# SEISMIC DISPLACEMENT ANALYSIS OF FREE-STANDING HIGHWAY BRIDGE ABUTMENTS

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## ABSTRACT

Seismic displacements of two typical highway bridge abutments used in Taiwan are examined based on the input ground accelerations suggested by both new and old aseismic design codes. A pseudo-static-based multi-wedge method is used in conjunction with Newmark's sliding block theory to evaluate seismic displacement of these bridge abutments. It was found that (1) the design peak ground acceleration specified in the new code are significantly greater than those used in the old code for some near-fault areas in Taiwan; (2) for the gravity-type bridge abutment, the seismic displacement under both level 2 and level 3 design earthquakes are beyond the permissible displacement suggested in the literature, indicating the vulnerability of the gravity-type bridge abutments to medium-large earthquakes; (3) A vertical-to-horizontal ground acceleration ratio, $\lambda = 0.67$ used in the new code gives slightly conservative seismic displacement evaluations compared to those calculated using $\lambda = 0.25$ measured at near-fault seismographs; (4) the passive resistance in front of the gravity-type bridge abutment may significantly reduce the seismic displacement of the abutment. Regular integrity inspections for passive zone or pre-earthquake reinforcement program is suggested for the gravity-type abutments to avoid excessive horizontal seismic displacements.

*Key words:* seismic design, seismic displacement, bridge abutment, pseudo-static method, Newmark's sliding block method.

## 1. INTRODUCTION

A great number of free-standing highway bridge abutments founded on shallow spreading footing were damaged during the 1999 Chi-Chi earthquake ($M_L = 7.3$). This damage was associated with large relative displacement and/or settlement between the bridge deck and the approach highway embankment. Damage and/or failure of bridge abutments associated with excessive abutment displacements has also been reported by Buckle (1994) and Housner and Theil (1995) in recent earthquakes. Fishman and Richards (1996) reported that a major part of the 40,000 bridge abutments in New York State are of the free standing-type and more than half are founded on shallow spread footing as the one focused in the present study. Fishman and Richards (1996) analyzed fifty representative bridge abutments with computed static safety factors for sliding and overturning all greater than 1.5 and 2.0, respectively, using modified coupled equations of motion proposed by Siddharthan *et al.* (1992) and a theory describing reduction of bearing capacity of foundation soil subjected to seismic loading developed by Richards *et al.* (1993). They found that all analyzed bridge abutments over 6 meters high had a critical (or threshold) horizontal ground acceleration ($a_y$) less than 0.2 g (g: gravitational acceleration) with many being less than 0.15 g and the abutments over 7.5 m had even smaller values of $a_y < 0.1$ g. Their results indicated that a significant amount of bridge abutments are vulnerable to seismic-induced displacement and remedial measures for the high seismic risk abutments are required. Siddharthan and El-Gamal (1996) reported that about 80% of the bridge abutment fills near the epicentral area in the 1994 Northridge earthquake ($M_L = 6.7$) were subjected to measurable differential settlement between the abutment and the approach embankment.

Highway bridge abutments constitute an important link between the approach embankment of the highway and the deck of the bridge. The integrity of the bridge abutment during strong ground disturbance may minimize the cost and time of post-earthquake retrofits. A possible measure to ensure the integrity of bridge abutments is to limit the relative horizontal displacement of the abutment under the seismic force induced by the inertia of the backfill and abutment and the thrust from the deck applied to the seat at the crest of the abutment. This requires an accurate evaluation of the seismic displacement of the bridge abutments in the aseismic design. Bridge abutments are structurally and functionally similar to the conventional soil retaining walls except that a superstructure loads from the bridge deck are applied at the seat of the bridge abutment. For the bridge abutments, the aseismic design and displacement analysis procedures are similar in principles to those for the soil retaining walls (*e.g.*, Whitman, 1990). Under current aseismic design guidelines for highway-related structures in the North America, force-based design of abutments and soil retaining walls still prevails, and displacement-based design is not mandatory even for an essential abutment located in a seismically active area (namely, a design catagory '*D*', the highest priority among four design categories, see American Association of State Highway and Transportation Officials, AASHTO, 2002)

In Japan, until the shock of the 1995 Hyogoken-Nambu earthquake ($M_L = 7.2$) in the Kobe area, the railway authority first adopted two-level input earthquake intensity in the aseismic design of reinforced and unreinforced structures, including earth retaining walls, bridge abutments and earth embankments (Tatsuoka *et al.*, 1996, 1998; Japan Railway Technical Research Institute, JRTRI, 1999). Furthermore, displacement-based damage levels were specified for related soil structures under corre-

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sponding input intensities of ground shaking. A typical example of the damage levels to the railway bridge abutments used in Japan is shown in Table 1. The necessity of displacement analysis arises when a very high level of ground shaking (*e.g.*, an earthquake with a 2500 years of return period) is considered in the aseismic design. Fulfilling all the stability requirements considered in a force-equilibrium-based design, generally leads to an over-conservative design outcome (the sizes of the structure and/or structural components). A compromise between construction costs and the extent of earthquake damage must be achieved by allowing a certain displacement (or deformation) of the structure based on the serviceability requirements.

A pioneer study based on the sliding block theory proposed by Newmark (1965) on the displacement-based aseismic design of earth retaining walls was performed by Richard and Elms (1979). Since then, numerous methods for evaluating translational and/or rotational movements of various soil retaining structures and slopes based on Newmark's sliding block theory have been proposed (*e.g.*, Siddharthan and El-Gamal, 1996; Cai and Bathurst, 1996; Ling and Leshchinsky, 1998; Tatsuoka *et al.*, 1998; Huang *et al.*, 2003; Huang, 2005).

A pseudo-static method termed the 'multi-wedge method' incorporating with Newmark's sliding block theory is used in the following. This method was developed by Huang *et al.* (2003) and Huang and Chen (2004), and was validated by analyzing four geosynthetic-reinforced modular block walls in the 1999 Chi-Chi earthquake (Huang *et al.*, 2003), two leaning-type soil retaining walls situated on slopes (Huang and Chen, 2004; Huang, 2005), a geosynthetic-reinforced railway embankment survived the 1995 Hyogoken-Nambu earthquake (Huang and Wang, 2005a) and some reduce-scaled reinforced and unreinforced soil model walls subjected step-wise increased ground excitations using a shaking table (Huang *et al.*, 2000; Kato, 2001; Wu, 2005). In addition, similar pseudo-static-based approaches have been used for calculating seismic displacement of soil retaining walls, *e.g.*, Richards and Elms (1979), Whitman (1990), Cai and Bathurst (1996), Ling and Leshchinsky (1998) and Tatsuoka *et al.* (1998). The present study focuses on the following issues:

(1) Comparisons of design ground accelerations in the old and new aseismic design guidelines in Taiwan.

(2) Investigations of the seismic stability and seismic displacement for typical bridge abutments, namely, the cantilever-type and gravity-type abutments.

(3) Investigations into the effect of vertical ground acceleration on the seismic displacement of the bridge abutment. Studies into the effect of vertical ground acceleration on the behavior of conventional and/or reinforced soil walls based on measured ground acceleration records suggested that the horizontal and vertical peak ground accelerations are out of phase and the stability of the structures can be evaluated based on the assumption of $k_v = 0$ (Seed and Whitman, 1970; Wolfe *et al.*, 1978; Madabhushi, 1996; Tatsuoka *et al.*, 1998; Bathurst and Alfaro, 1996; Huang and Wang, 2005b; Huang, 2005). This is significantly different from that suggested by Steward *et al.* (1994) and Ling and Leshchinsky (1998). In their analyses, peak values of horizontal and vertical ground accelerations, namely $a_{h_{\max}}$ and $a_{v_{\max}}$, respectively, despite the fact that they were out of phase, were used for deriving a parameter of $\lambda$ $(= a_{v_{\max}} / a_{h_{\max}})$ for the seismic stability analysis.

Although bearing capacity failure may constitute a major cause of damage to the bridge abutment because of the drastic decrease in the bearing capacity induced by inclined loading at the footing as reported by Richard *et al.* (1993), Tatsuoka *et al.* (1998) and Huang and Chen (2004). It is assumed in this study that the bridge abutments are placed on competent ground. Therefore, the issue of bearing capacity failure is beyond the scope of this study.

**Table 1 Criteria proposed for displacement-based aseismic design of soil retaining walls (compiled from Huang, 2005)**

| | Eurocode 8 (1994) | Wu and Prakash (1996) | JRTRI (1999) | AASHTO (2002) |
|---|---|---|---|---|
| Permissible horizontal displacement | $300 \cdot a_{\max}$ (in mm) $a_{\max}$ = maximum design ground acceleration (g) | 2% of the wall height | — | $250 \cdot a_{\max}$ (in mm)[c] $a_{\max}$ = maximum design ground acceleration (g) |
| Failure horizontal wall displacement | — | 10% of wall height | — | — |
| Permissible differential settlement | — | — | 0.1 to 0.2 m[a] | — |
| Severe differential settlement | — | — | > 0.2 m[b] | — |

[a] Damage level 3, requiring a minor retrofit measure for bridge abutments.

[b] Damage level 4, requiring a long-term retrofit measure for bridge abutments.

[c] Expected seismically induced horizontal displacement of abutments. Provisions should be made to accommodate this displacement when minimal damage is desired at abutment supports.

## 2. COMPARISONS BETWEEN OLD AND NEW DESIGN CODES

Old seismic design codes (SDC) for highways and bridges in Taiwan were issued in 2000 by the Construction and Planning Agency, Ministry of the Interior, (CPAMI, 2000). In which, two seismic zones with different design values of horizontal peak ground acceleration ($HPGA_{design}$, see Fig. 1(a)) were used *i.e.*, $HPGA_{design}$ = 0.23 g for Taipei and Kaohsiung areas; 0.33 g for other areas. (CPAMI, 2000). As to the demands for a more sophisticated design guideline, which reflects the progress in the seismic-related studies, the new SDC for buildings was enforced in July 2005 (CPAMI, 2005). The new SDC has the following features which distinguish it from the old one: (1) horizontal spectral acceleration coefficients (an example for short-period structures denoted as $S_S^D$ is shown in Fig. 1(b)) are used to determine the design seismic force for structures, (2) seismic zones with different horizontal spectral acceleration coefficients for rather detailed geographical or administrative units in Taiwan, (3) near-fault corrective factors ($N_A$ = 1.0 ~ 1.42) are considered for near-fault areas (within 15 km from the active faults) (4) increased design vertical component of peak ground acceleration ($VPGA_{design}$). (5) three levels of $HPGA_{design}$ and safety requirements, namely, Level 1: the structure should remain in an elastic condition for medium/small earthquakes with a 30-year return period, Level 2: the structure should remain in an allowable ductile state under a design earthquake with a 475-year return period, Level 3: the structure should be able to avoid reaching an ultimate ductility state (or reaching ultimate collapase state) under a design earthquake with a 2500-year return period (CPAMI, 2005). For earth retaining walls including the bridge abutment, both old and new design guidelines suggested a design horizontal seismic coefficient ($k_h$) equal to $(0.5 \cdot HPGA_{design})/g$ ($g$: gravitational acceleration) Seismic displacement calculations were not required in both design codes. It is noted that the bridge abutments to be discussed herein are classified as "short-period structure" in new SDC based on a numerical study conducted by Richardson and Lee (1975) on reinforced soil retaining walls. The fundamental period $T_1$ (in second/cycle) for a wall with a total height of $H$ (in meter) can be expressed by:

$$T_1 = (0.0018 - 0.003) \times H \tag{1}$$

For a 10 m-high bridge abutment, $T_1$ = 0.018 – 0.03 (sec/cycle). This is much smaller than 1 sec/cycle and is classified as short-period structures in the SDC.

Figures 1(a) and 1(b) show the seismic zones with respective $HPGA_{design}$ in old SDC and an example of spectral acceleration coefficient ( $S_S^D$ ) in new SDC respectively. A comparison of the $HPGA_{design}$ in the old and new SDC is shown in Table 2. The $HPGA_{design}$ for major bridges in three highly populated zones of west Taiwan and other three areas near active faults in Taiwan were selected for comparison as shown in Fig. 2 and Table 3. This table shows that the values of $HPGA_{design}$ in the new SDC are generally 18% ~ 63% greater than those in the old SDC.

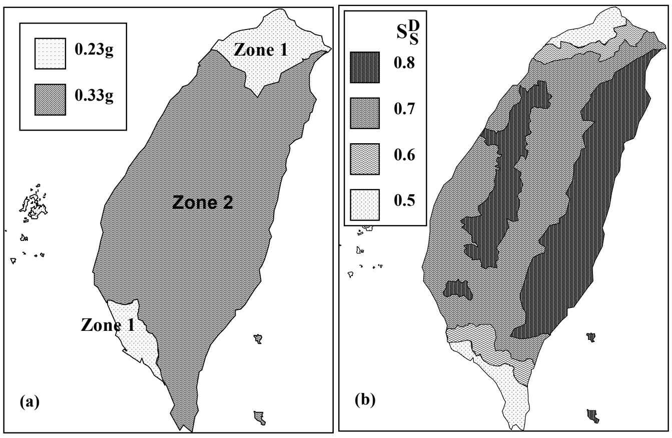


**Fig. 1 Seismic zones of Taiwan suggested in the aseismic design guidelines: (a) seismic zones and peak horizontal ground accelerations (HPGA) in SDC, 2000; (b) an example of seismic zones and horizontal spectral coefficients for short period structures under Level 2 earthquake**

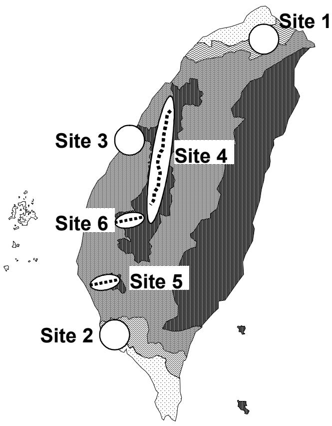


**Fig. 2 Six representative sites in west Taiwan used in the present study**

**Table 2 The comparisons of the design ground accelerations in old and new SDC**

| | Old SDC | | New SDC | |
|---|---|---|---|---|
| Earthquake Levels | $HPGA_{design}$ for other bridge | $HPGA_{design}$ for essential bridges | $HPGA_{design}$ for other bridges | $HPGA_{design}$ for essential bridges |
| Level 1 (30 years return period) | $= \alpha \cdot Z$ | $= 1.2 \cdot \alpha \cdot Z$ | $= 0.095 \cdot S_{DS} \cdot g$ | $= 0.114 \cdot S_{DS} \cdot g$ |
| Level 2 (475 years return period) | $= Z$ | $= 1.2 \cdot Z$ | $= 0.4 \cdot S_{DS} \cdot g$ | $= 0.48 \cdot S_{DS} \cdot g$ |
| Level 3 (2500 years return period) | Not Considered | | $= 0.4 \cdot S_{MS} \cdot g$ | $= 0.48 \cdot S_{MS} \cdot g$ |

Note: (1) $Z$ is equivalent to the values of HPGA as shown in Fig. 1(a); α is the reduction factor for small-medium earthquakes.
(2) $S_{DS}$ is the 'design spectral coefficient' which is a factored $S_S^D$ as shown in Fig. 1(b).
(3) $S_{MS}$ is the 'maximum spectral coefficient' for short period structures subjected to a Level 3 earthquake.

**Table 3 Comparisons of different levels of ground accelerations used in the old and new SDC for major bridges**

| Site | Area | Old SDC (g) | | New SDC (g) | | | New/Old (%) | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 30[(1)] years | 475[(2)] years | 30[(3)] years | 475[(4)] years | 2500[(5)] years | (3)/(1) | (4)/(2) | (5)/(2) |
| 1 | Taipei Basin | 0.054 | 0.276 | 0.069 | 0.288 | 0.384 | 128 | 104 | 139 |
| 2 | Kaohsiung City | 0.054 | 0.276 | 0.088 | 0.370 | 0.475 | 163 | 134 | 172 |
| 3 | Taichung City | 0.077 | 0.396 | 0.091 | 0.384 | 0.480 | 118 | 97 | 121 |
| 4 | Che-Lung-Pu Fault | 0.077 | 0.396 | 0.112 | 0.472 | 0.600 | 145 | 119 | 151 |
| 5 | Hsin-Hwa Fault | 0.077 | 0.396 | 0.112 | 0.472 | 0.619 | 145 | 119 | 156 |
| 6 | Mei-Shan Fault | 0.077 | 0.396 | 0.125 | 0.526 | 0.624 | 162 | 133 | 158 |

It is obvious that for the new SDC, the values of $HPGA_{design}$ are significantly different from those in the old SDC. Many bridge abutments designed and constructed based on the old SDC must be examined using new seismic loadings specified in the new SDC. For those which exhibit excessive seismic displacements, pre-earthquake remedial work must be implemented to mitigate possible disasters in the future.

## 3. METHODOLOGY

A multi-wedge pseudo-static method which was developed by Huang *et al.* (2003) is used in the following analyses. The multi-wedge failure mechanism, as shown in Fig. 3, includes a two-wedge failure (wedge F & wedge B) behind the wall, a sliding failure along the base of the wall and a passive failure in front of the wall (Wedge P). Based on the limit equilibrium for all wedges, a factor of safety against horizontal sliding at the wall base, $F_s$, can be derived (Fig. 3):

$$F_s = (S_f + P_{PH}) / (P_{FH} + W_W \cdot k_h) \tag{2}$$

where,

$k_h$ : Horizontal seismic coefficient (= $a_h/g$, $a_h$: horizontal ground acceleration, $g$: gravitational acceleration)

$S_f$ : Ultimate shear resistance of soils beneath the wall ($S_f = P_{bv} \times \tan\phi_b + c \times B_W$, $\phi_b$: Soil friction angle at the base of the wall, $c$: cohesion of soil, $B_W$: Width at the base of the wall, $P_{bv}$: Normal force acting on the wall base)

$P_{PH}$ : Horizontal component of seismic passive earth resistance in front of the wall ($P_p$) based on the limit equilibrium of wedge '*P*' using an input seismic coefficient, $k_h$.

$P_{FH}$ : Horizontal component of seismic active earth pressure behind the wall ($P_F$) based on the limit equilibrium of wedges '*B*' and '*F*' using an input seismic coefficient, $k_h$.

$W_W$ : Weight of retaining wall.

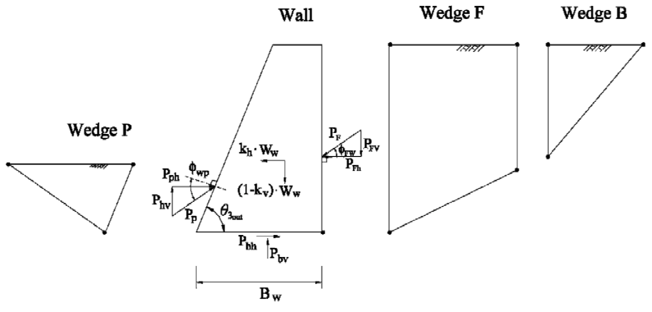


**Fig. 3 Failure mechanism and force equilibrium for the wall**

The seismic active earth pressure coefficient, $K_{AE}$ (= $(2 \cdot P_F) / (\gamma \cdot H^2)$, γ: unit weight of soil, $H$: total height of the wall) and seismic passive earth pressure coefficient, $K_{PE}$ (= $(2 \cdot P_p) / (\gamma \cdot H^2)$) calculated above are verified using the Mononobe-Okabe (M-O) theory (Mononobe, 1924; Okabe, 1924) and the experimental results reported by Fang *et al.* (1997) as described in detail by Huang *et al.* (2003) and Huang and Chen (2004).

## 4. SEISMIC DISPLACEMENT OF FREE-STANDING BRIDGE ABUTMENTS

A typical cantilever-type highway bridge abutment provided by Chen (2002) and a typical gravity-type highway bridge abutment currently used in Taiwan are adopted for calculating horizontal seismic displacement (Figs. 4(a) and 4(b)) in this study. Table 4 summarizes the safety factors against sliding and overturning instabilities under static and seismic ($k_h$ = 0.2 and $k_v$ = 2/3 $k_h$ = 0.13) conditions for the gravity-type and cantilever-type bridge abutments as shown in Figs. 4(a) and 4(b). It can be seen that the instability of these abutments is controlled by the sliding mode and the gravity-type abutment is only marginally stable when the passive resistance in front of the wall was not taken into account (*i.e.*, $P_p$ = 0). The merit of the cantilever-type abutment in terms of seismic stability partially comes from its wider base width (6.5 m) compared to 4.7 m for the gravity-type abutment. This wider base containing the backfill above the heel of the base slab forms a semi-rigid monolith and behaves reliably against seismic loads. A generally higher seismic stability for cantilever-type soil retaining walls than that for the gravity-type wall has also been reported by Tatsuoka *et al.* (1998) in the post-earthquake reconnaissance of 1995 Hyogoken-Nambu earthquake. The calculations are based on the value of $k_{h_{cr}}$ obtained in Fig. 5, the sliding-block theory proposed by Newmark (1965) and four ground acceleration records obtained in 1999 Chi-Chi ($M_L$ = 7.3, $M_L$ = Magnitude in Richter scale), 1995 Hyogoken-Nambu ($M_L$ = 7.2), 1989 Loma Prieta ($M_L$ = 7.1) and 1940 El-Centro earthquakes ($M_L$ = 7.1) as shown in Figs. 6(a) ~ 6(d). It can be seen in Fig. 5 that for the gravity-type of bridge abutment the influence of passive resistance to the values of $k_{h_{cr}}$ is significant ($k_{h_{cr}}$ = 0.265 vs. $k_{h_{cr}}$ = 0.152). The measurable influence (about 37% difference in $k_{h_{cr}}$) of the passive resistance to the stability of cantilever-type abutment expressed by the values of $k_{h_{cr}}$ can also be detected. The importance of the passive resistance to the seismic displacement of the soil retaining wall has

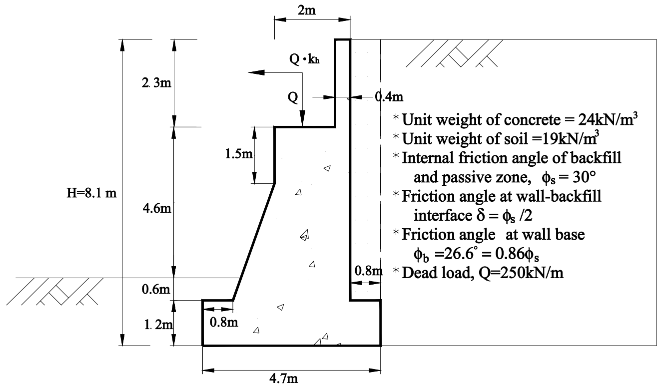


**Fig. 4(a) Cross-section of the gravity abutment analyzed in the present study**

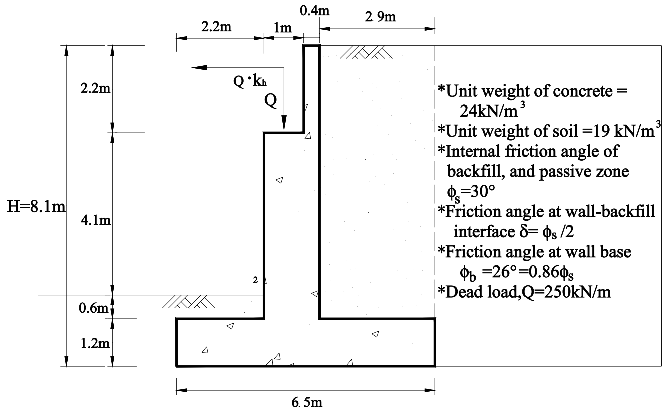


**Fig. 4(b) Cross-section of the highway bridge abutment analyzed in the present study**

been pointed out by Huang and Chen (2004) and Huang (2005) in a post-earthquake investigation of two collapsed leaning-type soil retaining walls used to support highway embankments in slope areas. In the practical design of soil retaining structures, the passive resistance at the toe of the wall is usually ignored. This may lead one to falsely conclude that the passive resistance to the stability of the wall is not important and a regular examination of the integrity of the passive zone is not required. However, the great influence of passive resistance to the values of $k_{h_{cr}}$ and also to the seismic displacements to be discussed later indicates that the insurance of the integrity of the passive zone can be vital to the disaster mitigation of free-standing bridge abutments.

It is also noted that in the following displacement calculations for the typical bridge abutments shown in Figs. 4(a) and 4(b), a single value of internal friction angle of soil, *i.e.*, $\phi_s = 30°$ is used. This value of $\phi_s$ represents a factored soil strength in the design taking into account the uncertainties in the soil properties and the possible strength deterioration associated with progressive failure in the backfill. Therefore, it is considered that $\phi_s = 30°$ may represent the residual strength of the soil, inferring that the displacements of the walls calculated in the following are conservative. Note that these recorded accelerations from the above-mentioned four earthquakes are scaled to $a_{\max} = \text{HPGA}_{\text{design}}$; $a_{\max}$: horizontal peak ground acceleration in earthquake records, $\text{HPGA}_{\text{design}}$: the design values of horizontal peak ground acceleration specified in old and new SDC.

**Table 4 Results of static and pseudo-static seismic stability analyses for two bridge abutments**

| | | Static condition * (DL = 0, $q$ = 20 kN/m)※ | | Seismic condition ** (DL > 0, $q$ = 0)※ | |
|---|---|---|---|---|---|
| | | $P_p = 0$ | $P_p > 0$ | $P_p = 0$ | $P_p > 0$ |
| Cantilever-type | $F_s$ against horizontal sliding | 2.4 | 6.5 | 1.2 | 1.4 |
| | $F_s$ against overturning | 7.1 | 8.2 | 2.2 | 2.3 |
| Gravity-type | $F_s$ against horizontal sliding | 1.7 | 4.6 | 1.1 | 1.3 |
| | $F_s$ against overturning | 4.1 | 4.7 | 1.6 | 1.6 |

* $\delta_{s-s} = \phi_s$ and $\delta_{s-c} = \phi_s/2$ ($\delta_{s-s}$: friction angle on the assumed failure lines in soil, $\delta_{s-c}$: friction angle on the soil-concrete interface)
** $\delta_{s-s} = \phi_s$, $\delta_{s-c} = \phi_s/2$, $k_h = 0.2$ and $k_v = 2/3\ k_h = 0.13$
※ DL ($Q$ = 250 kN/m): dead load applied at the seat of bridge abutment; $q$: uniform surcharge applied on the top of approach embankment

**Table 5 Calculated horizontal seismic displacements of cantilever bridge abutments**

| SDC | Site | Representative Sites | $\text{HPGA}_{\text{design}}$ (g) | Calculated seismic horizontal displacement, $\delta_h$(mm)* | | | |
|---|---|---|---|---|---|---|---|
| | | | | Chi-Chi | Hyogoken-Nambu | El-Centro | Loma-Prieta |
| Old SDC 475 years return period | – | Taipei & Kaohsiung | 0.276 | 0 | 0 | 0 | 0 |
| | – | Others | 0.396 | 3 ~ 12 | 2 ~ 14 | 1 ~ 4 | 2 ~ 6 |
| New SDC 475 years return period | 1 | Taipei Basin | 0.288 | 0 | 0 | 0 | 0 |
| | 2 | Kaohsiung City | 0.370 | 1 ~ 7 | 1 ~ 8 | 0 ~ 2 | 1 ~ 4 |
| | 3 | Taichung City | 0.384 | 2 ~ 9 | 1 ~ 10 | 0 ~ 3 | 1 ~ 5 |
| | 4 | Che-Lung-Pu Fault | 0.472 | 11 ~ 43 | 12 ~ 44 | 4 ~ 13 | 6 ~ 13 |
| | 5 | Hsin-Hwa Fault | 0.472 | 11 ~ 43 | 12 ~ 44 | 4 ~ 13 | 6 ~ 13 |
| | 6 | Mei-Shan Fault | 0.526 | 27 ~ 88 | 28 ~ 79 | 8 ~ 23 | 11 ~ 19 |
| New SDC 2500 years return period | 1 | Taipei Basin | 0.384 | 2 ~ 9 | 1 ~ 10 | 0 ~ 3 | 1 ~ 5 |
| | 2 | Kaohsiung City | 0.475 | 11 ~ 45 | 13 ~ 46 | 4 ~ 13 | 6 ~ 13 |
| | 3 | Taichung City | 0.480 | 12 ~ 47 | 14 ~ 49 | 4 ~ 14 | 7 ~ 14 |
| | 4 | Che-Lung-Pu Fault | 0.600 | 64 ~ 174 | 64 ~ 131 | 19 ~ 42 | 18 ~ 29 |
| | 5 | Hsin-Hwa Fault | 0.619 | 78 ~ 203 | 75 ~ 147 | 22 ~ 49 | 20 ~ 32 |
| | 6 | Mei-Shan Fault | 0.624 | 82 ~ 211 | 78 ~ 151 | 23 ~ 50 | 20 ~ 33 |

* Lower bound values shown in the ranges of $\delta_h$ represent the condition assuming full mobilization of passive resistance ($P_p > 0$); upper bound values of $\delta_h$ represent the condition of no passive resistance ($P_p = 0$).

The calculated values of horizontal seismic displacements ($\delta_h$) for cantilever-type bridge abutments located in various seismic areas are summarized in Table 5. In these calculations, $\lambda = 0$ is assumed for simplicity, and the effect of '$\lambda$' on the seismic displacement will be discussed later. It is observed that for sites 4, 5, and 6, significant differences in $\delta_h$ are obtained for $P_p = 0$ and $P_p > 0$ conditions under level 2 and 3 earthquakes in new SDC. Figure 7 highlights the displacements for level 3 earthquake and $P_p = 0$ conditions shown in Table 5. It can be seen that the $\delta_h$ calculated by using 1999 Chi-Chi and 1995 Hyogoken-Nambu records are generally 3 ~ 5 times greater than those of El-Centro and Loma-Pieta. This may be attributable to their different frequency contents as indicated by their fundamental periods (T) shown in Figs. 6(a) ~ 6(d), and also to their different number of pulses having $a_h > a_y$. The above-mentioned factors have dominant effects on the calculated values of $\delta_h$ using Newmark's double integration (sliding block) method. Table 5 shows that for the cantilever-type abutment, the maximum calculated seismic displacement is 88mm which is obtained for Site 6 using the 1999 Chi-Chi acceleration record scaled to $a_{\max}$ = HPGA$_{\text{design}}$ for a level 2 earthquake. When comparing the value of $\delta_h/H$ (= 0.088/8.1 = 1.0%) with the various criteria suggested in Table 1, it is found that the above horizontal displacement of the abutment is within the permissible level of damage (= $300 \cdot \alpha_d = 300 \cdot 0.526 = 157$ mm) suggested by Eurocode 8 (1994), Wu and Prakash (1996), and is also within the expected displacement suggested by AASHTO (2002) for free-standing abutments ($\delta_h = 250 \cdot \alpha_d = 250 \cdot 0.526 = 131$ mm). The maximum value of $\delta_h$ calculated for the cantilever-type bridge abutment based on the level 3 earthquake in the new SDC is also obtained at Site 6 ($\delta_h$ = 211 mm or $\delta_h/H$ = 0.211/8.1 = 2.6%). This value is also within the failure limit ($\delta_h/H$ = 10%) suggested by Wu and Prakash (1996).

Table 6 summarize the calculated values of $\delta_h$ for the gravity-type abutment. When comparing the values of $\delta_h$ with those shown in Table 5 it can be seen that the seismic displacements for the gravity-type abutment are generally 4-8 times greater than those for the cantilever-type abutments under similar input conditions. This demonstrates the vulnerability of the gravity-type bridge abutment against level 2 and 3 earthquake.

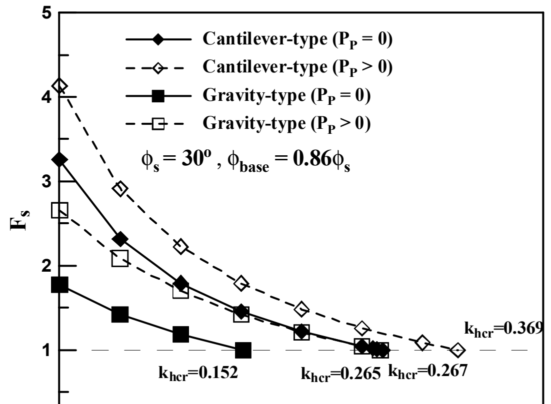


**Fig. 5** ***$F_s$* versus *$k_h$* relationships for two types of abutment considering**

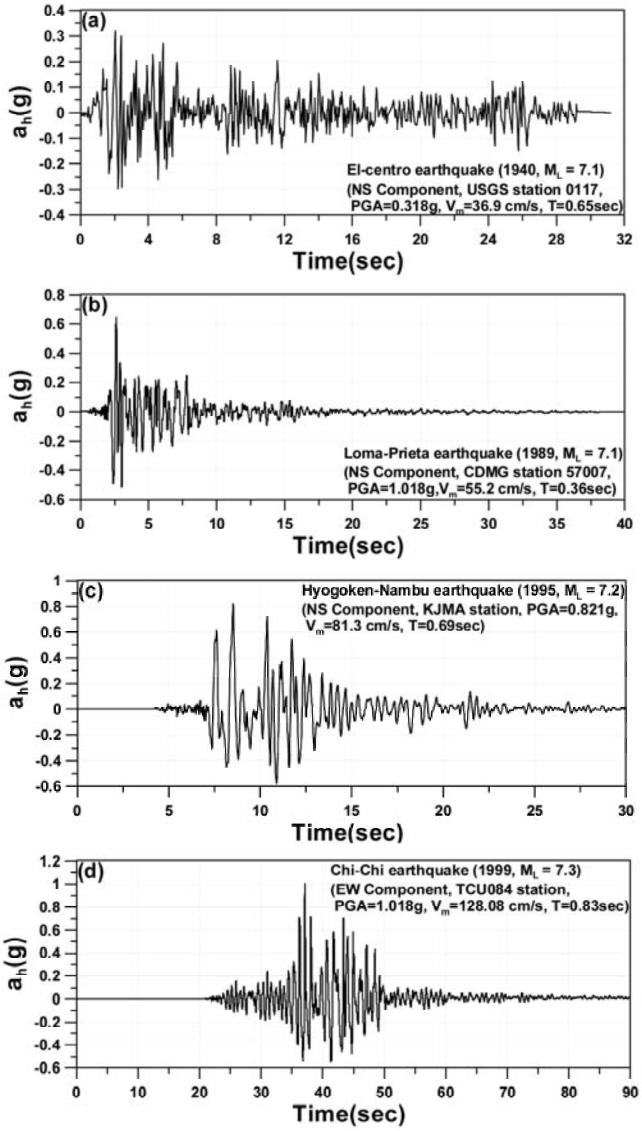


**Fig. 6 Four representative time histories of horizontal ground acceleration: (a) El-centro earthquake; (b) Loma-Prieta earthquake; (c) Hyogoken-Nambu earthquake; (d) Chi-Chi earthquake**

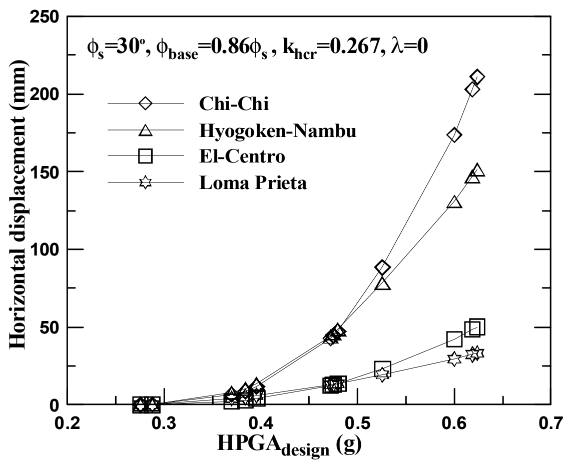


**Fig. 7 Seismic horizontal displacement of the cantilever-type abutment calculated using four ground acceleration records scaled to $a_{\max}$ = HPGA$_{\text{design}}$ for level 3 earthquake**

**Table 6 Calculated horizontal seismic displacements of gravity-type bridge abutments**

| SDC | Site | Representative Sites | Design HPGA (g) | Calculated seismic horizontal displacement, $\delta_h$ (mm) | | | |
|---|---|---|---|---|---|---|---|
| | | | | Chi-Chi | Hyogoken-Nambu | El-Centro | Loma-Prieta |
| Old SDC 475 years return period | – | Taipei & Kaohsiung | 0.276 | 0 ~ 29 | 0 ~ 29 | 0 ~ 8 | 0 ~ 8 |
| | – | General Sites | 0.396 | 13 ~ 213 | 14 ~ 128 | 4 ~ 47 | 6 ~ 27 |
| New SDC 475 years return period | 1 | Taipei Basin | 0.288 | 0 ~ 39 | 0 ~ 37 | 0 ~ 11 | 0 ~ 10 |
| | 2 | Kaohsiung City | 0.37 | 7 ~ 147 | 8 ~ 99 | 2 ~ 34 | 4 ~ 21 |
| | 3 | Taichung City | 0.384 | 9 ~ 176 | 11 ~ 113 | 3 ~ 40 | 5 ~ 24 |
| | 4 | Che-Lung-Pu Fault | 0.472 | 44 ~ 422 | 46 ~ 214 | 13 ~ 90 | 13 ~ 42 |
| | 5 | Hsin-Hwa Fault | 0.472 | 44 ~ 422 | 46 ~ 214 | 13 ~ 90 | 13 ~ 42 |
| | 6 | Mei-Shan Fault | 0.526 | 92 ~ 636 | 81 ~ 292 | 23 ~ 138 | 20 ~ 59 |
| New SDC 2500 years return period | 1 | Taipei Basin | 0.384 | 7 ~ 176 | 11 ~ 113 | 3 ~ 39 | 5 ~ 24 |
| | 2 | Kaohsiung City | 0.475 | 46 ~ 432 | 47 ~ 218 | 13 ~ 92 | 13 ~ 43 |
| | 3 | Taichung City | 0.48 | 49 ~ 449 | 50 ~ 224 | 14 ~ 97 | 14 ~ 44 |
| | 4 | Che-Lung-Pu Fault | 0.6 | 179 ~ 967 | 134 ~ 400 | 43 ~ 209 | 30 ~ 85 |
| | 5 | Hsin-Hwa Fault | 0.619 | 209 ~ 1069 | 150 ~ 430 | 50 ~ 230 | 33 ~ 94 |
| | 6 | Mei-Shan Fault | 0.624 | 218 ~ 1096 | 154 ~ 443 | 51 ~ 235 | 34 ~ 96 |

In addition, order of magnitude differences in the $\delta_h$ when using different earthquake events can be observed despite the fact that each input ground acceleration has been scaled to $a_{max}$ = HPGA$_{design}$. Table 6 shows the comparison of $\delta_h$ calculated for gravity-type abutment under $P_p = 0$ and $P_p \neq 0$ conditions. Significant difference of $\delta_h$ between the cases of $P_p = 0$ and $P_p \neq 0$ are apparent, indicating a routine integrity examination for the passive zone of gravity-type bridge abutment is vital to the mitigation of disaster in a major earthquake. Table 6 also shows the effect of code changes on the calculated values of seismic displacement ($\delta_h$) of bridge abutments. The input values of HPGA$_{design}$ suggested in new SDC generate extraordinarily great values of $\delta_h$ for gravity-type bridge abutments located in near-fault areas based on 475 and 2500 years return-period earthquake intensities. This potential of severe damage hasn't been considered in old SDC.

Figure 8 highlights the calculated values of $\delta_h$ for the gravity-type abutment using level 3 earthquake and $P_p = 0$ conditions shown in Table 6. A maximum $\delta_h$ = 636 mm ($\delta_h/H$ = 7.8%) is obtained using the Chi-Chi record under HPGA$_{design}$ = 0.526 g for a level 2 earthquake. This value is much greater than the permissible displacements discussed above. Figure 8 also shows that $\delta_h$ = 1096 mm ($\delta_h/H$ = 13.5%) is obtained for HPGA$_{design}$ = 0.625 g for a level 3 earthquake. This value exceeds the failure limit ($\delta_h/H$ = 10%) suggested by Wu and Prakash (1996), indicating a possible catastrophic failure in a major earthquake for gravity — type abutments when passive resistance is not available.

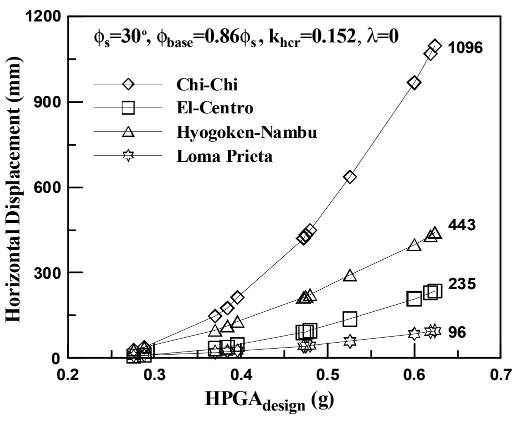


**Fig. 8 Seismic horizontal displacement of gravity-type abutment calculated using four ground acceleration records adjusted to $a_{max}$= HPGA$_{design}$ for level 3 earthquake**

## 5. EFFECT OF VERTICAL GROUND ACCELERATION

Table 7 shows a comparison of the design values of VPGA (VPGA$_{design}$) in old and new SDC. It indicates that the ratio between the VPGA$_{design}$ and HPGA$_{design}$, herein defined as "λ", is 0.33 ~ 0.67 in old SDC and 0.5 ~ 0.67 for new SDC. This indicates that the lower bound design value of 'λ' is increased in the new SDC and the design values of 'λ' fall within a narrow range of $0.5 \leq \lambda \leq 0.67$. Figures 9(a) through 9(c) show the horizontal ground accelerations ($a_h$), the vertical ground accelerations, ($a_v$), and the real-time values of λ (= $a_v$ / $a_h$) obtained at station CHY028 during the 1999 Chi-Chi earthquake. For the sake of simplicity, only the pulses with $a_h \geq 0.2$ g are focused. Data obtained at the same elapsed time are marked using solid symbols. For all the points examined, the values of 'λ' range between −0.606 (= $\lambda_{min}$: maximum value of λ in an earthquake record) and 0.906 (= $\lambda_{min}$: minimum value of λ in an earthquake record). However, at the peak accelerations for four major pulses as indicated in Fig. 9(a), their corresponding values of 'λ' range between −0.022 and 0.169. The fact that the peak values of λ and $a_h$ are out of phase has also been pointed out by Tatsuoka *et al.* (1998) and Huang (2005). A value of λ = 0.2 has been suggested by Huang *et al.* (2003) in analyzing four near-fault reinforced soil walls with modular block facing, and good agreements between the observed and calculated displacements of the wall were obtained. Figure 10 shows the measured values of 'λ' using horizontal and vertical ground acceleration records obtained from a total of 11 near-fault seismographs (within 15 km to the Che-Lung-Pu fault) in the 1999 Chi-Chi earthquake in Taiwan. It can be seen that, for all the records obtained from the near-fault stations in the 1999 Chi-Chi earthquake, the values of 'λ' at peak horizontal accelerations ($a_h$) larger than 0.2 g ranged between – 0.25 to +0.25. This supports the use of λ = 0.2 in the seismic displacement analysis performed by Huang *et al.* (2003) and also indicates a conservatism in the new SDC. Figure 11 shows the influence of λ on the $k_{h_{cr}}$ for both types of bridge abutments. It can be seen that for the cantilever-type, the difference in the values of $k_{h_{cr}}$ obtained based on λ = 0.25 ($k_{h_{cr}}$ = 0.262) and λ = 2/3 (or λ = 0.67, $k_{h_{cr}}$ = 0.234) differs by about 20%, while for

**Table 7 Vertical peak ground acceleration (VPGA) coefficients considered in SDC**

| Old SDC | New SDC |
|---|---|
| $VPGA_{design} = 1/3 \cdot HPGA_{design}$ for Zone 2 | $VPGA_{design} = 1/2 \cdot HPGA_{design}$ for non near-fault areas including Taipei Basin |
| $VPGA_{design} = 2/3 \cdot HPGA_{design}$ for Zone 1 | $VPGA_{design} = 2/3 \cdot HPGA_{design}$ for near-fault areas |

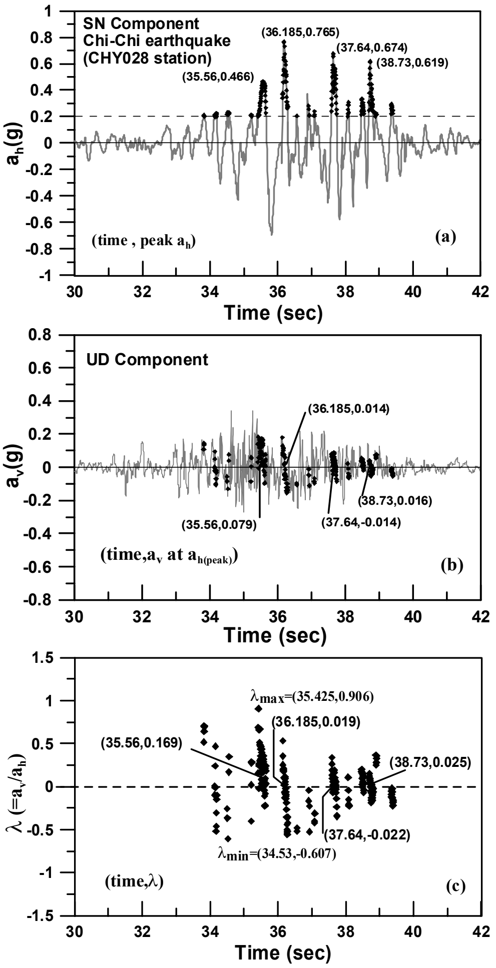


**Fig. 9 Observed values of (a) horizontal ground accelerations, $a_h$; (b) vertical ground acceleration, $a_v$; and (c) λ at station CHY028 in the 1999 Chi-Chi earthquake (after Huang, 2005)**

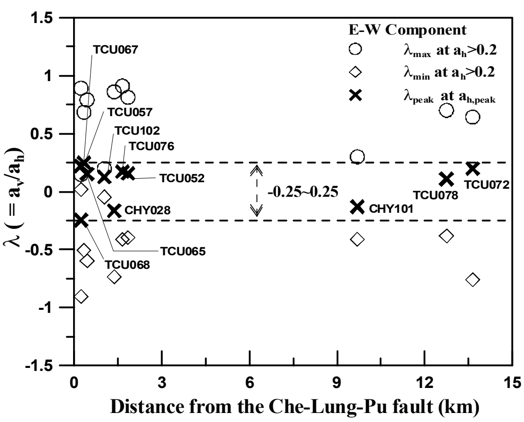


**Fig. 10 Measured values of λ for 11 near-fault seismographs in the 1999 Chi-Chi earthquake**

the gravity-type, the difference is only 8%. The larger effect of λ on the calculated values of $k_{h_{cr}}$ for the cantilever-type may be due to its larger mass of the wall including the soil above the heel slab. Further studies into this effect is necessary in the future. The influence of λ to the calculated displacement ($\delta_h$) is shown in Fig. 12. For a level 3 earthquake at site 6 the uses of λ = 0.25 and λ = 0.67 result in $\delta_h$ = 225 mm ($\delta_h/H$ = 2.7%) and $\delta_h$ = 330mm ($\delta_h/H$ = 4.0%), respectively. These values are all well-below the failure limit suggested by Wu and Prakash (1996). It is also seen in Fig. 12 that for a level 2 earthquake based on new SDC for Site 6 (the Mei-Shan fault area, $HPGA_{design}$ = 0.526 g) the difference in the values of $\delta_h$ calculated using λ = 0.25 and λ = 0.67 are 100 mm and 155 mm, respectively. The use of λ = 0.67 results in slightly conservative values of $\delta_h$. Figure 13 shows that the differences in $\delta_h$ calculated for the gravity-type abutment at site 6 subjected to level 3 and level 2 earthquakes induced by the difference in input value of 'λ' (namely, λ = 0.25 and λ = 0.67) are only about 125 mm and 65 mm, respectively. Due to the large calculated $\delta_h$, the influence of λ on the predicted values of $\delta_h$ is relatively small.

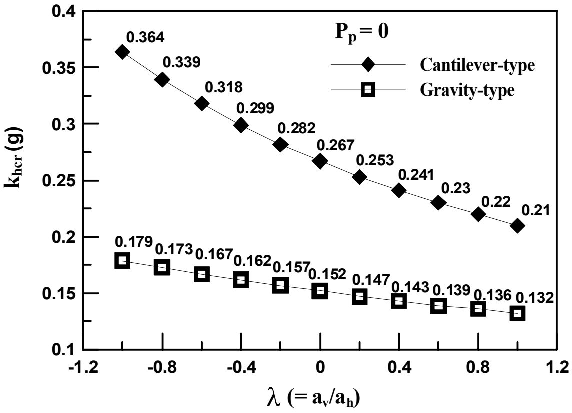


**Fig. 11 Influences of the values of 'λ' to the critical seismic coefficient, $k_{h_{cr}}$**

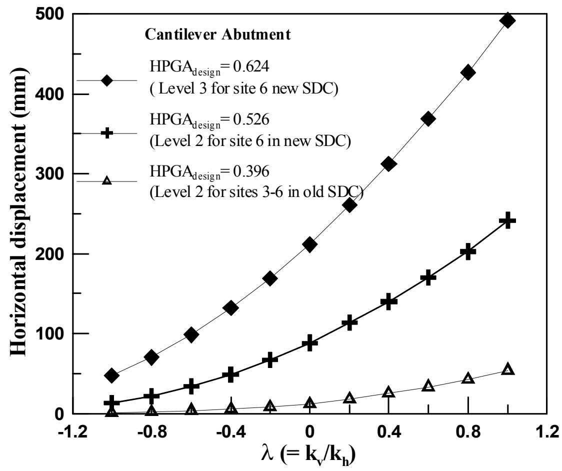


**Fig. 12 Influence of 'λ' to the horizontal displacement using 1999 Chi-Chi earthquake record (TCU084 station)**

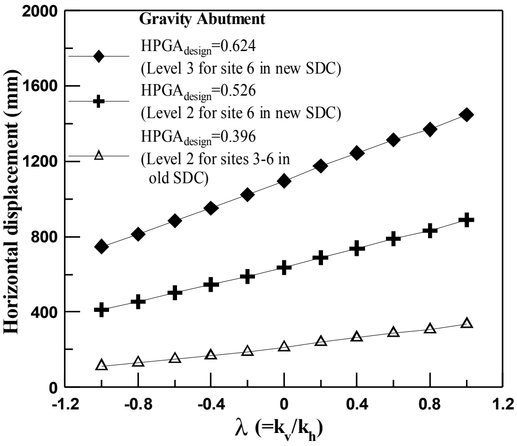


**Fig. 13 Influence on horizontal displacement when λ is adopted in the analysis using 1999 Chi-Chi earthquake record (TCU084 station)**

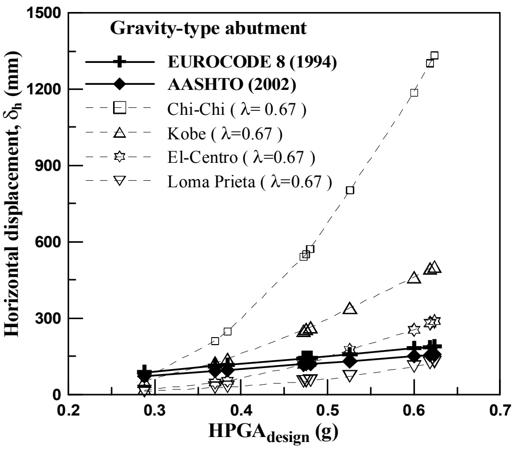


**Fig. 14(b) Horizontal seismic displacement of gravity-type abutment subject to design PGA of Chi-Chi earthquake with different λ values compared with criteria**

Figure 14(a) summarizes the values of $\delta_h$ for the cantilever-type abutment ($P_p$ = 0) for the six sites shown in Table 5. For comparison purposes, permissible displacements specified in the Eurocode (1994) and the expected displacement suggested by AASHTO (2002) are also shown. It can be seen that a major part of the predicted values of $\delta_h$ are within permissible ranges except a few sites for which the $HPGA_{design} \geq 0.6$ g. This infers the robustness of the cantilever-type bridge abutment. The values of $\delta_h$ which exceed the permissible range are not likely to be associated with a catastrophic failure because their values do not differ from the permissible ones by a order-of-magnitude difference. Figure 14(b) shows the calculated $\delta_h$ for the gravity-type abutment assuming $P_p$ = 0 for the six sites shown in Table 6. By comparing Figs. 14(a) and 14(b), the vulnerability of the gravity-type abutment when subjected to level 2 and level 3 earthquakes in almost all sites can be detected. This suggests that a pre-earthquake reinforcement program for the gravity-type bridge abutments used as major traffic facilities is necessary for mitigating possible disasters caused by major earthquakes in the future. The calculated values of $\delta_h$ shown in Tables 5 and 6 also suggests that to take advantage of the stability contribution by the passive resistance in front of the gravity-type abutment, a regular integrity inspection for the passive zone in front of these structures can also be considered as a part of pre-earthquake disaster mitigation program.

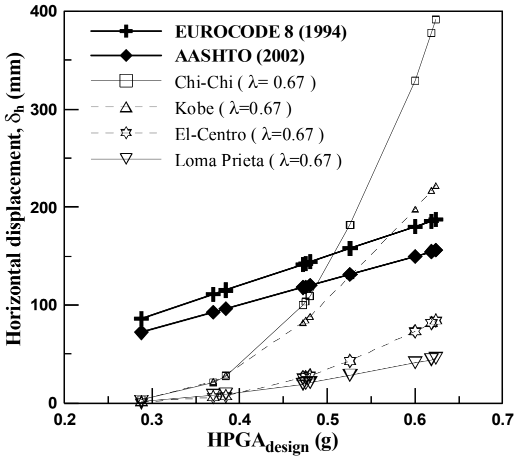


**Fig. 14(a) Horizontal seismic displacement of cantilever-type abutment subject to design PGA of Chi-Chi earthquake with different λ values compared with criteria**

## 6. CONCLUSIONS

A multi-wedge method is used to calculate the seismic displacement of two typical examples of cantilever-type and gravity-type bridge abutments based on the design ground accelerations specified in old and new seismic design codes in Taiwan. The following conclusions relevant to these two typical examples of highway bridge abutment can be drawn:

(1) The design values of horizontal ground acceleration ($HPGA_{design}$) are generally 4% to 33% higher in the new seismic design code (SDC) for a level 2 design earthquake with a 475 year return period, the increase in $HPGA_{design}$ from level 3 to level 2 design earthquake is 39% to 72%, and seismic displacement analysis must be performed for level 2 and level 3 design earthquakes to avoid an over-conservative design for the size and geometry of the structures.

(2) Vertical ground accelerations specified in the old and new SDC are generally larger than those observed at near-fault stations during the 1999 Chi-Chi earthquake. The use of λ = 0.67 specified in the new SDC results in slightly conservative values of calculated seismic displacement for the gravity and cantilever bridge abutments when compared with those calculated using λ = 0.25 which is a possible upper limit observed in the near-fault seismographers during the 1999 Chi-Chi earthquake in Taiwan.

(3) Based on the displacement calculations for six sites of geographical importance in west Taiwan using four acceleration records with peak acceleration scaled to the level 2 design horizontal ground acceleration, the maximum horizontal seismic displacement calculated for the cantilever-type bridge abutment are all within the permissible ranges proposed in the literature; for the level 3 design horizontal ground acceleration, the calculated displacement are well below the failure criterion suggested in the literature. However, this is not the case for the gravity-type abutment, indicating the vulnerability of the gravity-type abutment when

subjected to level 2 and 3 earthquakes.

(4) The existence of passive resistance in front of the wall increases the seismic stability (or reduces the seismic displacement) of the gravity-type bridge abutment to a large degree. It is suggested to perform regular integrity inspections for the passive zone in front of gravity-type abutments or pre-earthquake reinforcement using soil nailing, soil anchoring or ground improvement techniques to avoid any catastrophic failure of the wall in a major earthquake.

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