UPDATING RELIABILITY OF INSTRUMENTED GEOTECHNICAL SYSTEMS VIA SIMPLE MONTE CARLO SIMULATION

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ABSTRACT

In this research, a new method is proposed to update reliability based on data recorded by instruments and sensors installed on geotechnical systems. The method is founded on Bayesian analysis and Monte Carlo simulation and is capable of estimating the functional relationship between the updated failure probability and the monitoring value. It is shown that as long as the probability distribution of the uncertainties and the mathematical model of the target system are given, this relationship can be obtained prior to the monitoring process. This new method may be applied to safety monitoring of in-construction geotechnical systems and monitoring of existing geotechnical systems.

Key words: Reliability, Bayesian analysis, reliability update, monitoring, maximum entropy.

1. INTRODUCTION

Uncertainties are abundant in geotechnical engineering. Major sources of these uncertainties may include uncertainties of geo-material properties, spatial variability, model uncertainties, and measurement uncertainties. Reliability analyses (Benjamin and Cornell, 1970; Madsen, *et al*. 1986; Thoft-Christensen and Murotsu, 1986; Ang and Tang, 1984) are the main tool of quantifying these uncertainties.

However, it is sometimes the case that the amount of uncertainties associated with geotechnical systems is so significant that the resulting failure probability is quite large. For one of the examples in this paper (a deep excavation case study), where the probability distributions of the uncertainties are reasonably chosen according to the laboratory test results and previous research, the probability that the maximum ground settlement is greater than 10 cm is as high as 30% (note that this probability should be interpreted as the degree of belief, rather than the actual relative frequency of failure). Such a high failure probability is usually not acceptable. Similar issues may exist in various geotechnical systems because they are subjected to a large amount of uncertainties. How to reduce the uncertainties in geotechnical systems can be an important research topic.

There are at least two ways of reducing geotechnical uncertainties: (a) obtain new information from the systems by monitoring them and (b) conduct more in-situ tests to reduce the uncertainties of the ground. This research focuses on the former approach: how to reduce uncertainties and update reliability by using monitoring data. Please note that it is usually not easy to measure the quantities that directly define failure. For instance, in a deep excavation problem, the failure is, for example, defined as the maximum ground settlement exceeding certain threshold. However, it is not possible to measure ground settlement if there are adjacent buildings nearby the site. On the other hand, in most deep excavation cases, deformation of diaphragm walls is measured, and the measured deformation contains certain amount of information about the ground settlement. Therefore, it is possible to reduce uncertainties in ground settlement by using the wall deformation data, *i.e.,* to update reliability.

Despite its importance, research focusing on updating reliability is rare in civil engineering literature: Beck and Au (2002) proposed adaptive Metropolis-Hastings algorithm to update reliability. A limitation of their approach is that the dimension of uncertainties cannot be too high. Ching and Beck (2006) proposed a method based on an efficient importance sampling technique developed by Au and Beck (2001) to update reliability of linear systems. Their approach is not constrained in the uncertainty dimension, but it can be only applied to linear systems with Gaussian uncertainties.

In this research, a new method is proposed to update reliability of general systems by using scalar monitoring data without the dimensionality and linearity constraints. This new method is based on Bayesian analysis (Ang and Tang, 1984; Gelman, *et al*. 1995) and Subset Simulation (SubSim), Monte Carlo simulation and update reliability of an instrumented system by using its monitoring data. In fact, as long as the probability distribution of the uncertainties and the mathematical model of the target system are given, the functional relationship between the updated failure probability and the scalar monitoring value can be obtained prior to the monitoring process. This means in real applications, it is not necessary to conduct the new algorithm in an online manner. Instead, the relationship can be calculated a priori so that the reliability update can be achieved right away once the monitoring data is obtained. It is expected that the new approach is useful for safety monitoring of in-construction geotechnical systems and monitoring of existing geotechnical systems.

The structure of the paper is as follows: In Section 2, the problem of updating reliability is formally defined. In Section 3, the algorithm of the new approach is described. In Section 4, examples are used to demonstrate the new approach, and in Section 5, discussions and conclusions will be given.

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2. PROBLEM DEFINITION

The goal of regular reliability analyses is to estimate failure probability given the probability distribution of the uncertainties in the target system and the mathematical model of the system, *i.e.,* compute $P(F|M)$, where F denotes the failure event, and M contains the mathematical model as well as the assumed probability distribution of the uncertainties. When new information φ is available, it is essential to incorporate it to reduce the uncertainties (*i.e.,* update reliability). This is especially the case if the new information φ is the direct measurement on the target geotechnical system: this measurement directly reflects system status and may contain much more information than usual field data. Therefore, it is desirable to develop a methodology to update reliability based on these measurements, *i.e.,* compute $P(F|\varphi, M)$. In this paper, φ is assumed to be a scalar. For vectoral φ, the problem of estimating $P(F|\varphi, M)$ is much more difficult, so it is left as future research.

A naive way of achieving the aforementioned task is as follows: Employ brute-force Monte Carlo simulation (MCS) to draw many samples of uncertain variables, each sample corresponds to a monitoring value. Suppose that there are m samples whose monitoring values are identical to the actual monitoring value and that among the m samples, there are n samples satisfying the prescribed failure condition (called the failure samples). The failure probability can therefore be updated as n/m. However, this approach is infeasible in practice since the chance that the sampled monitoring value is equal to the actual one is zero, so obtaining such m samples requires infinite amount of computational time.

Nevertheless, according to the Bayes' rule, we know

$$P(F \mid \varphi, M) = \frac{f(\varphi \mid F, M) P(F \mid M)}{f(\varphi \mid M)} \tag{1}$$

where $P(F|\varphi, M)$ is the updated failure probability; $f(\varphi|F, M)$ is the probability density function (PDF) of the monitoring value conditioned on the failure event; $P(F|M)$ is the failure probability without the monitoring information, called the prior failure probability; and $f(\varphi|M)$ is the prior PDF of the monitoring value. For our purpose, the goal is to find $P(F|\varphi, M)$. According to (1), if $f(\varphi|F, M)$, $P(F|M)$ and $f(\varphi|M)$ are all available, $P(F|\varphi, M)$ can be readily obtained. In the following section, the detailed descriptions for estimating $f(\varphi|F, M)$, $P(F|M)$ and $f(\varphi|M)$ are provided. For notational simplicity, the symbol M in the conditions will be dropped from all the following discussion. Readers should keep in mind that all results are conditioned on the assumed model M.

3. ESTIMATION OF *P*(*F*), *f*(φ) AND *f*(φ|*F*)

In principle, $P(F)$ can be estimated by MCS. In the process, samples distributed as $f(\varphi)$ and $f(\varphi|F)$ are obtained. Based on the samples, the unknown PDFs $f(\varphi)$ and $f(\varphi|F)$ can be estimated with the maximum entropy theory (Jaynes, 1957). The detailed procedures are described in the following.

3.1 Estimation of *P*(*F*)

The failure probability $P(F)$ can be estimated with any reliability analysis method. In this paper, MCS is employed due to its simplicity. Let Z denote the uncertain variables of the target system. First, N sets of Z samples $\{\hat{Z}^i : i = 1 \ldots N\}$ are drawn from the prescribed PDF of Z. According to the Law of Large Number, we have

$$P(F) \approx \frac{1}{N} \sum_{i=1}^{N} \left(R\left[\hat{Z}^i \right] > 1 \right) \tag{2}$$

where R denotes the limit-state function that defines failure event F, *i.e.,* the failure event is defined as $R[Z] > 1$.

Please note that in the process of MCS, samples distributed as $f(\varphi \mid F)$ and $f(\varphi \mid F^C)$ can be obtained (F^C denotes the non-failure event): Corresponding to the N sample sets $\{\hat{Z}^i : i = 1 \ldots N\}$ are the N samples of the monitoring value $\{\hat{\varphi}^i : i = 1 \ldots N\}$. Assuming that among the N samples, there are N_F failure samples, *i.e.,* samples satisfying $R(\hat{Z}^i) > 1$, so the corresponding φ samples are distributed as $f(\varphi|F)$. On the other hand, there are N-N_F non-failure samples, so the corresponding φ samples are distributed as $f(\varphi \mid F^C)$. With the samples, the unknown PDFs $f(\varphi)$ and $f(\varphi \mid F)$ can be estimated by using the maximum entropy theory, as described in the next section.

3.2 Estimation of *f*(φ) and *f*(φ|*F*): Maximum Entropy Theory

The entropy of a PDF quantifies the degree of surprise, *i.e.,* the information content of the PDF. With the limited information obtained from the samples of an unknown PDF, it is desirable to estimate the PDF based on the information. One way of achieving so is to find the PDF whose entropy is maximized subjected to the information constraint. In this study, the first four sample moments of the samples are calculated and are taken as the condensed information about the unknown PDF. The maximum entropy PDF is then calculated to maximize its entropy subjected to the moment constraints.

The maximum entropy theory is employed to estimate $f(\varphi|F)$ and $f(\varphi|F^C)$ based on the sample moments of the samples obtained from $f(\varphi|F)$ and $f(\varphi|F^C)$, *i.e.,* to solve the following optimization problem:

$$\max_{g(\cdot)} \left(-\int_L^U \log g(\varphi) \cdot g(\varphi)\, d\varphi \right)$$

$$\text{subjected to } \int_L^U g(\varphi)\, d\varphi = 1 , \quad \int_L^U \varphi^i g(\varphi)\, d\varphi = \mu_i , \quad i = 1 \ldots 4 \tag{3}$$

Please note that in the optimization problem, the variable is the entire $g(\varphi)$ function, where $g(\varphi)$ can be either $f(\varphi|F)$ or $f(\varphi|F^C)$; L and U are the upper and lower bounds, respectively, of the monitoring value φ (if they exist; otherwise, they are $-\infty$ and ∞); the quantity $-\int_L^U \log g(\varphi) \cdot g(\varphi)\, d\varphi$ is the entropy of $g(\varphi)$, and $\{\mu_k : k = 1 \ldots 4\}$ are the first four sample moments:

$$\mu_k = \frac{1}{N} \sum_{i=1}^{N} (\hat{\varphi}^i)^k \tag{4}$$

where $\{\hat{\varphi}^i : i = 1 \ldots N\}$ are the samples of φ.

When the maximum entropy theory is implemented, only the first four moments are considered for the following reasons: (a) the first four moments roughly contain the most important information of a PDF, *i.e.,* mean, variance, skewness and kurtosis; and (b) it is found that if higher moment information is considered (higher than the fourth moment), the maximum entropy algorithm provided in the appendix may have convergence problems due to numerical inaccuracy. L and U are usually taken to be 0 and infinity, respectively, since many monitoring values are positive real numbers.

To solve the optimization problem, let us define the Lagrangian function:

$$L(\lambda, g) = \int_L^U \log g(\varphi) \cdot g(\varphi)\, d\varphi + \lambda_0 \left[\int_L^U g(\varphi)\, d\varphi - 1 \right] + \sum_{i=1}^{4} \lambda_i \left[\int_L^U \varphi^i g(\varphi)\, d\varphi - \mu_i \right] \quad (5)$$

where λ_0, λ_1, … and λ_4 are the Lagrangian parameters. The optimal solution of the problem must satisfy the so-called saddle point condition, *i.e.,*

$$\frac{\partial L(\lambda, g)}{\partial g(\varphi)} = 0 = 1 + \log g(\varphi) + \sum_{i=0}^{4} \lambda_i \varphi^i \quad (6)$$

It is easy to show that the solution must have the following form:

$$g(\varphi) = e^{-(1+\lambda_0+\lambda_1\varphi+\lambda_2\varphi^2+\lambda_3\varphi^3+\lambda_4\varphi^4)} = e^{-\lambda_0 - \sum_{i=1}^{4}\lambda_i\varphi^i} \quad (7)$$

For convenience, the constant 1 has been absorbed into the λ_0 parameters in the above equation. Moreover, the Lagrangian parameters λ_0, λ_1, … and λ_4 must satisfy

$$\int_L^U g(\varphi) d\varphi = 1, \quad \int_L^U \varphi^i g(\varphi)\, d\varphi = \mu_i \quad i = 1 \ldots 4 \quad (8)$$

In (8), there are five unknowns with five nonlinear equations. The unknown Lagrangian parameters can be solved with the Newton method described in the appendix. Once the optimal Lagrangian parameters are found, plugging them into (7) will give us our maximum entropy estimate for the target PDF.

Since samples distributed as $f(\varphi \,|\, F)$ and $f(\varphi \,|\, F^C)$ are obtained in MCS, $f(\varphi \,|\, F)$ and $f(\varphi \,|\, F^C)$ can then be estimated by using the maximum entropy theory. Furthermore, $f(\varphi)$ can be estimated with the following equation:

$$f(\varphi) = P(F) \times f(\varphi \,|\, F) + P(F^C) \times f(\varphi \,|\, F^C) \quad (9)$$

where $P(F)$ has been estimated by MCS, and $P(F^C) = 1 - P(F)$.

Please note that the approach proposed in this research for reliability updating is based on MCS and the maximum entropy theory. The former is applicable to general linear or nonlinear systems whose uncertainty dimension can be arbitrarily large, while the latter is also applicable to general PDF. Therefore, the applicability of the proposed approach is quite broad, especially for geotechnical systems, which are usually quite nonlinear and uncertainty dimensionality is quite large.

Besides, with the proposed approach, the functional relationship between the updated failure probability and the scalar monitoring value can be obtained prior to the monitoring process as long as the probability distribution of the uncertainties is given. This means in real applications, it is not necessary to conduct a new analysis in an online manner. Instead, the relationship can be calculated a priori so that the reliability update can be achieved right away once the monitoring data is obtained. Let us take the first case study in Section 4 as an example, where the monitoring value is the height of the water table in the slope, and failure is defined as the sliding of the slope. Using our method, it is possible to estimate the functional relationship between the updated failure probability and the water table height (see the left-hand-side figure in Fig. 1). Suppose the height is monitored, and the daily data is shown in the upper-right figure in Fig. 1, we can use the functional relationship to compute the updated daily failure probability of the slope, as shown in the lower-right plot in Fig. 1. The lower-right plot has significant application: based on the plot, decisions can be made to maintain the slope to prevent possible failure.

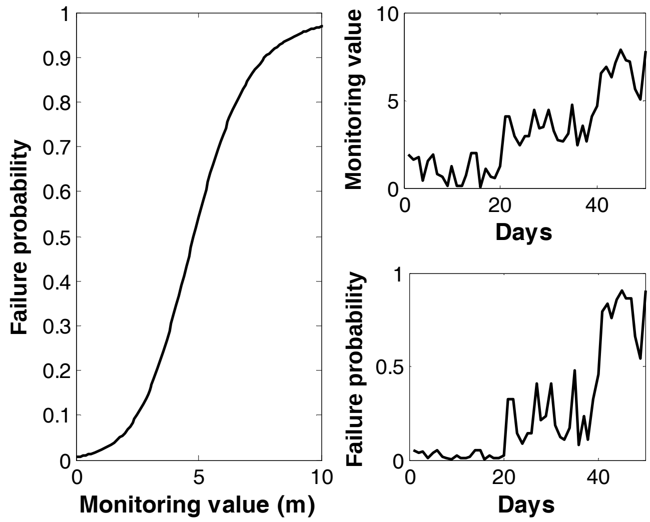

Fig. 1 An imaginary infinite slope example. The left-hand-side plot is the estimated $P(F \,|\, \varphi)$ function. The upper-right plot is the monitoring value, while the lower-right plot is the resulting failure probability time history.

4. EXAMPLES

Two examples are selected to demonstrate the use of the new method. The first example is an infinite slope, where the monitoring value is the height of the water table, and failure is defined as the sliding of the slope. The second example is a real case study, a deep excavation problem, where the monitoring value is the maximum diaphragm wall deformation, and failure is defined as the exceedance of the maximum ground settlement over a prescribed threshold. There are some common features for the two examples: (a) the uncertainties are so significant that the failure probability is quite large when no monitoring data is available; (b) the physical quantities that directly define failure cannot be monitored easily, but the monitored value contains certain information about those quantities. So the knowledge of the monitoring value is helpful in reducing the uncertainties associated with the defined failure.

4.1 Infinite Slope

In this hypothetical example, let us consider the infinite slope in Fig. 2, where H is the thickness of the soil layer, β is the slope angle, h is the height of the water table, γ is the unit weight of the soil, γ_{sat} is unit weight of the saturated soil, and ϕ is the friction angle of the cohesiveless soil. The failure event is defined as the sliding of the slope along the soil-rock interface, *i.e.,* downward sliding force is greater than the shear resistance along the interface. Therefore, the limit-state function is

$$R(Z)=\frac{\left[\gamma_{sat}h+\gamma(H-h)\right]\sin(\beta)\cos(\beta)}{\left[(\gamma_{sat}-\gamma)h+\gamma(H-h)\right]\cos^2(\beta)\tan(\phi)} \tag{10}$$

The height of the water table is uncertain and is uniformly distributed over the [0, H] interval, *i.e.,* $h = H\cdot U$, where U is uniformly distributed over the [0, 1] interval. For this example, Z contains all uncertainties, including γ, γ_{sat}, H, ϕ, U. It is assumed that the uncertain variables γ, γ_{sat}, H, ϕ are normally distributed with mean equal to [17 kN/m^3, 10 m, 19 kN/m^3, 35°] and standard deviation equal to [1.5 kN/m^3, 3 m, 1.5 kN/m^3, 3°]. The slope angle β is known and is equal to 20°. The monitoring value φ is the height of the water table h.

In order to estimate $P(F|\varphi)$, MCS is employed to estimate $P(F)$; the result shows that $P(F)$ is roughly 50%, indicating that the uncertainties are significant. Therefore, it is desirable to update the failure probability with the monitoring data so that the uncertainties can be reduced. During MCS, samples distributed as $f(\varphi|F)$ and $f(\varphi|F^C)$ are obtained, whose histograms are shown in Fig. 3. With the samples together with the maximum entropy theory, the unknown PDF $f(\varphi|F)$ and $f(\varphi|F^C)$ are estimated and are plotted in Fig. 3. With the Bayes' rule in (1) and the relationship in (8), the estimate for $P(F|\varphi)$ is obtained. The left-hand-side plot in Fig. 4 shows the analysis result of the proposed approach with a single MCS run with sample number N = 1000, which shows that the updated failure probability increases with increasing height of water table. The right-hand-side plot shows the average and 95% confidence interval of the results from independent 500 MCS runs.

To examine the analysis results, let us consider the following verifying procedure based on MCS: Employ MCS to generate numerous samples of Z, each sample corresponds to a monitoring value φ. Suppose the actual monitoring value is $\hat{\varphi}$, the samples whose monitoring values are close to $\hat{\varphi}$ (in the range of $\hat{\varphi}\pm 0.1$) are kept. Assume that there are m such samples and among them, H samples satisfy the failure definition. Then $P\left(F\,|\,\hat{\varphi}\right)$ can be roughly estimated as H/m. Figure 4 shows the examining results when $\hat{\varphi}$ is 1, 2, … and 10 m (including the estimate and 95% confidence interval). Comparisons between the analysis and examining results indicate that the proposed method provides consistent $P(F|\varphi)$ estimates. Nevertheless, the proposed approach requires much less computation than the examining MCS approach.

4.2 Deep Excavation

This real case of deep excavation is taken from Ou, *et al.* (1998). The site was located in the Taipei City in Taiwan. The diaphragm wall is 35 m deep and 0.9 m in thickness. The excavation process is divided into seven stages shown in Fig. 5.

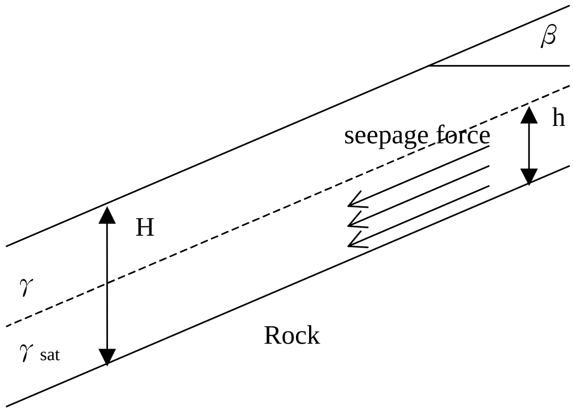

Fig. 2 An illustration of the infinite slope

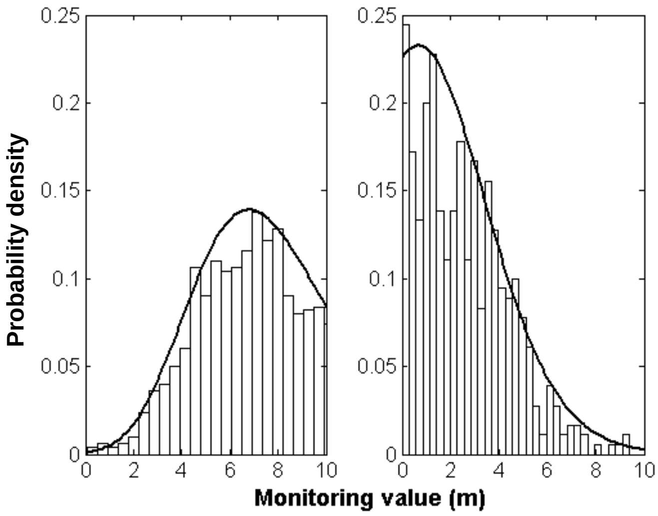

Fig. 3 The histograms of the $f(\varphi|F)$ (left) and $f(\varphi|F^C)$ (right) samples and the corresponding maximum entropy estimates of $f(\varphi|F)$ and $f(\varphi|F^C)$ (Example 1)

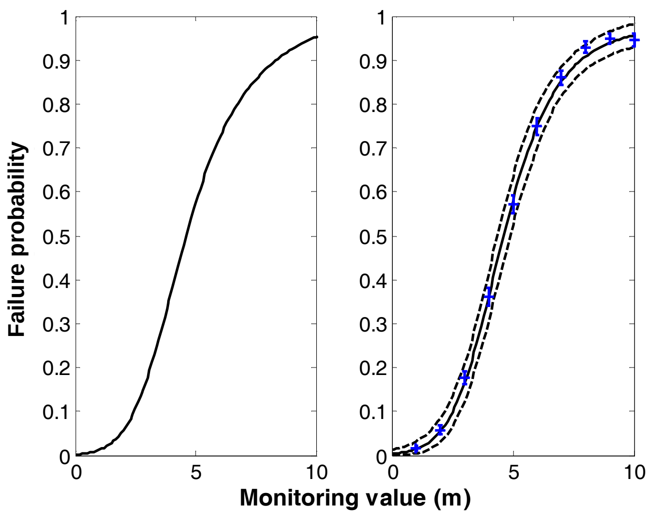

Fig. 4 Example 1 analysis results. The left-hand-side plot is the result from a single MCS run, while the right-hand-side plot compiles the results from 500 independent MCS runs.

The final excavation depth is 19.7 m. The excavation site mainly consists of three clayey soil layers and three sandy soil layers (the soil layering details can be found in Tables 1 and 2). Finite element analyses (the mesh is shown in Fig. 6, after Ou, *et al*. (1998)) are conducted to simulate the excavation process of this case study. Only the final stage of the excavation is analyzed and discussed.

In the finite element analysis, the modified hyperbolic model developed by Hsieh and Ou (1997) is adopted for the clayey soils, while the traditional hyperbolic model developed by Duncan and Chang (1970) is taken for sandy soils. The model developed by Hsieh and Ou (1997) is slightly different from Duncan and Chang's model: the stress-strain relationship is still hyperbolic:

$$\sigma_1 - \sigma_3 = \frac{\varepsilon}{\dfrac{1}{E_e(\varepsilon)} + R_f \dfrac{\varepsilon\,[1-\sin(\phi)]}{2c\cos(\phi) + 2\sigma_3 \sin(\phi)}} \tag{11}$$

where σ_1 and σ_3 are the principal stresses; ε is the axial strain; c and ϕ are effective cohesion and friction angle, respectively; R_f is called the failure ratio; $E_e(\varepsilon)$ is the elastic modulus. The major difference in Hsieh and Ou's model is that $E_e(\varepsilon)$ depending on the axial strain according to the following relationship:

$$\frac{E_e(\varepsilon)}{s_{uc}} = \begin{cases} \dfrac{E_i}{s_{uc}} - \dfrac{\varepsilon - 10^{-3}\%}{a + (\varepsilon - 10^{-3}\%)/b} & \varepsilon > 10^{-3}\% \\ \dfrac{E_i}{s_{uc}} & \varepsilon \le 10^{-3}\% \end{cases} \tag{12}$$

$$E_i = KP_a \left(\frac{\sigma_3}{P_a} \right)^n \tag{13}$$

where E_i is the initial elastic modulus; P_a = 101.4 kPa; K is the modulus parameter; n is the modulus exponent; s_{uc} is the undrained shear strength; a and b are parameters in the $E_e - \varepsilon$ relationship. This strain-dependent modulus model is based on the observations made by Wood (1990) and Jardine (1992): the small-strain modulus of clayey soils is considerably higher than that in common range. Hsieh and Ou (1997; 1998) found that the modified model outperforms the original model in predicting ground settlement in many deep excavation problems.

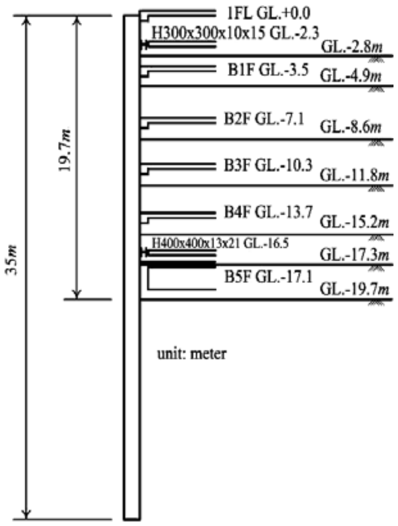

Fig. 5 An illustration of the deep excavation case study (after Ou, *et al*. (1998))

Table 1 Prior PDF parameters for the clayey soils in Example 3 (mean values from Ou, *et al*. (1998))

Depth (m)	Soil type	Normal distribution	
		Total unit weight (kN/m^3)	
		mean	c.o.v.
0.0 ~ 5.6	CL	18.25	7%
8.0 ~ 33.0	CL	18.93	7%
35.0 ~ 37.5	CL	18.15	7%

Table 2 Prior PDF parameters for the sandy soils in Example 3 (mean values from Ou, *et al*. (1998))

Depth (m)	Soil type	Normal distribution				Lognormal distribution	
		Total unit weight (kN/m^3)		ϕ (°)		K (MPa)	
		mean	c.o.v	mean	c.o.v	mean	c.o.v
5.6 ~ 8.0	SM	18.93	7%	31	10%	76	40%
33.0 ~ 35.0	SM	19.62	7%	31	10%	254	40%
37.5~ 46.0	SM	19.62	7%	32	10%	254	40%

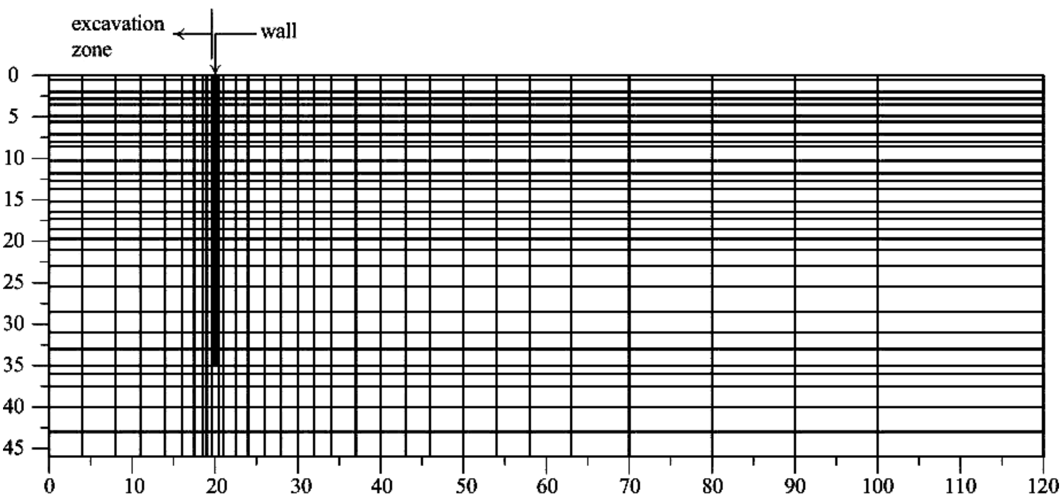

Fig. 6 The finite element mesh used in Example 2 (after Ou, *et al.* (1998))

The uncertainties associated with the real case study include the properties of the sandy and clayey soils, locations of water tables and the forces provided by the brace systems. In selecting the PDF of the uncertainties, most of the mean values of the uncertain soil properties are taken to be equal to the laboratory test results from Ou, *et al*. (1998) (see Tables 1 and 2), while c.o.v.s are chosen based on the recommendations made by Phoon (1995). All uncertainties are assumed to be independent. The failure ratio R_f of the sandy and clayey soils are taken to be uniformly distributed over [0.8, 0.9], and the modulus exponent n to be uniformly distributed over [0.4, 1.0]. The Poisson ratios v of all sandy soils are uniformly distributed over [0.2, 0.4], while those of all clayey soils are equal to 0.49. Cohesion c of all sandy soils are taken to be 0. According to Ou, *et al*.'s laboratory test results, E_i / s_{uc} of the clayey soils is roughly among 1800 and 2200, so we assume E_i / s_{uc} is uniformly distributed over [1800, 2200]. For the clayey soil layers, s_{uc} / σ_v' is assumed to be normally distributed with mean value described by the following equation:

$$\frac{s_{uc}}{\sigma_v'} = \begin{cases} 0.28\left(\dfrac{z}{z-3}\right)^{0.9} & z \ge 3.5 \\ 0.28 & z < 3.5 \end{cases} \tag{14}$$

where z is the depth, while the c.o.v. is set to be 20% (estimated based on the results in Ladd and Foote (1974)).

In Fig. 7, the laboratory test results from Ou, *et al*. (1998) regarding the small-strain modulus are shown as "○" in the figure, while the solid line describe the relationship in (12) with a and b set to be 6×10^{-8} and 1520, respectively. Therefore, $E_e(\varepsilon)/s_{uc}$ is assumed to be normally distributed, its mean value is described by the solid line in Fig. 7, and c.o.v. is roughly estimated to be 10%. Since there are more than one impermeable layer in the case study, two water tables are present, whose depths are uncertain and are uniformly distributed over [2 m, 3 m] and [14 m, 19 m]. Finally, the forces provided by the three supports are uncertain and uniformly distributed over [20000 kN/m, 22000 kN/m], [10000 kN/m, 12000 kN/m], and [5000 kN/m, 5400 kN/m] from bottom to top. For most uncertainties, their assumed distribution types, mean values, and c.o.v.s are shown in Tables 1 and 2.

For this case study, the deformation of the diaphragm wall is monitored, and the monitoring value φ is chosen to be the maximum lateral deformation of the wall. Several failure definitions are considered: the maximum ground settlement is greater than 5, 10, 15, 20, 25, and 30 cm, and the corresponding failure probabilities (without the monitoring data) estimated from MCS are roughly 80%, 30%, 10%, 6%, 5%, and 4%, respectively. In Fig. 8, the histograms of the samples distributed as $f(\varphi | F)$ and $f(\varphi | F^C)$ are shown, using the failure threshold of 10cm as an example, and the estimated $f(\varphi | F)$ and $f(\varphi | F^C)$ based on the maximum entropy theory are also shown in the same plot. With (1) and (8), $P(F | \varphi)$ for various failure thresholds can be estimated, shown in Fig. 9. Please note that in order to obtain the six curves in the figure, only a single $N = 2000$ MCS run is needed. From Fig. 9, it is obvious that failure probability grows with increasing monitoring values. The finite element analysis is quite time-consuming, so the confidence intervals are not computed, nor are the examining results.

Note that the results shown in Fig. 9 can only apply to the final excavation stage. For the earlier six stages of the excavation, the relationship between the updated failure probability and the maximum wall deformation will be different. Moreover, the results are only for this specific site. For other sites, the relationship between the updated failure probability and the maximum wall deformation will also be different.

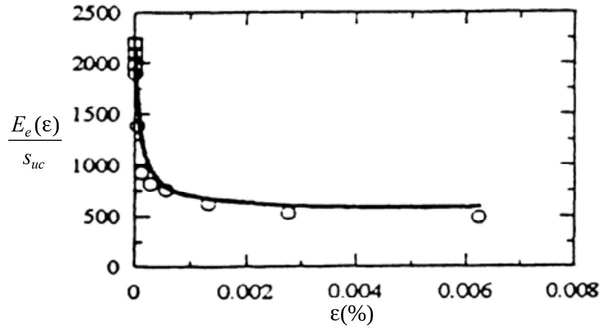

Fig. 7 $E_e(\varepsilon)/s_{uc}$ ***v.s.*** **ε relationship. The dots are obtained from the laboratory tests conducted by Ou, *et al.* (1998), and the solid line is the fit with (12), where *a* and *b* are taken to be 6×10^{-8} and 1520, respectively**

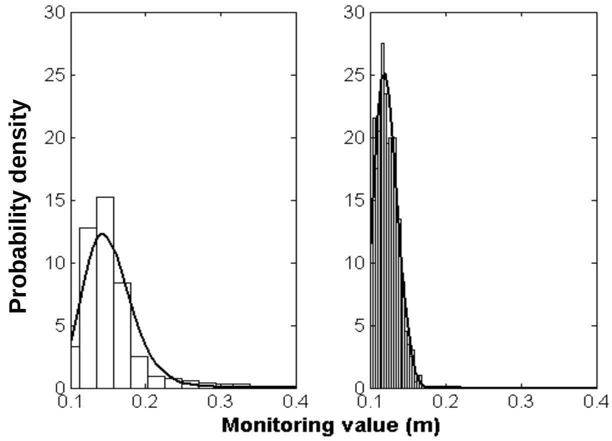

Fig. 8 The histograms of the $f(\varphi | F)$ (left) and $f(\varphi | F^C)$ (right) samples and the corresponding maximum entropy estimates of $f(\varphi | F)$ and $f(\varphi | F^C)$ (Example 2)

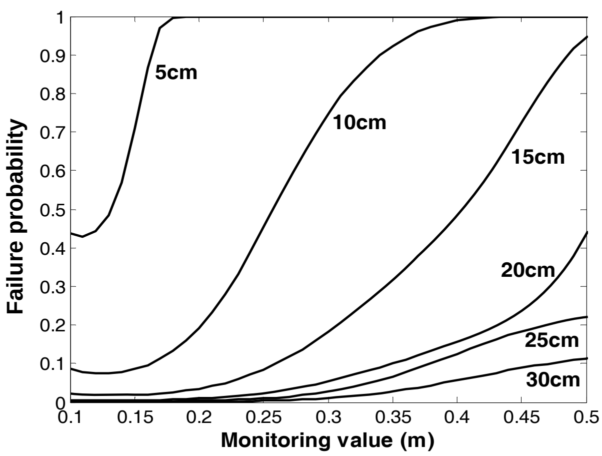

Fig. 9 Example 2 analysis results based on a single *N* = 2000 MCS run

5. DISCUSSIONS AND CONCLUSIONS

A new method of updating reliability of geotechnical systems is proposed in this paper. Geotechnical systems are usually very uncertain due to the fact that its materials, soils and rocks, are buried in the ground. The consequence is that in some cases, the failure probability can be unacceptably large. The proposed method is capable of reducing the uncertainties by using new information, *i.e.,* scalar monitoring data φ from the target system, to update the failure probability. It is based on simple Monte Carlo simulation and the maximum entropy theory, therefore, the methodology can be applied to general systems with high dimensional uncertainties. Moreover, the functional relationship between the updated failure probability and the monitoring value can be obtained prior to the monitoring process. This means in real applications, it is not necessary to conduct a new analysis in an online manner.

Two examples are studied in the paper to demonstrate the use of the new method. Between them, the results from one example are verified by crude Monte Carlo simulation. The verification shows that the new method produces consistent results. Nevertheless, the proposed approach requires much less computation than the examining MCS approach.

There are several major limitations of the new method: (a) because Monte Carlo simulation is employed, when very small failure probability is of interest, the new method is not efficient; (b) for the current setting of the new method, the monitoring value is a scalar. However, in real situation, there are usually many monitoring values. Let us take the deep excavation case as an example, the monitoring values include the entire wall deformation profile. When implementing the new method, the entire profile is converted into a single index, the maximum wall deformation. How to choose this conversion so that the correlation between the chosen single index and the failure event is maximized is a future research topic.

REFERENCES

Ang, A., and Tang, W. H. (1984). *Probability Concepts in Engineering Planning and Design, Vol. I, Basic Principles*, John Wiley & Sons.

Au, S. K., and Beck, J. L. (2001). “First excursion probability for linear systems by very efficient importance sampling.” *Probabilistic Engineering Mechanics*, 16(3), 193−207.

Beck, J. L., and Au, S. K. (2002). “Bayesian updating of structural models and reliability using Markov chain Monte Carlo simulation.” *Journal of Engineering Mechanics*, 128(4), 380−391.

Benjamin, J. R., and Cornell, C. A. (1970). *Reliability, Statistics and Decision for Civil Engineers*, McGraw Hill, New York.

Ching, J., and Beck, J. L. (2006). “Real-Time Reliability Estimation for Serviceability Limit States in Structures Using only Incomplete Output Data.” accepted by *Probabilistic Engineering Mechanics*.

Duncan, J. M., and Chang, C.-Y. (1970). “Nonlinear analysis of stress and strain in soil.” *ASCE Journal of the Soil Mechanics and Foundations*, 96(5), 1629−1653.

Gelman, A., Carlin, J., Stern, H., and Rubin, D. (1995). *Bayesian Data Analysis*. Chapman & Hall, London.

Hsieh, P. G., and Ou, C. Y. (1997). “Use of the modified hyperbolic model in excavation analysis under undrained condition.” *Geotechnical Engineering, SEGAS*, 28(2), 123−150.

Hsieh, P. G., and Ou, C. Y. (1998). “Shape of ground surface settlement profiles caused by excavation.” *Canadian Geotechnical Journal*, 35(6), 1004−1017.

Jardine, R. J. (1992). “Some observations on the kinematic nature of soil stiffness.” *Soil and Foundations*, 32(2), 111−124.

Jaynes, E. T. (1957). “Information theory and statistical mechanics.” *Physical Review*, 106, 620-630. Also in Rosenkrantz, R. D. and Jaynes, E. T. (1982), *Papers on Probability, Statistics and Statistical Physics*, Dordrecht, Holland, D. Reidel Publishing Co.

Ladd, C. C., and Foote, R. (1974). “A new design procedure for stability of soft clays.” *Journal of the Geotechnical Engineering Division*, ASCE, 100(7), 763−786.

Madsen, H. O., Krenk, S., and Lind, N. C. (1986). *Methods of Structural Safety*, Prentice-Hall, New York.

Ou, C. Y., Liao, J. T., and Lin, H. D. (1998). “Performance of diaphragm wall constructed using top-down method.” *Journal of Geotechnical and Geoenvironmental Engineering, ASCE,* 124(9), 987−808.

Phoon, K. K. (1995). “Reliability-based design of foundations for transmission line structures.” Ph.D. Dissertation, Department of Civil and Environmental Engineering, Cornell University.

Thoft-Christensen, P., and Murotsu, Y. (1986). *Application of Structural Systems Reliability Theory*, Springer Verlag, Berlin.

Wood, D. M. (1990). *Soil Behavior and Critical State Soil Mechanics*, Cambridge University Press.

APPENDIX
MAXIMUM ENTROPY DENSITY ESTIMATION

This appendix describes a procedure of estimating PDF based on the maximum entropy theory given the sample moments of the PDF. We only discuss the four-moment maximum entropy approximation in this appendix. As discussed in the main text, the resulting optimization problem subjected to moment constraints is as following:

$$\max_{g(\cdot)}\left(-\int_L^U \log g(\varphi)\cdot g(\varphi)\,d\varphi\right)$$
$$\text{subjected to}\quad \int_L^U g(\varphi)d\varphi = 1\ ,\quad \int_L^U \varphi^i g(\varphi)\,d\varphi = \mu_i \quad i = 1\ldots 4 \tag{15}$$

and the optimal solution has the following form:

$$g(\varphi) = e^{-\lambda_0 - \sum_{i=1}^{4}\lambda_i\varphi^i} \tag{16}$$

where λ_0, λ_1, …, and λ_4 must satisfy

$$\int_L^U g(\varphi)\,d\varphi = 1\ ,\quad \int_L^U \varphi^i g(\varphi)\,d\varphi = \mu_i \quad i = 1\ldots 4 \tag{17}$$

According to the above five nonlinear equations, the goal is to determine the five unknowns λ_0, λ_1, …, and λ_4.

The Newton method is employed to solve the five unknowns. For notational simplicity, let us denote

$$m_i(\lambda) = \int \varphi^i \cdot e^{-\sum_{i=0}^{4}\lambda_i\varphi^i}\,d\varphi \quad i = 0\ldots 4 \tag{18}$$

where λ encompasses λ_0, λ_1, λ_2, λ_3, λ_4. The original five equations can be written as

$$m(\lambda) \equiv \begin{bmatrix} m_1(\lambda) \\ m_2(\lambda) \\ m_3(\lambda) \\ m_4(\lambda) \\ m_5(\lambda) \end{bmatrix} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix} \equiv \mu \tag{19}$$

and the corresponding Newton algorithm is

$$\lambda^{(i+1)} = \lambda^{(i)} + \left[\nabla m\left(\lambda^{(i)}\right)\right]^{-1}\left[\mu - m\left(\lambda^{(i)}\right)\right] \tag{20}$$

$$\nabla m(\lambda) = \begin{bmatrix} \frac{\partial m_0}{\partial \lambda_0} & \frac{\partial m_0}{\partial \lambda_1} & \cdots & \frac{\partial m_0}{\partial \lambda_4} \\ \frac{\partial m_1}{\partial \lambda_0} & \frac{\partial m_1}{\partial \lambda_1} & \cdots & \frac{\partial m_1}{\partial \lambda_4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial m_4}{\partial \lambda_0} & \frac{\partial m_4}{\partial \lambda_1} & \cdots & \frac{\partial m_4}{\partial \lambda_4} \end{bmatrix}$$

$$\frac{\partial m_i}{\partial \lambda_j} = -\int \varphi^{i+j} \cdot e^{-\sum_{i=0}^{4} \lambda_i \varphi^i} d\varphi \tag{21}$$

The integral in the above equation can be calculated by using any numerical integration technique, *e.g.,* Gauss quadrature. Note $-\nabla m(\lambda)$ is always a positive definite matrix, so the solution of λ is unique. Plugging the solution back to $g(\varphi) = e^{-\lambda_0 - \sum_{i=1}^{4} \lambda_i \varphi^i}$ will give us the maximum entropy estimation of the target PDF.