PLACED ADJACENT TO THE SLOPE

Figure 6(a) shows an example of the effect of setback (b/B) on the bearing capacity coefficient (N_γ) for a vertically loaded footing placed near a slope consisting of a cohesionless soil with $\phi = 30^\circ$ and various slope angles (α). A footing width $B = 5$ m and a unit weight of soil, $\gamma = 17.6$ kN/m³ are used in the present

study to derive ultimate bearing capacity of footings. It has been verified that the values of N_γ obtained herein are not susceptible to the changes in the values of ‘ B ’ and ‘ γ ’ for wide ranges of $0.5 \text{ m} \leq B \leq 20 \text{ m}$ and $10 \text{ kN/m}^3 \leq \gamma \leq 20 \text{ kN/m}^3$. The effect of ‘ b/B ’ on N_γ is expressed by using a correction factor ‘ η_b ’ in Terzaghi’s bearing capacity formula:

$$q_{u(\alpha>0, b>0)} = \frac{1}{2} \cdot B \cdot \gamma \cdot N_{\gamma(\alpha>0, b=0)} \cdot \eta_b \tag{3}$$

or

$$\eta_b = \frac{N_{\gamma(\alpha>0, b>0)}}{N_{\gamma(\alpha>0, b=0)}} = \frac{q_{u(\alpha>0, b>0)}}{q_{u(\alpha>0, b=0)}} \tag{4}$$

in which,

$N_{\gamma(\alpha>0, b>0)}$ and $N_{\gamma(\alpha>0, b=0)}$: bearing capacity coefficients induced by the self-weight of soils for a given slope angle (α) under $b > 0$ and $b = 0$ conditions, respectively.

$q_{u(\alpha>0, b=0)}$: ultimate bearing capacity for a footing adjacent to the slope with a given slope angle (α) and a setback distance $b = 0$.

Figure 6(a) shows that for the cases investigated, threshold values of ‘ b/B ’, namely, $(b/B)_t$ can be clearly defined and the effect of b/B on N_γ is limited to the condition of $b/B \leq (b/B)_t$. For $b/B > (b/B)_t$, the bearing capacity of footing is controlled by a failure mechanism similar to that of level ground. The values of η_b can be related to ‘ b/B ’ using bi-linear curves consisting of a segment with a slope of ‘ S_b ’ and a flat segment:

$$\eta_b = S_b \cdot (b/B) + 1 \quad \text{for } b/B \leq (b/B)_t \tag{5}$$

and

$$\eta_b = S_b \cdot (b/B)_t + 1 \quad \text{for } b/B > (b/B)_t \tag{6}$$

Figures 6(b), 6(c), and 6(d) show similar analytical data to those shown in Fig. 6(a), except that Figs. 6(b), 6(c), and 6(d) are for $\phi = 35^\circ$, 40° , and 45° , respectively. Similar conclusions to those of Fig. 6(a) can be drawn, except that the values of $(b/B)_t$ tend to increase with the increase in ϕ . Figure 7 shows that the value of $(b/B)_t$ can be expressed as a linear function of ϕ , as follow:

$$(b/B)_t = b_1 + b_2 \cdot \phi \tag{7}$$

in which, $b_1 = -8.1$ and $b_2 = 0.332$ (1/degree)

In Fig. 7, the values of $(b/B)_t$ provided by Meyerhof (1957) based on the results shown in Fig. 5 are also plotted. The similarity between these two analytical results is clear.

Figure 8 shows the value of ‘ S_b ’ representing the slope of the lines for the η_b vs. b/B relationship which can be expressed using the following equations. The use of this equation is based on the result of a comparative study on various types functions, such as, polynomial, logarithmic, and exponential functions. The comparative study shows that curve-fittings using Eq. (8) generate relatively great values of correlation coefficient compared with the cases using other types of function.

$$S_b = x \cdot (\tan \alpha)^z \tag{8}$$

in which, ‘ x ’ and ‘ z ’ are functions of ϕ as shown in Figs. 9 and 10:

$$x = x_1 + x_2 \cdot \phi \tag{9}$$

in which, $x_1 = 3.657$ and $x_2 = -0.03$ (1/degree)

The value of z in Eq. (8) can be expressed as (see Fig. 10):

$$z = z_1 + z_2 \cdot \phi \tag{10}$$

in which, $z_1 = 0.135$ and $z_2 = 0.04$ (1/degree).

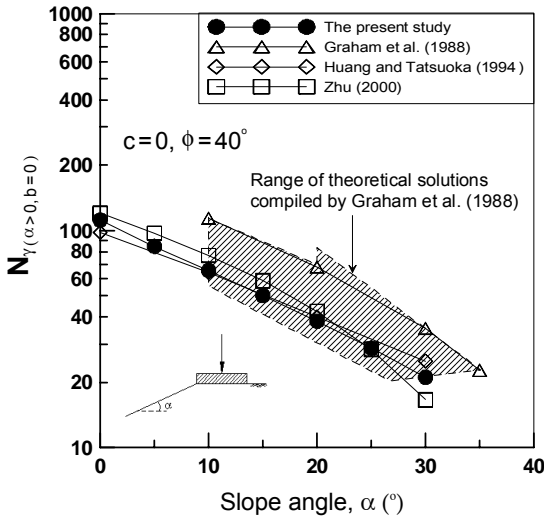


Fig. 4 Comparisons of analytical values of N_γ for a rigid footing adjacent to the slope obtained in various studies (compiled from Graham, et al., 1988)

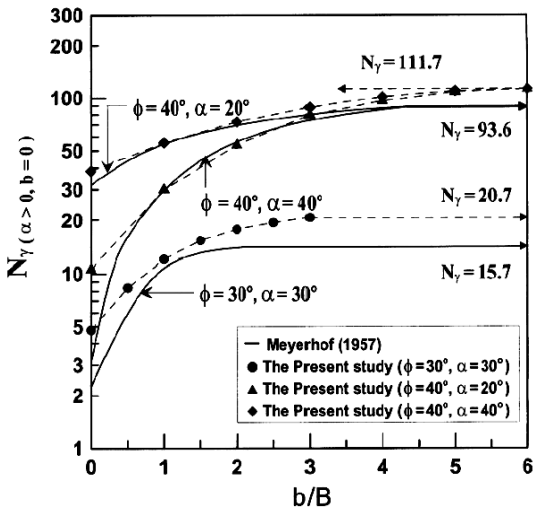


Fig. 5 Comparisons of theoretical values of N_γ obtained by Meyerhof (1957) and the present study

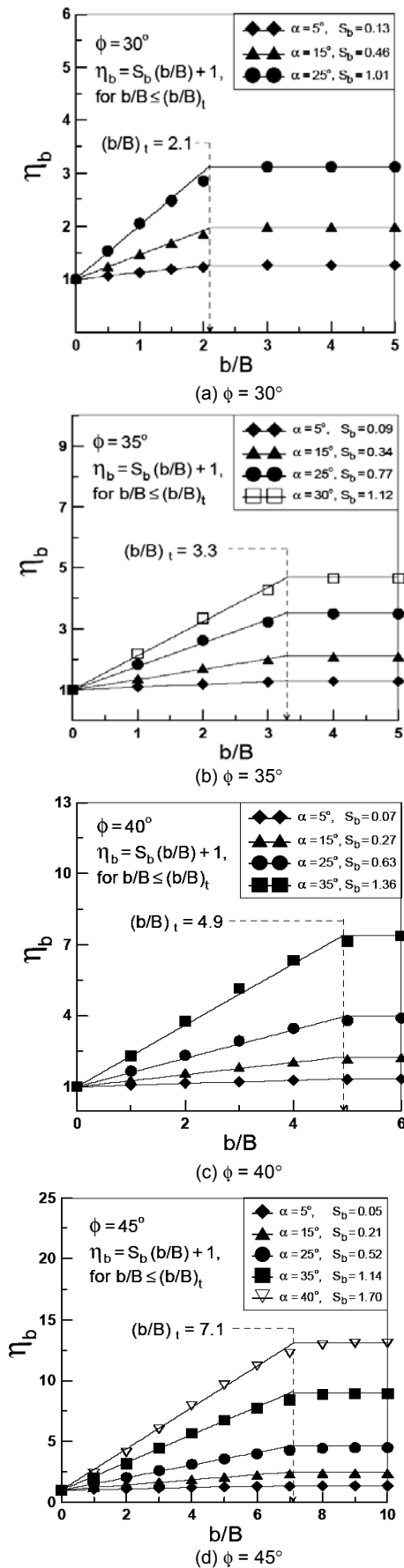


Fig. 6 Correction factors η_b for ultimate bearing capacity of footings placed near the slope with various ϕ and slope angles

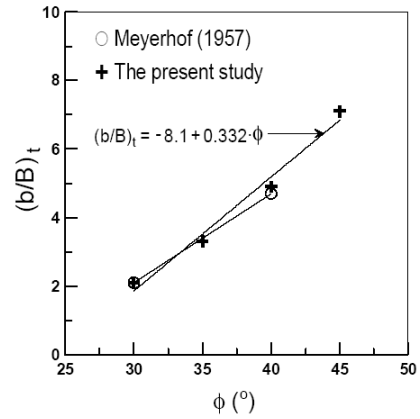


Fig. 7 Relationships between $(b/B)_t$ and internal friction angle of soil, ϕ

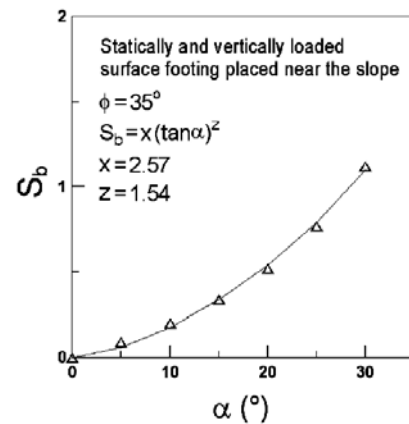


Fig. 8 Relationships between S_b and slope angles, α for $\phi = 35^\circ$

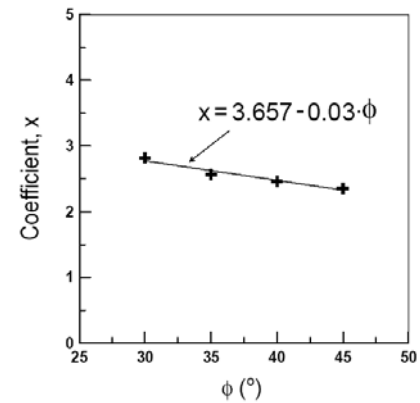


Fig. 9 Values of coefficient x as function of ϕ

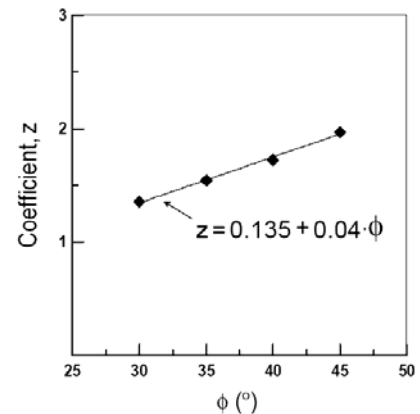


Fig. 10 Values of coefficient z as function of ϕ

4. COMPARATIVE STUDY ON THE EFFECT OF SETBACK

Table 1 summarizes the empirical equations for η_b discussed earlier. Correction factors ' η_b ' calculated using empirical Eqs. (5) ~ (10) are compared with the analytical value of η_b provided by Meyerhof (1957), Graham, *et al.*, (1988) and the present study, as shown in Fig. 11. In general, values of η_b obtained in the present study agree well with those reported by Graham, *et al.*, (1988) for the case of $\phi = 45^\circ$ and $\alpha = 26^\circ$; they also agree well with those reported by Meyerhof (1957) for the case of $\phi = 40^\circ$ and $\alpha = 20^\circ$.

Figure 12 shows a comparison between the values of η_b obtained in two ways. The first are based on the direct approach as proposed in the present study and the second are those based on the indirect approach using synthesized bearing capacity coefficients ' $N_{rq(\alpha>0, b=0)}$ ' which takes into account the combined effect of ' N_γ ' and ' N_q ' expressed by:

$$\begin{aligned} N_{\gamma q(\alpha>0, b>0)} &= \frac{2 \cdot q_{u(\alpha>0, b>0)}}{\gamma \cdot B} \\ &= \frac{2 \cdot q_{u(\alpha>0, b=0)}}{\gamma \cdot B} + \frac{2 \cdot \gamma \cdot D_f \cdot N_{q(\alpha>0, b=0)}}{\gamma \cdot B} \\ &= N_{\gamma(\alpha>0, b=0)} + \frac{2 \cdot D_f}{B} \cdot N_{q(\alpha>0, b=0)} \end{aligned} \quad (11)$$

or

$$N_{\gamma q(\alpha>0, b>0)} = N_{\gamma(\alpha=0)} \cdot g_\gamma + \frac{2 \cdot D_f}{B} \cdot N_{q(\alpha=0)} \cdot g_q \quad (12)$$

in which,

$N_{\gamma q(\alpha>0, b>0)}$: bearing capacity coefficient of footing on the crest of a slope ($b > 0$, $\alpha > 0$), taking into account the combined effect of self-weight and surcharge of soils.

$N_{\gamma(\alpha=0)}$: N_γ for a footing placed on level ground

$N_{q(\alpha=0)}$: N_q for a footing placed on level ground

g_γ : correction factor for $N_{\gamma(\alpha=0)}$ the effect of sloped ground

g_q : correction factor for $N_{q(\alpha=0)}$ the effect of sloped ground

D_f : virtual depth of footing embedment corresponding to a certain footing setback ($D_f = b \cdot \tan \alpha$ is used in the present study)

Table 1 Correction factors for the setback of footing adjacent to the slope under vertically and statically loaded conditions

Number of equations	Equation	Parameters
5	$\eta_b = S_b(b/B) + 1$ for $b/B < (b/B)_t$	See below
6	$\eta_b = S_b(b/B)_t + 1$ for $b/B \geq (b/B)_t$	See below
7	$(b/B)_t = b_1 + b_2 \cdot \phi$	$b_1 = -8.1$ $b_2 = 0.332$ (1/degree)
8	$S_b = x \cdot (\tan \alpha)^z$	See below
9	$x = x_1 + x_2 \cdot \phi$	$x_1 = 3.657$ $x_2 = -0.03$ (1/degree)
10	$z = z_1 + z_2 \cdot \phi$	$z_1 = 0.135$ $z_2 = 0.04$ (1/degree)

The values of $N_{\gamma q(\alpha=30^\circ, b=B)}$ and $N_{\gamma q(\alpha=30^\circ, b \geq B)}$ shown in Table 2 for Vesic (1973) and Hansen (1970) are calculated using Eq. (12) and bearing capacity coefficients suggested in the respective studies. For the value of η_b calculated using indirect approaches, η_b is defined as follows:

$$\begin{aligned} \eta_b &= \frac{N_{\gamma q(\alpha>0, b>0)}}{N_{\gamma(\alpha>0, b=0)}} = \frac{N_{\gamma(\alpha>0, b=0)}}{N_{\gamma(\alpha>0, b=0)}} + \frac{2 \cdot D_f}{B} \cdot \frac{N_{q(\alpha>0, b=0)}}{N_{\gamma(\alpha>0, b=0)}} \\ &= 1 + \frac{2 \cdot D_f}{B} \cdot \frac{N_{q(\alpha=0)} \cdot g_q}{N_{\gamma(\alpha=0)} \cdot g_\gamma} \end{aligned} \quad (13)$$

Values of η_b in Table 2 and Fig. 12 were calculated using the following equations Table suggested by Hansen (1970):

$$N_{\gamma(\alpha=0)} = 1.5 [N_{q(\alpha=0)} - 1] \tan \phi \quad (14)$$

$$\text{and } g_\gamma = g_q = (1 - 0.5 \tan \alpha)^5 \quad (15)$$

Values of η_b in Table 2 and Fig. 12 were also calculated using equations suggested by Vesic (1973):

$$N_{\gamma(\alpha=0)} = 2 [N_{q(\alpha=0)} + 1] \tan \phi \quad (16)$$

$$\text{and } g_\gamma = g_q = (1 - \tan \alpha)^2 \quad (17)$$

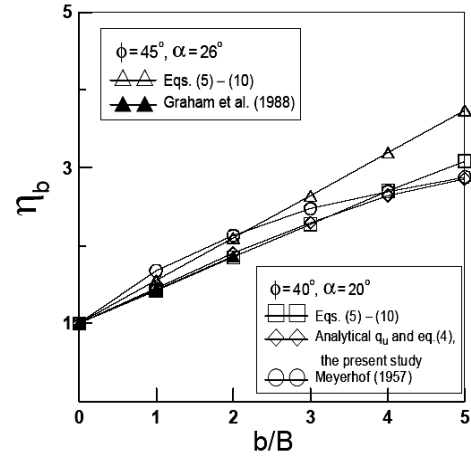


Fig. 11 Comparisons of analytical values of η_b obtained in various analytical studies

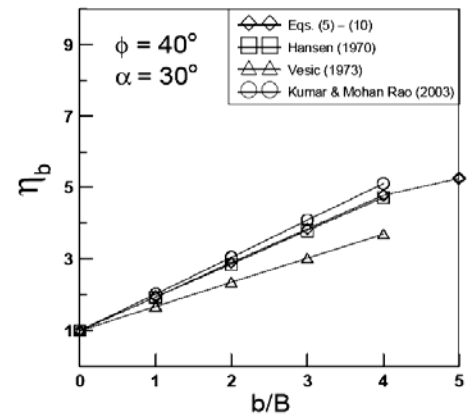


Fig. 12 Comparisons of η_b based on analytical values of N_γ and empirical values of $N_{\gamma q}$

Table 2 Comparisons the values of N_{γ} for $\phi = 40^\circ$

	$N_{\gamma(\alpha=0)}$	$N_{\gamma(\alpha=30^\circ, b=0)}$	$N_{\gamma(\alpha=30^\circ, b=B)}$ or $N_{\gamma q(\alpha=30^\circ, b=B)}$	$N_{\gamma(\alpha=30^\circ, b=2B)}$ or $N_{\gamma q(\alpha=30^\circ, b=2B)}$
The present study	111.7	21.1	40.7 ²	61.7 ²
Hansen (1970)	79.5	14.5 ¹	28.0 ³	41.5 ³
Vesic (1973)	109.4	19.6 ¹	32.9 ³	46.2 ³
Graham, <i>et al.</i> (1988)	135	35.5	–	–
Zhu (2000)	120.2	16.6	–	–
Kumar and Mohan Rao (2003)	87.3	14.7	29.8 ⁴	44.9 ⁴

¹ Corrected values of $N_{\gamma(\alpha=0)}$ using g_{γ} expressed by Eq. (15) or Eq. (17)
² Analytical values of N_{γ} based on direct approach for the effect of b/B
³ $N_{\gamma q(\alpha>0, b>0)}$ calculated using Eq. (12) and the equations of $N_{\gamma(\alpha=0)}$, $N_{q(\alpha=0)}$, g_{γ} and g_q suggested by various authors, Eqs. (14) ~ (18)
⁴ $N_{\gamma q(\alpha>0, b>0)}$ calculated using Eq. (11) and values of $N_{\gamma(\alpha>0, b=0)}$ and $N_{q(\alpha>0, b=0)}$ reported by Kumar and Mohan Rao (2003)

in Eqs. (12), (13), (14), and (16)

$$N_{q(\alpha=0)} = e^{\pi \tan \phi} \tan^2 \left(45^\circ + \frac{\phi}{2} \right) \quad (18)$$

It can be seen in Eq. (13) that the accuracy of calculated values of η_b based on Hansen’s and Vesic’s solutions are not influenced by the values of g_{γ} and g_q , because both proposed $g_{\gamma} = g_q$ as shown in Eqs. (15) and (17).

The values of $N_{\gamma q(\alpha>0, b=0)}$ for Zhu (2000) and Kumar and Mohan Rao (2003), as summarized in Table 2, are calculated using Eq. (11) and values of $N_{\gamma(\alpha>0, b=0)}$ and $N_{q(\alpha>0, b=0)}$ reported in the respective studies. Note that the present study is not intended to examine in-detail the background leading to the widely varied theoretical and/or empirical solutions of N_{γ} and $N_{\gamma q}$ provided by various methods. In stead, the present study focuses on the comparison of ‘ η_b ’ provided by various analytical and/or empirical methods. It is well-known that theoretical solutions of N_{γ} (for identical values of ϕ and α) can span a wide range of $\pm 100\%$ from the averaged theoretical values of N_{γ} obtained using various methods. This fact is exemplified in Fig. 4 and has also been discussed by Tatsuoka, *et al.*, (1989) and Huang and Tatsuoka (1994). The essence of Table 2 is that theoretical solutions of N_{γ} for the level ground ($\alpha = 0$) and that for $\alpha = 30^\circ$ derived here are always close to the averaged values of N_{γ} obtained using various methods. It seems that values of $N_{\gamma(\alpha=30^\circ, b=B)}$ and $N_{\gamma(\alpha=30^\circ, b=2B)}$ obtained here are somewhat greater than those obtained by other empirical solutions. The greater values of $N_{\gamma(\alpha=30^\circ, b=B)}$ and $N_{\gamma(\alpha=30^\circ, b=2B)}$ for about 25% ~ 30% compared to other solutions indicate that composed values of $N_{\gamma q(\alpha=30^\circ, b=B)}$ and $N_{\gamma q(\alpha=30^\circ, b=2B)}$ based on various empirical approaches are somewhat conservative. For the theoretical solutions provided in the first two columns of Table 2, the ones provided in the present study deviate from the averaged values within +7% and –20%. This is considered small when compared with the range of $\pm 100\%$ for various theoretical solutions as discussed previously. Figure 12 shows that the values of

η_b calculated using the indirect approach, as shown in Eq. (13), and the direct approach as proposed herein are comparable for $b/B \leq 4$, suggesting that conventional indirect approaches also work well as an alternative analytical approach.

5. CONCLUSIONS

In this study, a limit equilibrium method incorporated with Janbu’s slice method is used to evaluate the effect of setback on the ultimate bearing capacity of a surface footing placed near the shoulder of a slope. This method eliminates possible shortcomings that may be associated with conventional approaches which employ overburden pressure and Terzaghi’s bearing capacity coefficient for surcharge, N_q , to indirectly evaluate the effect of setback. The analytical results show that the bearing capacity of the footing increases almost linearly with an increase in setback distance up to certain threshold values denoted by a dimensionless setback-to-footing width ratio, $(b/B)_t$. Beyond these threshold values, the ultimate bearing capacity remains constant like that of a footing placed on a semi-infinite level ground. The results show that the value of ‘ $(b/B)_t$ ’ is a linear function of the internal friction angle of the foundation soil (ϕ) regardless the value of slope angle (α) ranged between 0° and 35° . The results also show that the gradient (S_b), characterizing the linear relationship between the effect of setback (in terms of η_b) and the normalized setback (b/B), can be expressed as functions of ϕ and α . The correction factors, η_b which were obtained in the present study are comparable with those obtained in two previously documented analytical studies. It is also shown that an indirect approach which takes into account the effect of setback on the ultimate bearing capacity of footing by using equivalent surcharges on the slope surface also works well in the sense that the indirect approach examined herein generates comparable values of η_b as those provided by straightforward analytical solutions derived in the present study.

APPENDIX A

FORMULAS AND PROCEDURES FOR CALCULATING SEISMIC BEARING CAPACITY OF FOOTINGS ADJACENT TO THE SLOPE

The soil mass confined by the slip lines shown in Fig. A-1 is divided into n vertical slices (width of slice, $B_s = 0.1$ m) with base inclinations α_i ($i = 1, \dots, n$) in the present study. These slices are grouped into two categories, namely, slices subjected to the footing load, P_i and Q_i , at their top surfaces and the slices without footing load at their top surfaces. The following assumptions are made.

1. Assume inter-slice thrust heights to be 1/3 of the inter-slice heights and calculate θ_i for all slices (see Fig. A-1 for the definition of θ_i).
2. The ultimate footing loads P_{fi} and Q_{fi} are uniformly distributed on the surface of the slices located directly under the footing, *i.e.*, $P_{f1} = P_{f2} = \dots = P_{fm} = P_f$, and $Q_{f1} = Q_{f2} = \dots = Q_{fn} = Q_f$ (m : number of slices directly subjected to the footing load is ‘ n_f ’; $n_f = 50$ in the present study).
3. $Q_f = P_f \cdot \tan \beta$, β : angle of load inclination on the footing base.

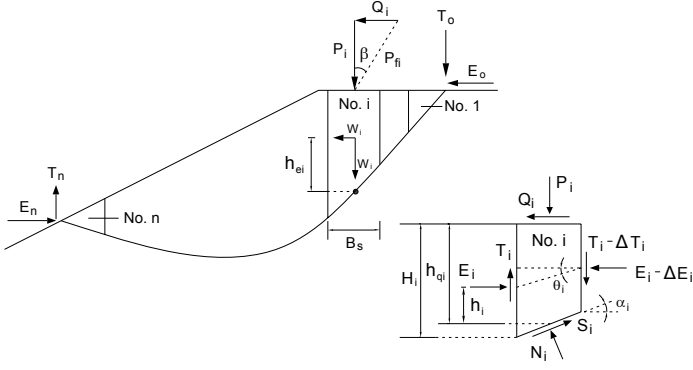


Fig. A-1 Schematic figure of forces and boundary conditions used in the slice method

According to: (1) the force equilibrium in horizontal and vertical directions for slice Nos. 1 – n , (2) Mohr-Coulomb's failure criterion, $S_i = (S_{fi}/F_s) = (N_i \cdot \tan \phi)/F_s$ (assuming cohesionless soils and no pore water pressure; F_s : safety factor against shear failure; $F_s = 1.0$ in the present study), and (3) $\sum_1^n E_i = E_n - E_o$, ultimate footing load on the surface of the slice, P_f and the differential between inter-slice forces, $\Delta E_i (= E_i - E_{i-1})$ can be obtained as:

$$P_f = \frac{E_n - E_o + \sum_1^{n_f} D_{fi} + \sum_1^n D_i + \sum_1^{n_f} G_{fi} + \sum_1^n G_i}{\sum_1^{n_f} H_{fi}} \quad (A1)$$

$$\Delta E_i = -A_i + B_i \quad (A2)$$

The ultimate bearing capacity (q_u) and the bearing capacity coefficient (N_γ) can be obtained using:

$$q_u = \frac{P_f \cdot n_f}{B} \quad (A3)$$

$$N_\gamma = \frac{2 \cdot q_u}{\gamma \cdot B} \quad (A4)$$

in Eq. (A2),

$$A_i = S_{fi} \cdot \sec \alpha_i \quad (A5)$$

$$B_i = (P_i + W_i - \Delta T_i) \cdot \tan \alpha_i + Q_i \quad (A6)$$

$$S_{fi} = \frac{C_i + (W_i + P_i - \Delta T_i) \cdot \tan \phi \cdot \sec \alpha_i}{1 + \frac{\tan \phi \cdot \tan \alpha_i}{F_s}} \quad (A7)$$

$$\Delta T_i = T_i - T_{i-1}$$

in Eq. (A1),

$$D_{fi} = \frac{C_i \cdot \sec \alpha_i + (W_i - \Delta T_i) \cdot \tan \phi \cdot \sec^2 \alpha_i}{1 + \tan \phi \cdot \tan \alpha_i} \quad (A8)$$

$$D_i = \frac{C_i \cdot \sec \alpha_i + (W_i + P_i - \Delta T_i) \cdot \tan \phi \cdot \sec^2 \alpha_i}{1 + \tan \phi \cdot \tan \alpha_i} \quad (A9)$$

$$G_{fi} = (W_i - \Delta T_i) \cdot \tan \alpha_i \quad (A10)$$

$$G_i = (P_i + W_i - \Delta T_i) \cdot \tan \alpha_i + Q_i \quad (A11)$$

$$H_{fi} = \tan \beta + \frac{\tan \alpha_i - \tan \phi}{1 + \tan \phi \cdot \tan \alpha_i} \quad (A12)$$

Taking the moment equilibrium about the center of the slice base yields the following equation:

$$T_i = E_i \cdot \tan \theta_i - \Delta E_i \cdot \tan \theta_i + \frac{\Delta T_i}{2} + \frac{-\Delta E_i \cdot h_i + Q_i \cdot h_{qi}}{B_s} \quad (A13)$$

Assuming that the width of slice is small, $\Delta T_i \rightarrow 0$ and $\Delta E_i \rightarrow 0$, we can rewrite Eq. (A13) as:

$$T_i = E_i \cdot \tan \theta_i + \frac{Q_i \cdot h_{qi}}{B_s} \quad (A14)$$

In the present study, Eq. (A14) instead of Eq. (A13) is used.

The computer algorithm for calculating ultimate footing load P_{fi} is as follows:

1. Assume $\Delta T_i = 0$.
2. Calculate the first approximated value of P_f using Eq. (A1).
3. Calculate ΔE_i and E_i ($i = 1, 2, \dots, n$) using Eq. (A2).
4. Calculate T_i ($i = 1, 2, \dots, n$) using Eq. (A14).
5. Calculate an improved value of P_f using Eq. (A1).
6. Repeat Steps (4) to (6) until the convergence of P_f is achieved (the convergence criterion for P_f was $|\text{New } P_f - \text{Old } P_f| / (\text{Old } P_f) \leq 0.3\%$ in the present study.)

NOTATIONS

- A_i : term used in Eq. (A5) (N/m)
 B : width of footing (m)
 b : distance of setback (m)
 B_s : width of slice (m)
 B_i : term used in Eq. (A6) (N/m)
 $(b/B)_t$: threshold value of b/B that transform from a near-slope bearing capacity into a level-ground bearing capacity (dimensionless)
 C_i : Cohesive shear resistance for slice No. i (N/m)
 D_f : virtual depth of footing embedment (m)
 D_{fi} : term used in Eq. (A9) (N/m)
 D_i : term used in Eq. (A8) (N/m)
 ΔE_i : differential value of E_i (N/m)
 E_i ($i = 0 \dots n$): horizontal interslice force (N/m)
 G_{fi} : term used in Eq. (A10) (N/m)
 G_i : term used in (A11) (N/m)

g_q : correction factor on N_q ($\alpha=0$) for the effect of sloped ground (dimensionless)

g_γ : correction factor on N_γ ($\alpha=0$) for the effect of sloped ground (dimensionless)

H_{fi} : term used in Eq. (A12) (N/m)

h_i : arm of rotation for horizontal inter-slice force (m)

h_{qi} : arm of rotation for horizontal seismic force at the top of slice (m)

n_i : number of slices directly under the footing

N_i : normal force on the slice base (N/m)

N_q : Terzaghi's bearing capacity coefficient for footing embedment or surcharge (dimensionless)

N_γ : Terzaghi's bearing capacity coefficient for self-weight of soils (dimensionless)

$N_{q(\alpha>0, b=0)}$: bearing capacity coefficient due to surcharge for a footing adjacent to the slope with a slope angle α (dimensionless)

$N_{\gamma(\alpha=0)}$: N_γ for a footing placed on a horizontal ground (dimensionless)

$N_{q(\alpha=0)}$: N_q for a footing placed on a horizontal ground (dimensionless)

$N_{\gamma(\alpha>0, b=0)}$: N_γ for a footing adjacent to the slope with a slope angle α (dimensionless)

$N_{\gamma(\alpha>0, b>0)}$: N_γ for a footing placed on the crest of a slope with a setback ($b > 0$) (dimensionless)

$N_{\gamma q(\alpha>0)}$: bearing capacity coefficient of footing adjacent to the slope ($b = 0$, $\alpha > 0$), taking into account the combined effect of self-weight and surcharge of soils (dimensionless)

P_{fi} ($i = 1 \dots m$): footing load on the top of slice No. i ; m : number of slices directly subjected to the footing load (N/m)

P_i : vertical load at the top of slice No. i (N/m)

P_f : total footing load (N/m)

Q_{fi} ($i = 1 \dots m$): horizontal force exerted by the footing (N/m)

Q_i : horizontal load at the top of slice No. i (N/m)

q_u : Ultimate bearing capacity of footings (N/m²)

$q_{u(\alpha>0, b=0)}$: ultimate bearing capacity for a footing adjacent to the slope with a given slope angle, α (N/m²)

r : radius of a logarithmic spiral from the toe of footing (m)

r_o : radius of a logarithmic spiral at the interface between the active wedge and the transitional zone (m)

S_b : gradient of the linear η_b vs. b/B relationship (dimensionless)

S_{fi} ($i = 1 \dots m$): shear force at the base of slice No. i directly subjected to the footing load (N/m)

S_i : shear force at the base of slice No. i (N/m)

ΔT_i : differential value of T_i (N/m)

W_i : self-weight of slice No. i (N/m)

α : slope angle (degree)

α_i : base inclination for slice No. i (degree)

β : footing load inclination (degree)

ϕ : internal friction angle of soils (degree)

γ : unit weight of soils (kN/m³)

η : parameter dictating the curvature of a log-spiral (degree)

η_b : correction factor on N_γ for a footing setback (dimensionless)

μ : angle between r_o and r for a log-spiral determining the transitional zone bounded by a log-spiral (radian)

θ_i : thrust line inclination for slice No. i (degree)

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