BOUNDARY ELEMENT ANALYSIS OF TUNNELING THROUGH A WEAK ZONE

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ABSTRACT

Generally, a weak zone can be considered as a layered system composed of formations with different material properties. In order to understand the potential mechanism for material instability in relation to the material contrast, it is essential to consider the problem as a multi-layered system. For the analysis of multi-layered elastic media, a linear variation displacement discontinuity method was developed based on a scheme superposing two sets of bonded half-planes and subtracting one infinite plane. The model was verified before applied to the study of the fracture zone behaviour of tunneling through a weak zone. The results show that the influence of the material contrast on the stress distribution might be insignificant. However, the fracture zone behaviour, especially the fracture patterns, can be quite different for different material contrasts.

Key words: tunnel mechanics, numerical simulation, boundary element method, rock mechanics.

1. INTRODUCTION

Boundary element models are widely used in geomechanics problems for computing stresses and displacements around underground excavations. Most of these models assume the rock mass to be a homogeneous, isotropic, linearly elastic solid, although inhomogeneity and anisotropy can also be analyzed by the boundary element method (Banerjee and Butterfield, 1981; Crouch and Starfield, 1989; Napier and Ozbay, 1993). An important development of the boundary element approach is that half-plane and bonded half-plane problems can be solved without making additional numerical approximations (Frasier and Rongved, 1957; Crouch and Starfield, 1989; Pan *et al*., 1998). Problems involving layers of finite thickness are more difficult to treat as closed form fundamental solutions are difficult to derive. However, the fundamental solution for a multi-layered medium has been derived using the principle of superposition (Shou and Napier, 1999; Shou, 2000).

Many important rock engineering problems involve analysis of the stress fields of multi-layered systems. For the problem of mining a tabular orebody with different hangingwall and footwall, in order to understand the potential instability in relation to the material contrast, it is necessary to consider the problem as a layered system composed of formations with different material properties. A stope approaching a dyke with different properties from the host rock, which may trigger slip along the interface or may induce seismic activity related to rockburst, is another typical problem involving material contrast and interface. Similarly, a weak zone encountered in tunneling can be considered as a layered system composed of formations with different material properties. The material contrast and the existence of interface(s) play important roles on the behaviour of the rock mass near the interface approached by the tunneling face.

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2. BOUNDARY ELEMENT METHODS

2.1 Solution for a Three-Layer Medium

The solution of a displacement discontinuity element within a three-layer medium was derived by a superposing procedure, which is based on the principle of superposition and the bonded half-planes solution (Shou, 2000). A three-layered elastic region, with Young's modulus E_1, E_2, and E_3, can be obtained by superposing two sets of bonded half-plane regions and an infinite plane. It is accomplished by introducing two sets of bonded half-planes solution with different elastic modulus sets (E_2, E_3) and (E_1, E_2), and subtracting a supplementary infinite domain solution, as shown in Fig. 1. The complete solution can be written as:

$$(u_i)_{[A]} = (u_i)_{[B1]} + (u_i)_{[B2]} - (u_i)_{[C]}$$
$$(\sigma_{ij})_{[A]} = (\sigma_{ij})_{[B1]} + (\sigma_{ij})_{[B2]} - (\sigma_{ij})_{[C]} \quad (1)$$

For each layer, the solution can be expressed in more detail as:

(1) Layer 1 ($y > H$)

$$u_i^{[1]} = (u_i)_{[B1U]} + (u_i)_{[B2U]} - (u_i)_{[C]}$$
$$\sigma_{ij}^{[1]} = (\sigma_{ij})_{[B1U]} + (\sigma_{ij})_{[B2U]} - (\sigma_{ij})_{[C]} \quad (2)$$

(2) Layer 2 ($H \geq y \geq 0$)

$$u_i^{[2]} = (u_i)_{[B1U]} + (u_i)_{[B2L]} - (u_i)_{[C]}$$
$$\sigma_{ij}^{[2]} = (\sigma_{ij})_{[B1U]} + (\sigma_{ij})_{[B2L]} - (\sigma_{ij})_{[C]} \quad (3)$$

(3) Layer 3 ($y < 0$)

$$u_i^{[3]} = (u_i)_{[B1L]} + (u_i)_{[B2L]} - (u_i)_{[C]}$$
$$\sigma_{ij}^{[3]} = (\sigma_{ij})_{[B1L]} + (\sigma_{ij})_{[B2L]} - (\sigma_{ij})_{[C]} \quad (4)$$

where subscript [B1] represents the bonded half-planes with parameters (E_2 , ν_2) and (E_3 , ν_3), [B2] represents the bonded half-planes with parameters (E_1 , ν_1) and (E_2 , ν_2), and [C] repre-

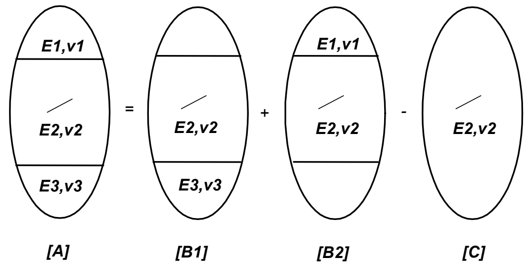


Fig. 1 The superposition scheme to obtain the solutions for a three-layered medium

sents the infinite plane with parameters (E_2, ν_2). The additional subscripts U and L represent the solutions for upper and lower half-planes, as the bonded half-planes solutions are different for the upper and lower half-planes.

2.2 Numerical Implementation

Based on the developed fundamental solution, a solution procedure can be applied to establish the displacement discontinuity method for boundary value problems. To illustrate the solution procedure for the displacement discontinuity method, a simple problem for a line crack in an infinite layered body is adopted. For this problem, we divide the crack into *N* line segment (2*N* nodal points), each of which is represented by a displacement discontinuity element (see Fig. 2).

For the *j*-th nodal point, the shear and normal displacement discontinuities applied to this node are denoted as $\overset{j}{D}_s$ and $\overset{j}{D}_n$ and the actual stresses as $\overset{j}{\sigma}_s$ and $\overset{j}{\sigma}_n$. The actual stresses, $\overset{j}{\sigma}_s$ and $\overset{j}{\sigma}_n$, are induced by the discontinuity values arising on all *N* segments (2*N* nodal points). With suitable coordinate transformations to account for the orientations of the line segments, we can express the stresses $\overset{i}{\sigma}_s$ and $\overset{i}{\sigma}_n$ (i = 1 to 2*N*) at suitable collocation points within each segment as:

$$\overset{i}{\sigma}_s = \sum_{j=1}^{2N} \overset{ij}{A}_{ss} \overset{j}{D}_s + \sum_{j=1}^{2N} \overset{ij}{A}_{sn} \overset{j}{D}_n + \overset{i}{P}_s$$

$$\overset{i}{\sigma}_n = \sum_{j=1}^{2N} \overset{ij}{A}_{ns} \overset{j}{D}_s + \sum_{j=1}^{2N} \overset{ij}{A}_{nn} \overset{j}{D}_n + \overset{i}{P}_n \qquad i = 1 \quad \text{to} \quad 2N \qquad (5)$$

where $\overset{ij}{A}_{ss}$, *etc.*, are the boundary stress influence coefficients, and $\overset{i}{P}_s$ and $\overset{i}{P}_n$ are the original field stress components. The coefficient $\overset{ij}{A}_{ss}$, for example, gives the shear stress at the *i*-th collocation point, $\overset{i}{\sigma}_s$, due to a unit applied shear displacement discontinuity at the *j*-th collocation point ($\overset{j}{D}_s = 1$).

Introducing prescribed boundary stresses into Eq. (5), we can obtain a system of 4*N* algebraic equations to determine the applied displacement discontinuity $\overset{j}{D}_s$ and $\overset{j}{D}_n$ for j = 1 to 2*N*. We can then obtain the solutions for other points within the domain since the solutions can be expressed as linear combinations of the displacement discontinuities. The foregoing equa tions were used to develop the linear displacement discontinuity model DD3L for solving two-dimensional boundary value problems.

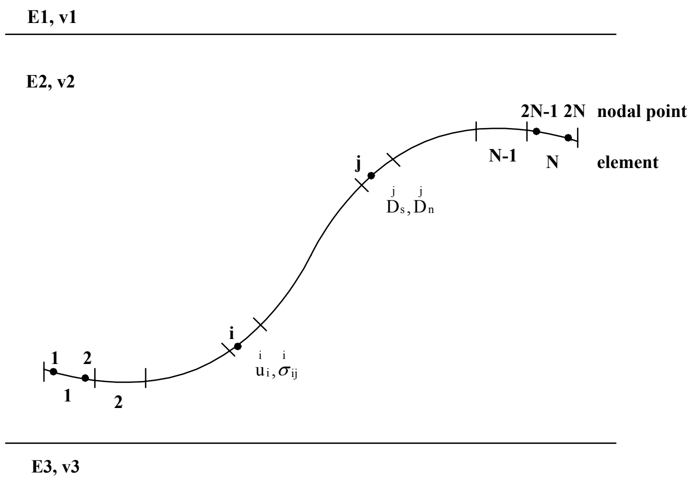


Fig. 2 Displacement discontinuity method for the problem of a crack in a layered medium

2.3 Strength Factor

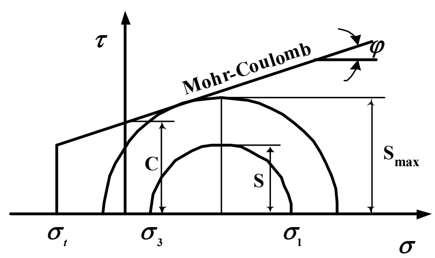
In this study, strength factor (***S.F.***) is adopted to illustrate the stress concentration as well as the potential of failure (Curran and Corkum, 1992). The strength factor is calculated by dividing the rock strength in terms of the maximum internal shear stress for a given confining pressure at a point, by the maximum induced internal shear stress at that point due to the excavation. It can be defined as below (see Fig. 3):

$$\boldsymbol{S.F.} = \boldsymbol{S}_{\max} / \boldsymbol{S} \qquad (6)$$

where

$$\boldsymbol{S}_{\max} = c\ \cos\varphi + \sin\varphi\ (\varphi_1 + \varphi_3)/2$$

and $\boldsymbol{S} = (\sigma_1 - \sigma_3)/2$.



$$\boldsymbol{S.F} = \frac{\boldsymbol{S}_{\max}}{\boldsymbol{S}}$$

Fig. 3 Strength factor corresponding to the Mohr-Coulomb failure criterion

3. RESULTS

Using the developed model, a series of analyses was performed by considering a weak zone in host rock as a layered system with different material properties. A tunnel approaching a weak zone, with different material properties from the host rock, may trigger slip along the interface or may induce stress concentrations leading to collapse as well as micro seismic activity. An example problem of a tunnel approaching and penetrating a weak zone from different angles (see Fig. 4), with the properties as in Table 1, is analyzed. The approaching angle θ is defined as the angle between the mining direction and the weak zone. We let the thickness of the weak zone be $t = 5$ m, and the distance from the mining face to the first interface of host rock and weak zone be $d = 5$ m.

The results in Figs. 5(a) ~ 5(d) shows the significant influence of the approaching angle on the strength factor distribution. The higher strength factor along the interface, especially for the hanging wall side (see Figs. 5(a) and 5(b)) reveals that the difference in the stresses is maximized for the approaching angle nears 45° to 60°. And, according to the stress state and failure criterion, the failure mode near the interface might change from shear to tensile when the approaching angle getting smaller (Shou and Napier, 1999).

For a tunnel penetrating a weak zone, different phases of the tunneling process were analyzed. For simplicity, we only consider the case of tunnel mining through a weak zone at 90°. The results for different stages in Figs. 6(a) to 6(d) always show the concentration at the mining face as well as the interface. And the effect of energy release is quite significant when the mining face reach the interfaces.

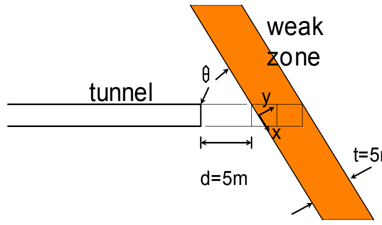
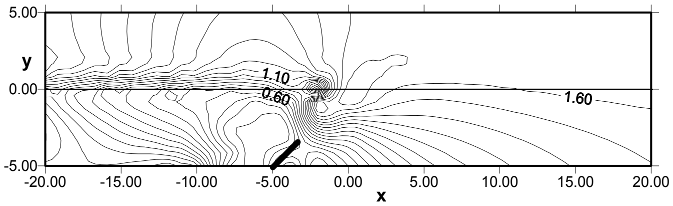


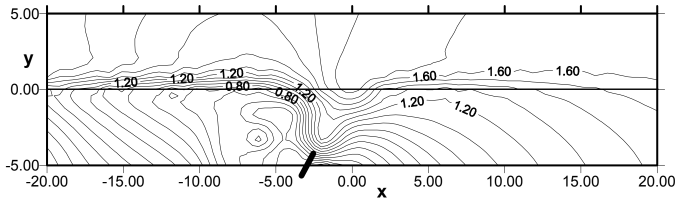
Fig. 4 The problem of a tunnel approaching a weak zone

Table 1 Input data for the problem of a tunneling penetrating a weak zone

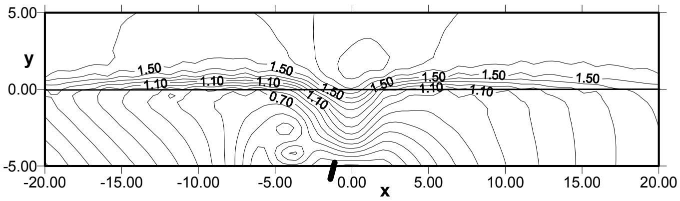
E (host rock)	78 GPa
ν (host rock)	0.21
E (weak zone)	7.8 GPa
ν (weak zone)	0.21
σ_v (vertical in situ stress)	54 MPa
σ_h (horizontal in situ stress)	27 MPa (*i.e.* $k = 0.5$)
t (width of weak zone):	5 m
d (distance from mining face to weak zone)	5 m
θ (angle between the stope and interface)	45°, 60°, 75°, 90°



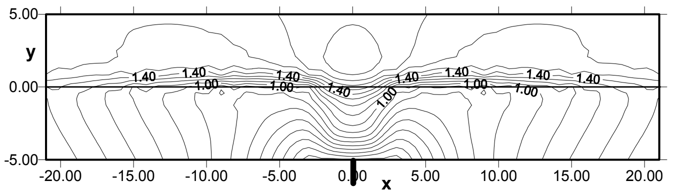
(a) approaching angle 45°



(b) approaching angle 60°

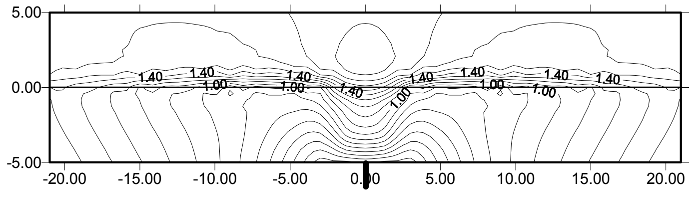


(c) approaching angle 75°

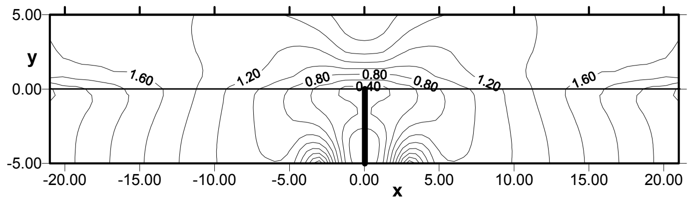


(d) approaching angle 90°

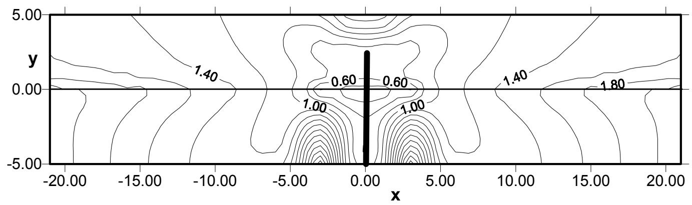
Fig. 5 The influence of the approaching angle on the distribution of strength factor for a tunnel mined toward a weak zone



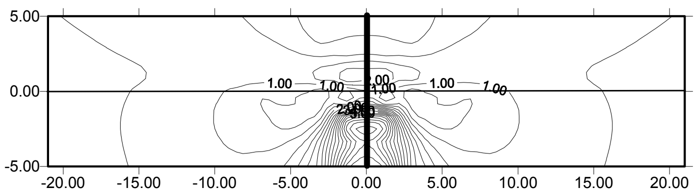
(a) 5 m ahead of the interface



(b) right at the interface of the weak zone



(c) 2.5 m penetrating in the weak zone



(d) penetrating through the weak zone

Fig. 6 The distribution of strength factor for a tunnel being mined through a weak zone

4. CONCLUSION AND SUGGESTION

In this study, a weak zone in host rock was considered as a layered system composed of formations with different material properties. For the analysis of multi-layered system, a linear variation displacement discontinuity method based on a superposition scheme was adopted. And the results show that the boundary element approach works well in describing the mutual interaction of multiple layers.

The material contrast and the existence of interface(s) play important roles on the behavior of the rock mass near the interface approached by the tunneling face. The results show that the influence of the material contrast on the stress distribution might not so significant except at the interfaces. However, the fracture zone behavior, especially the fracture patterns, can be quite different for different material contrasts.

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