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FINITE ELEMENT ANALYSES OF GEOSYNTHETIC-REINFORCED SOIL GROUND UNDER SURCHARGE AND PROBABILISTIC ESTIMATION OF THE BEARING CAPACITY

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ABSTRACT

The soft stratum has always been a concern for embankment and pavement loadings. A widely accepted method to cope with this problem is to use geosynthetics materials to reinforce the soft ground. The effect of geosynthetics on the deformation characteristics of the reinforced soil layer, especially under a probabilistic framework, is of interest in this study. In this paper, the finite element analysis was performed to identify the effect of key parameters, including soil properties, geosynthetics material properties, depth and spacing of geosynthetics installation, on the surcharge-deformation characteristics of geosynthetic reinforced soft ground. It is found that the application of stronger soil and geosynthetic material can reduce the probability of weak geosynthetic reinforced foundation fails to meet the prerequisite settlement tolerance while the most important design parameter is the better interaction between soil and geosynthetics. A series of numerical analyses of deformation of geosynthetics reinforced soft ground were conducted. The analysis results were inventoried and regressed to establish a performance model in the form of second-order model (SOM) equation. With the incorporation of developed simplified SOM equation and Monte-Carlo simulation, the bearing capacity under certain soil and geosynthetic material properties for specified deformation criterion can be estimated.

Key words: Geosynthetics, soft ground, bearing capacity, second-order model equation, numerical analysis, Monte-Carlo simulation.

1. INTRODUCTION

In western Taiwan, the soil stratum is usually composed of thick layer of flooding deposition and fine soil weathered from mudstone or laterite. Since high quality in-situ soils are not always available locally, most infrastructures are constructed on weak subsurface strata. The presence of such soft strata often results in concerns about bearing capacity as well as settlements for overlying structures. For example, Fig. 1 shows some cases of excessive settlement, uneven surface, or cracks on connection road, country road and city highway in western Taiwan. The enhancement of bearing capacity of the marginal soil is an important issue to improve the quality of structures constructed over them.

The concept of reinforced soil as construction material was first introduced in the 1960s (Vidal 1978). The merits of introducing of reinforcement into soil are based on the strength properties of the reinforcement, and the existence of soil-reinforcement interaction due to tensile strength and frictional mechanisms. Since then, reinforcements have been widely used in geotechnical engineering practice. Specifically, geosynthetics reinforcements are widely recognized as an effective, cost saving, and durable reinforcements for various geotechnical related constructions (Horvath *et al.* 2011; Bouazza *et al.* 2013; Indraratna *et al.* 2013; McWatters *et al.* 2016). Among them, improving foundations on soft ground is one of the most important applications (Dash *et al.* 2013; Uttam *et al.* 2013). To increase the bearing capacity and reduce settlement of a soft foundation, a layer of geosynthetics is placed on the weak stratum and then covered with granular fill. The use of geosynthetic reinforcement could effectively confine the soil movement both above and below the geosynthetic reinforcement and dissipate the concentrated load to the soft ground.

A lot of researches have been conducted on the topics of geosynthetics reinforced soft ground. For example, several experiments (Binquiet and Lee 1975a; Guido 1986; Sakti and Das 1987; Mandal *et al.* 1992; Michalowski 2003; Huang 2007) and numerical simulations (Boutrup *et al.* 1982; Yetimoglu *et al.* 1994; Bergado *et al.* 2002; Sawwaf 2009) have shown the positive effect of using geosynthetics to increase the soil's bearing capacity. In summary, experimental and numerical analyses have been extensively conducted to verify this application, to understand the mechanism, and to qualify and quantify the reinforcing effect of bearing capacity of soft soil layers. However, a significant deficit is found after a thorough review of these previous studies. That is, most previous studies mainly focused on deterministic analyses; there have been few studies on probabilistic analyses especially in term of deformation characteristics of soft ground surface under application of surcharge.

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(a) Country road in Hsinchu county

(b) City highway in Taichung

(c) Connection road in industrial zone in Kaohsiung city

Fig. 1 Pictures of road settlement

The aim of this research is to perform a probabilistic analysis of deformation characteristics of geosynthetics reinforced soil layer. First, a numerical model of geosynthetic reinforced soil ground is built and validated. A parametric analysis to identify important design parameters is conducted. A series of numerical analysis cases involving changing of input values of parameters and recording the system deformation responses are accomplished. The second order method is then used to develop an approximate equation for estimating the bearing capacity of soft ground with reinforcement. This approximate equation is further advanced by incorporating with Monte Carlo simulation analysis for probabilistic estimation of bearing capacity under certain settlement performance criterion.

2. DEVELOPMENT AND VERIFICATION OF PLAXIS NUMERICAL MODEL

In this paper, the finite element program PLAXIS (version 8) was used to build the numerical model of geosynthetic reinforced soil. PLAXIS is a versatile analysis tool in cooperating multiple material properties and constitutive models to simulate geotechnical problems. It has been commonly used as the analysis tool for verifying or evaluating the effect of geosynthetics on improving soft foundation numerically (Bergado *et al.* 2002; Ling *et al.* 2003; Howard *et al.* 2006; Sawwaf 2009).

Two-dimensional plane strain model was adopted to simulate the performance of strip footing over reinforced soil. The model is composed by one soft sand layer reinforced by multiple layers of reinforcements. The model width is 3 m with roller boundary set on sides while a fixed boundary was set at the bottom side. The finite element geometry with boundary condition is shown in Fig. 2. The soil material is simulated by 15-node isotropic triangular elements of linear elastic model with Mohr-Coulomb failure criterion. This 15-node triangle is considered a very accurate element that has produced high quality stress results for difficult problems (Mutalik *et al.* 2016). The geosynthetics are simulated by two-node membrane elements which allow for development of tension but not for bending or compression. The contact interfaces which allow for relative slip were set between the geosynthetic and the soil material around it. The strip footing is regarded as rigid, so applying surcharge on the footing is equal to applying uniform vertical downward displacements at the nodes immediately underneath the footing (Yetimoglu *et al.* 1994). The mesh coarseness was set finer at the vicinity of pressure application. The model is surcharged gradually until the strain at the footing reaches 30 mm.

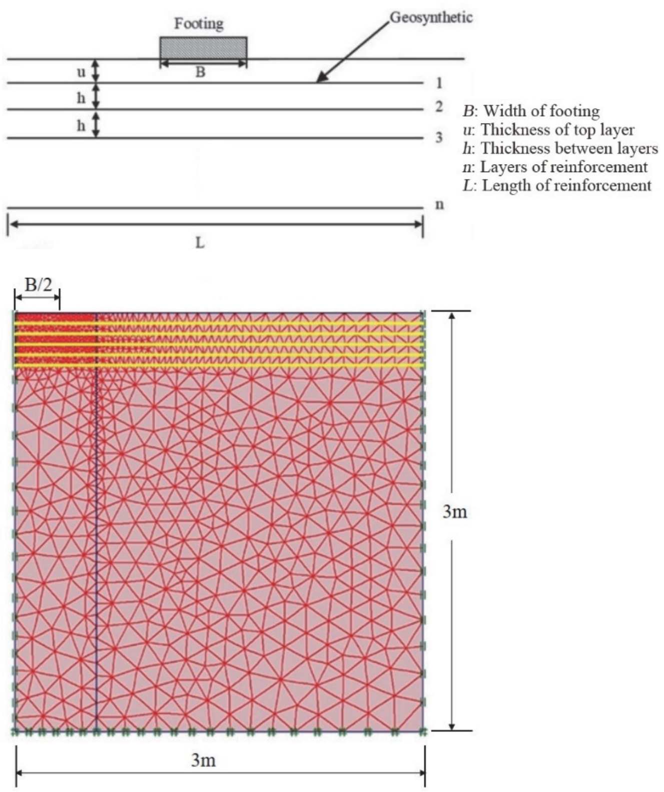


Fig. 2 Illustration and finite element mesh of numerical model of strip footing on geosynthetic-reinforced soil

In the loading process, the footing settlement corresponding to the surcharge was recorded until 10% of the footing width (s/B = 10%, s is the footing settlement, B is the footing width) at the footing center was taken as the ultimate bearing stress (Yoo 2001).

The suitability of the numerical models developed in this analyses were examined by comparing with the analysis results reported on three documents (Love *et al.* 1987; Patra *et al.* 2005; Huang 2007). Huang (2007) used ABAQUS software to investigate the effects of placing geosynthetics on the interface of subgrade aggregates of subgrade layer. This study rebuilt the numerical model by adopting the dimension, composition and material properties presented in his research. The comparison between the numerical simulation results of Huang (2007) and this study is shown in Fig. 3. The comparison shows a similar trend but with slight discrepancy. The discrepancy might be attributed to the different parameter values used between ABAQUS and PLAXIS. Patra *et al.* (2005) conducted a laboratory model test of geosynthetic reinforced sand under surcharge. The test was in plane strain condition with specimen width of 0.365 m and height of 0.7 m. The parameters provided on this paper were input into the numerical model developed in this analysis. The comparisons between numerical simulation results and test measurements are shown in Fig. 4. The comparison indicates the laboratory test results can be numerically simulated closely as long as accurate parameter values were input. Love *et al.* (1987) conducted experimental and analytical solutions on unreinforced and reinforced soil foundation. The effectiveness of geogrid reinforcement (Polypropylene), placed at the base of a layer of granular fill on the surface of soft clay, has been studied by small-scale model tests (width and height are 50 cm × 45 cm) in the laboratory. They also provided analytical results of these experiments. The comparison between these results and PLAXIS simulation results is shown in Fig. 5. It shows these results match pretty well, except for the large deformation stage. This inconsistence might be attributed to the linear elastic model is set for reinforcement and for soil material, and the creep effect is not considered in the PLAXIS model developed in this study.

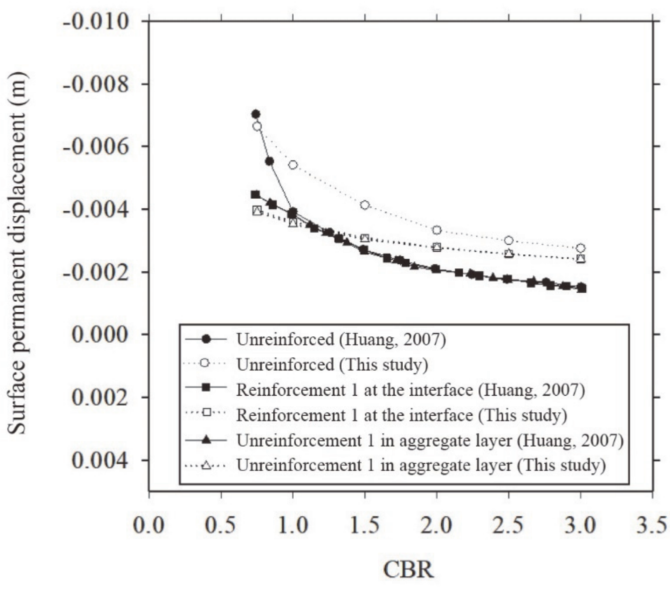


Fig. 3 Comparison between model simulation results in this study and Huang (2007)

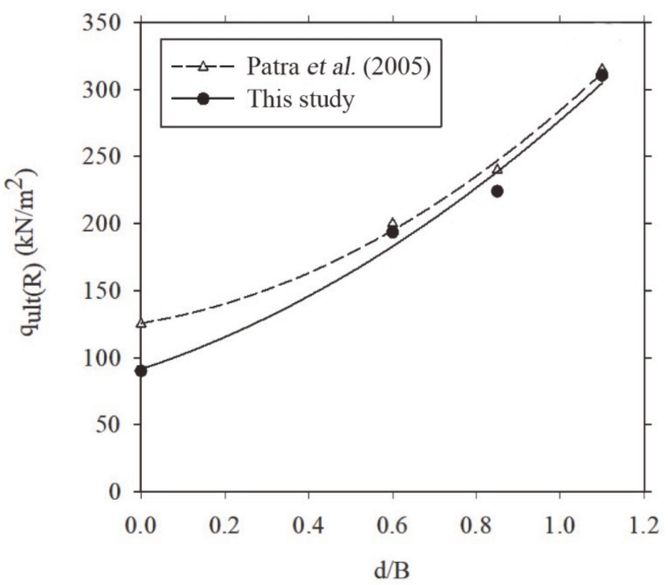


Fig. 4 Comparison between model simulation results obtained in this study and experimental results obtained by Patra *et al.* (2005)

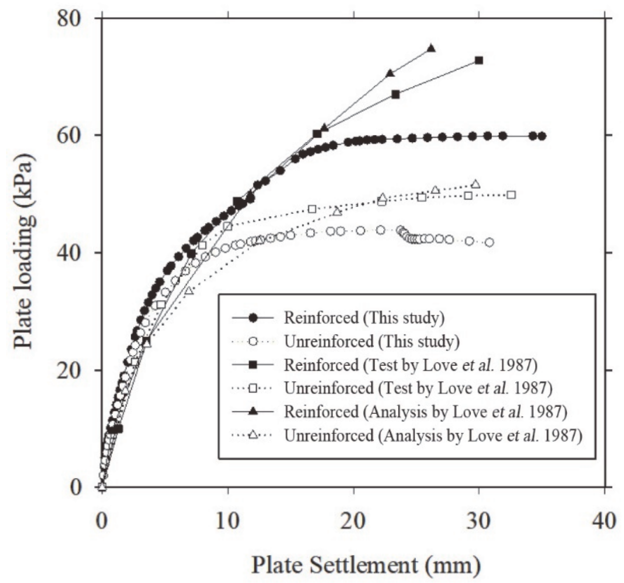


Fig. 5 Comparison between model simulation results obtained in this study, analytic and experimental results obtained by Love *et al.* (1987)

In general, the PLAXIS model simulates the laboratory model test or other numerical model satisfactorily on the premise of accurate constitutive models and input parameters of materials. These favorable comparisons verify the numerical model developed in this study is suitable for the successive simulation of geosynthetic reinforced soils under surcharge

3. PARAMETRIC ANALYSES

Because of the wide variety of soil types, geosynthetic types, and geosynthetics layout conditions, laboratory model and field tests are limited. With the aid of numerical simulations, it is now possible to investigate the mechanical behavior of geosynthetic reinforced weak foundation under variety combination of geometry, layout, materials and surcharging conditions.

The following parameters: Young's modulus of soil (E), cohesion of soil (c), friction angle of soil (ϕ), interface coefficient between geosynthetics and soil (IN), tensile stiffness of geosynthetics (E_t), layers of geosynthetics (n), normalized depth of placement (u/B), normalized interval between geosynthetic layers (h/B) are treated as important to the mechanical behavior of geosynthetic reinforced soft foundation. They are selected as variables composing the numerical model. Based on the investigation by Institute of Transportation (IOT), Ministry of Transportation and Communications (MOTC) in Taiwan, the definition of soft soil is SPT-N $\leqq 10$ for silty sand, which is commonly encountered in sediment region in western Taiwan. Therefore, the silty sand of small SPT-N (< 10) is used as the soil material in the numerical model. Wang (2001) collected a lot of test data in Taiwan and correlated the empirical formula between modulus (E) of silty sand and SPT-N is E = 650.72 (kN/m^2). The friction angle (ϕ) of sand is obtained from the famous empirical formula (ϕ = 27 + 0.3 N) proposed by Peck (1953). In most cases, geogrid or geotextile reinforcements are the one selected for soil reinforcement. A wide range between 625 and 2500 kN/m was set as the stiffness of geosynthetic because it varies depending on the thickness and materials. Liu *et al.* (2009) conducted large scale direct shear test on geosynthetic and soil interface and found the coefficient of interface friction ranges between 0.7 and 1.05, also dependent on the type of soil and geosynthetics. In this study, 0.6 ~ 1.0 was set as the interface coefficient. These values associated with the configuration of the numerical model to perform parametric study for identifying parameter effect on reinforced soil layers are listed in Table 1.

For each numerical simulation, the footing settlement during surcharging process was recorded. Figure 6 presents the footing settlement behaviors of unreinforced SPT-N = 10 soil and geosynthetic reinforced soils. It shows the unreinforced soil reaches its ultimate bearing capacity as footing settlement is small, about 30 mm, while reinforced soil is more ductile. The inclusion of geosynthetics also improves the bearing capacity. The improvement effect is more significant for more layers and stiffer geosynthertic placed in soil. In this study, an index, BCR, is defined as the ratio of the bearing capacity of reinforced soil to unreinforced soil corresponding to footing settlement of 30 mm, which is 10 % of footing width, Through the BCR, the effect of geosynthetic reinforcement of bearing capacity can be examined.

Table 1 Parameters of numerical analysis

Parameters	Range	Unit
Cohesion of soil, c	0.5(SPT-N = 4, 7, 10)	kPa
Friction angle, ϕ	28.2(SPT-N = 4), 29.1(SPT-N = 7), 30.0(SPT-N = 10)	Degree
Possion ratio, v	0.3	–
Young's modulus of soil, E	2656(SPT-N = 4), 4650(SPT-N = 7), 6640(SPT-N = 10)	kPa
Unit weight of soil, γ_s	19.5	kN/m^3
Interface coefficient, IN	0.6, 0.8, 1.0	–
Layers of geosynthetics, n	1, 2, 3, 5	layer
Stiffness of geosynthetics, E_t	625, 1250, 2500	kN/m
Ratio of depth, u/B	0.25, 0.5, 1.0	–
Ratio of layer interval, h/B	0.25, 0.5, 1.0	–
Ratio of settlement, Δ/B	10	%

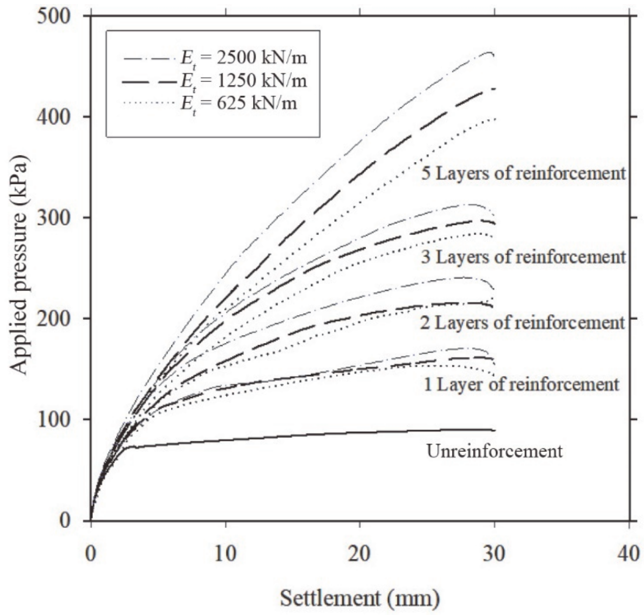


Fig. 6 Applied pressure versus footing settlement for different stiffness and layers of geosynthetics (SPT-N = 10, u/B = 0.25, h/B = 0.25, IN = 1)

Some laboratory model tests (Mandal *et al.* 1992; Ramaswamy and Purushothaman 1992; Adams and Collin 1997) and numerical simulation (Yetimoglu *et al.* 1994) of the geosynthetic reinforced soil layer reach the conclusion that the effect of bearing capacity increase is less significant as reinforcement is placed deeper than the optimum depth. It is noted that the suggested optimum depth, in general between 0.175 ~ 0.5 u/B, is different on different research. For example, Ramaswamy and Purushothaman (1992) conducted laboratory test on cohesive soil indicated that the optimal depth for placing geogrid is 0.5 u/B. Mandal *et al.* (1992) conducted similar test on clay and concluded that the bearing capacity is maximum corresponding to reinforcement was placed at 0.175 u/B. Adams and Collin (1997) ran large-scale reinforced sandbox test and found that when 3 layers of geogrid were placed, bearing capacity increased the most as they were placed at the depth of 0.25 u/B. Yetimoglu *et al.* (1994) performed FEM analysis on reinforced sand layer and found the optimal depths for placing 1 and 3 layers reinforcement were 0.3, and 0.25 u/B, respectively. Figure 7 shows how the BCR increases with a smaller u/B. In general, the analysis results are consistent with the previous researches that the geosynthetic reinforcement effect is more significant as geosynthetic placed in shallower depth. The optimal depth for placing different layers of geogrid of different stiffness is smaller than 0.5 u/B. The interaction between soil and geosynthetic is an important factor to the geosynthetic reinforcement. The BCR increases with a larger interface coefficient (Fig. 8).

4. DEVELOPING BEARING CAPACITY FUNCTION USING SECOND-ORDER MODEL

A series of numerical experiment was conducted to produce a comprehensive database containing bearing capacity (dependent) corresponding to different combinations of design parameters of reinforced soft silty sand. In the experiment, the SPT-N value for the weak silty soil is set as 4, 7, and 10, respectively. The mechanical

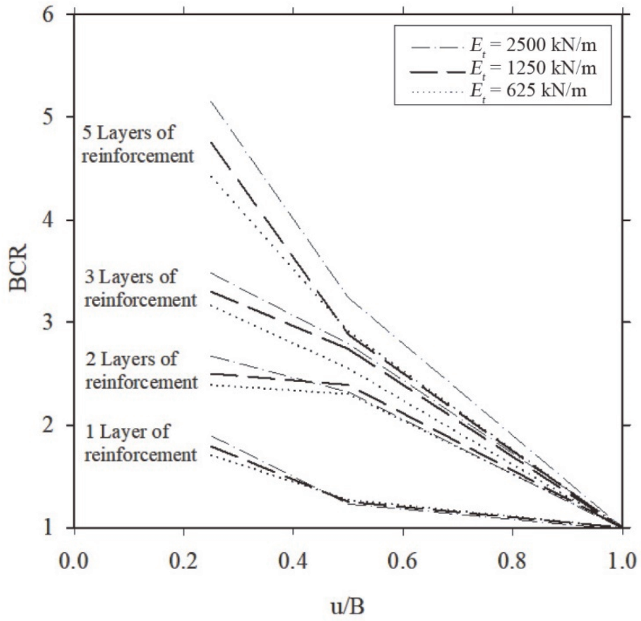


Fig. 7 BCR versus u/B for different stiffness and layers of geosynthetics (SPT-N = 10, h/B = 0.25)

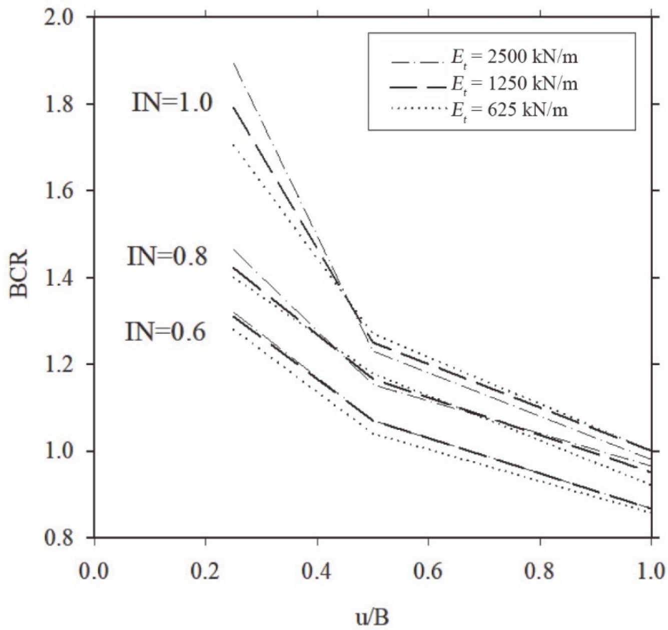


Fig. 8 BCR versus u/B for different stiffness and interface of geosynthetics (SPT-N = 10, h/B = 0.25)

properties (E and ϕ) corresponding to SPT-N value follow Wang (2001) and Peck (1953). The parameter and its value applied in the experiments are listed in Table 2. In total, an experiment set consisting of 810 simulations by varying design parameters is conducted. The combination of parameter values covers wide range of material properties and configuration could be encountered in a general geosynthetic reinforced soft silty sand. The dependent variable of the dataset is expressed in a form as $y = \ln(q/E)$ where q is the bearing capacity corresponding to 30 mm footing settlement, which is equivalent to 10% of footing width, considered as the limitation of footing serviceability. The independent were the design parameters H, B, c, ϕ, E, E_t, u, h, d, n, Δ, and IN as shown in Table 2.

Table 2 Range of parameters for SOM equation

Parameters	Range	Unit
Footing width, B	0.3	m
Cohesion of soil, c	0.5	kPa
Young's modulus of soil, E	2656, 4650, 6640	kPa
Height of model, H	3	m
Friction of soil, ϕ	28.2, 29.1, 30.0	Degree
Depth of reinforcement placement, d	0.075, 0.15, 0.225, 0.3, 0.375, 0.45, 0.6, 0.675, 0.75, 0.9, 1.275	m
Elastic modulus of geosynthetics, E_t	625, 1250, 2500	kN/m
Thickness between layers, h	0, 0.075, 0.15, 0.3	m
Interface between soil and geosynthetics, IN	0.6, 0.8, 1.0	–
Number of reinforcement, n	1, 2, 5	–
Thickness of top layer, u	0.075, 0.15, 0.3	m
Displacement, Δ	0.03	m

The experiment results were analyzed by non-linear multiple regression function, follow response surface methodology rule. This approximte equation is called the Second-Order Model (SOM). There are several reasons used in SOM equation from response surface in this study. First, the second-order model is flexible, and the results can approximate the true response surface well. Second, it is easy to estimate the parameters using the least squares method. Third, there is considerable application in geotechnical problems indicating that approach performs well in solving real problems (*e.g.* Babu *et al.* 2007; Youssef *et al.* 2008; Wu *et al.* 2013). The SOM equation is derived by regressing the dependent and design parameters in the database. The SOM equation were in the form as:

$$\{C\} = [A] \times \{\beta\} + \varepsilon \tag{1a}$$

$$\{C\} = \ln\left(\frac{q}{E}\right) \tag{1b}$$

where $\{C\}$ is the dimensionless independent matrix $\ln(q/E)$, while $[A]$ is the independent parameter matrix and $\{\beta\}$ is the multiple regression coefficient corresponding to independent parameters $[A]$. As learned from the parametric analysis results (Figs. 6 to 8) that the bearing capacity of a geosynthetic reinforced soft soil is a non-linear response to design parameters, different forms of $[A]$ has been tested for the best regression results. The analysis results reveal that $[A]$ is composed of the following terms fits best. This SOM equation model assumes that the ε is uncorrelated random variables.

The SOM equation is full quadratic specification with six variables (x_1 to x_6) based on the 810 numerical simulation results, as proposed in Eq. (2):

$$x_1 = \ln\,(n \times IN) \tag{2a}$$

$$x_2 = \frac{h}{B} \tag{2b}$$

$$x_3 = \ln\left(\frac{\tan(\phi) \times c}{E}\right) \tag{2c}$$

$$x_4 = \ln\left(\frac{E_t \times IN}{d \times c}\right) \tag{2d}$$

$$x_5 = \ln\left(\frac{E_t \times n}{\Delta \times \mathrm{E}}\right) \tag{2e}$$

$$x_6 = \ln\left(\frac{H}{u}\right) \tag{2f}$$

$$\begin{aligned} y = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 \\ & + \beta_7 x_1^2 + \beta_8 x_2^2 + \beta_9 x_3^2 + \beta_{10} x_4^2 + \beta_{11} x_5^2 + \beta_{12} x_6^2 \\ & + \beta_{13} x_1 x_2 + \beta_{14} x_1 x_3 + \beta_{15} x_1 x_4 + \beta_{16} x_1 x_5 + \beta_{17} x_1 x_6 \\ & + \beta_{18} x_2 x_3 + \beta_{19} x_2 x_4 + \beta_{20} x_2 x_5 + \beta_{21} x_2 x_6 \\ & + \beta_{22} x_3 x_4 + \beta_{23} x_3 x_5 + \beta_{24} x_3 x_6 \\ & + \beta_{25} x_4 x_5 + \beta_{26} x_4 x_6 \\ & + \beta_{27} x_5 x_6 + \varepsilon \end{aligned} \tag{2g}$$

The numerical result of equation coefficient values calculated from regression analysis are presented in Table 3.

The result of proposed bearing capacity are presented in Fig. 9, which shows the good match between the calculated bearing capacity from PLAXIS and predicted bearing capacity from SOM equation, the mean is one. It is noted that error term ε, called the model uncertainty must be considered into the SOM equations to modify predict accurate. The model uncertainty (ε), refers to the variance between SOM equation prediction and numerical simulation result. The analysis results indicate that there is no bias between the SOM equation prediction and numerical simulation results while the estimated coefficient of variation (*i.e.,* the model uncertainty, ε), is 0.13.

Table 3 Coefficients of SOM equation in soft sand

Statistical parameters	Coefficient	Parameters
β_0	10.02689	1
B_1	0.119428	X_1
β_2	1.172170	X_2
β_3	2.871991	X_3
β_4	0.539036	X_4
β_5	– 0.53100	X_5
β_6	1.382430	X_6
β_7	0.23489	$X_1 \times X_2$
β_8	0.44655	$X_1 \times X_3$
β_9	0.49127	$X_1 \times X_4$
B_{10}	– 0.45446	$X_1 \times X_5$
β_{11}	0.42583	$X_1 \times X_6$
β_{12}	0.08822	$X_2 \times X_3$
β_{13}	0.04348	$X_2 \times X_4$
β_{14}	– 0.04835	$X_2 \times X_5$
β_{15}	– 0.07052	$X_2 \times X_6$
β_{16}	0.18972	$X_3 \times X_4$
β_{17}	– 0.17011	$X_3 \times X_5$
β_{18}	0.08706	$X_3 \times X_6$
β_{19}	– 0.18342	$X_4 \times X_5$
β_{20}	0.11603	$X_4 \times X_6$
β_{21}	– 0.08911	$X_5 \times X_6$
β_{22}	0.61621	X_1^2
β_{23}	– 0.25545	X_2^2
β_{24}	0.17382	X_3^2
β_{25}	0.09026	X_4^2
β_{26}	0.08122	X_5^2
β_{27}	– 0.19962	X_6^2

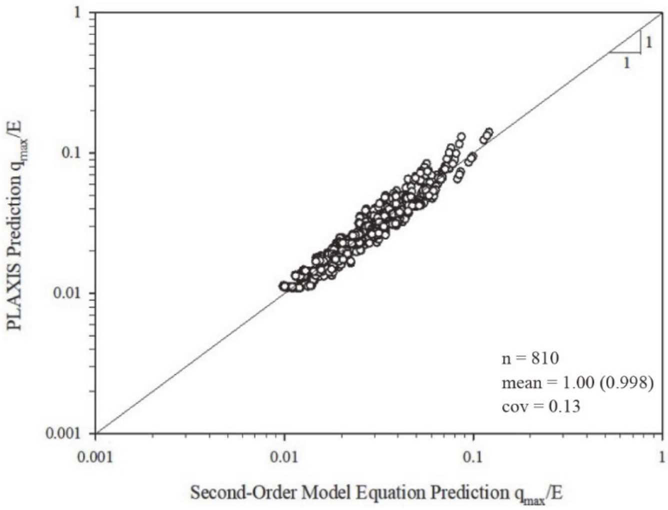


Fig. 9 Numerical results versus SOM equation prediction

5. PROBABILISTIC ANALYSIS OF BEARING CAPACITY OF GEOSYNTHETICS REINFORCED SOFT SOIL

The prediction of the bearing capacity of soft silty sand can be made deterministically from Eq. (2) using best estimates of the input parameter values. However, the limitation of this deterministic value is that it does not contain a quantitative estimate of the uncertainty in this prediction. The sources of uncertainty are from the choice of parameter value. Among them, the soil properties and geosyntetic stiffness are the most common ones. Numerous researches (Haldar and Tang 1979; Popescu 1995; Popescu *et al.* 1996; Lacasse and Nadim 1996; Popescu 1997; Fenton 1999) have worked on identifying these uncertainties. The uncertainty in the SOM equation is accounted for by considering the uncertainties of parameters in it. The attributes and statistical properties including distribution type and magnitude of coefficient of variance of these parameters as summarized in Table 4, refer to the publications including Phoon and Kulhawy 1999; Duncan 2000; Walters *et al.* 2002; Chen *et al.* 2005; Huang and Bathurst 2009. It is worth noting that the advanced statistical characteristics of soil properties including random field concept and inter-correlation among soil parameters are not considered in this research.

Table 4 Statistical properties of parameter for base case

Parameters	Distribution type	Mean value	Coefficient of variation (COV)
E	Lognormal	4656 kN/m^2	0.2
IN	Normal	0.8	0.2
ϕ	Normal	29.1°	0.07
E_t	Lognormal	1250 kN/m	0.1
c	Normal	0.5 kPa	0.2
n	Deterministic	2	–
B	Deterministic	0.3 m	–
u	Deterministic	0.075 m	–
d	Deterministic	0.15 m	–
Δ	Deterministic	0.03 m	–
h	Deterministic	0.075 m	–
H	Deterministic	3 m	–

These variables which exist in the SOM equation are treated as random variables in the subsequent study under probabilistic framework. Probabilistic estimates of the bearing capacity can be carried out using Monte Carlo simulation which involves substituting values sampled from a probability distribution for each input parameter that has inherent uncertainty into SOM equation. Note that the SOM equation has a simple form which can be calculated with spreadsheet or by computer program, the Monte Carlo simulation with multiple realizations can be manipulated easily.

The histogram of 100,000 simulations of bearing capacity of base-case (Table 4) is shown in Fig. 10. It can be fitted with a probability density function (PDF) of lognormal distribution. The lognormal PDF fits the histogram of raw data well. The characteristic parameters (shape and scale parameters) λ and ζ for this lognormal distribution can be calculated by the following equations:

$$\lambda = \frac{1}{n}\sum_{i}^{n} \log(q / E) \tag{3a}$$

$$\varsigma = \sqrt{\frac{\sum_{i}^{n}[\log(q / E)_i - \lambda]^2}{n-1}} \tag{3b}$$

where n is the number of realizations. For the base-case distribution, λ and ζ were 3.36 and 0.20, respectively.

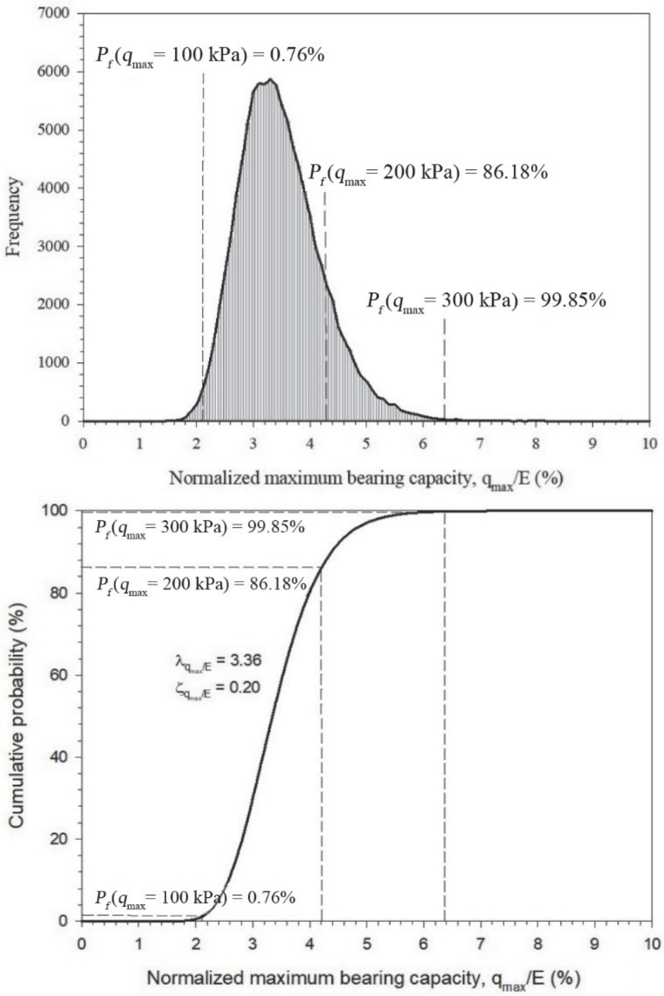


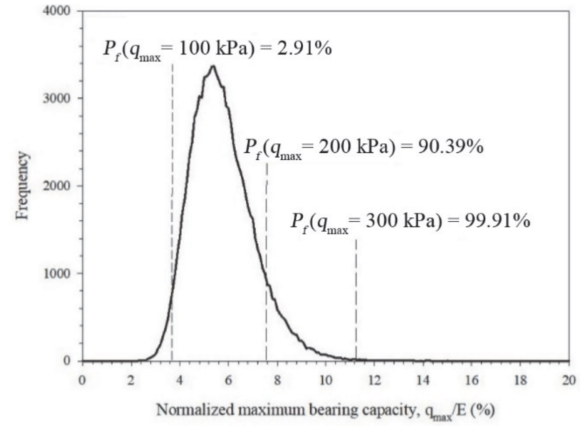
Fig. 10 Frequency and cumulative probability function based on SOM equation prediction and 100,000 MC simulations using base case

For a soft foundation under the applied surcharge, the probability that applied surcharge ($q_{applied}$) exceeds bearing capacity (q) corresponding to certain prerequisite performance condition can be calculated according to the probability distribution transformation of lognormal density function:

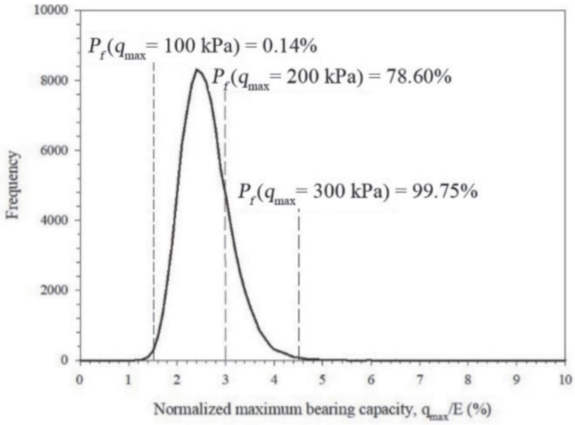
$$P\left[\left(\frac{q}{E}\right) < \left(\frac{q_{applied}}{E}\right)\right] = \Phi\left(\frac{\log\left(\frac{q_{applied}}{E}\right) - \lambda}{\varsigma}\right) \tag{4}$$

where $P[(q/E) < (q_{applied}/E)]$ is the probability of its argument while $\Phi\left(\frac{\log\left(\frac{q_{applied}}{E}\right) - \lambda}{\varsigma}\right)$ is the standard normal cumulative distribution function. For example, there is a reinforced soft foundation has the same configuration and material properties as the base case. If this foundation is surcharged by 100 kPa, as shown in Fig. 10, indicating a large probability that the applied surcharge is smaller than the bearing capacity of this soil. The probability can be calculated by using Eq. (4), which is equal to 0.76%. If the applied surcharge increases to 200 kPa or even 300 kPa, one can easily observe that the exceeding probability increasing dramatically (Fig. 10). The failure probabilities for applying 200 and 300 kPa surcharge are 86.1% and 99.85%. respectively.

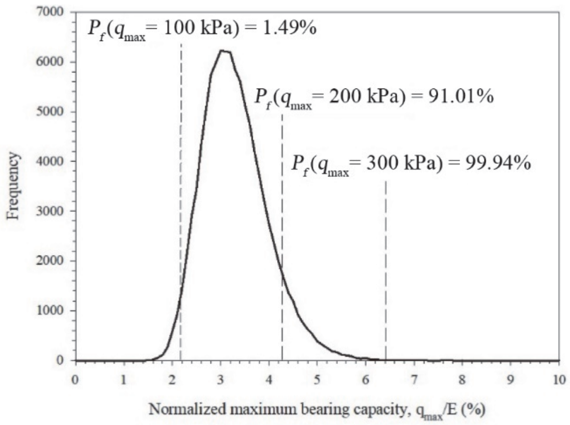
It is of interest in investigating the relative significance of different design parameters on the probabilistic characteristics of bearing capacity. To achieve the parametric analysis, the process of conducting 100,000 realizations of Monte Carlo Simulation for base-case is similarly performed for 8 cases. In each case, one parameter is changed from the base-case value. These cases are believed to be sufficient to demonstrate the main features of the influence of important parameters (E, ϕ, E_t, and IN) on bearing capacity of reinforced soft soil. For each case, the obtained histogram can be well fitted with lognormal distribution function and they are statistically analyzed to calculate the shape and scale parameters for best fitted lognormal distribution (Figs. 11). The analysis results along with failure probabilities for applying 100, 200, and 300 kPa surcharge are presented in Table 5 for reference. It shows that for all cases, there is a significant increase of failure probability with an increase in surcharge magnitude. As expected, the parametric study reveal that a stronger soil (E and ϕ, cases 1 ~ 4) reduces the failure probability. It demonstrates the importance of compaction of backfill soil in controlling the settlement of reinforced soft ground. The use of a stronger geosynthetic reinforcement (E_t, cases 5, 6) reduce the failure probability, too. For the soil which is reinforced by stiffer material or is placed by more layers of reinforcement, it is more resistant to the occurrence of lateral deformation, thus the settlement. It is found that the failure probability decreases with an increase of interaction between soil and geosynthetic (IN, cases 7, 8), and the improvement is significant than the selection of stronger soil or stiffer geosynthetics. The interaction between soil and geosynthetic is a key factor for the engineering properties of soil/geosynthetic composite. Many researchers have been conducted through laboratory analyses (for example, Liu *et al.* (2009) used direct shear test, Ezzein and Bathurst (2014) used pull-out test, Liu *et al.* (2014) used large scale plane strain compression test) to evaluate the contribution of interaction on improving the soil/geosynthetic composite. The



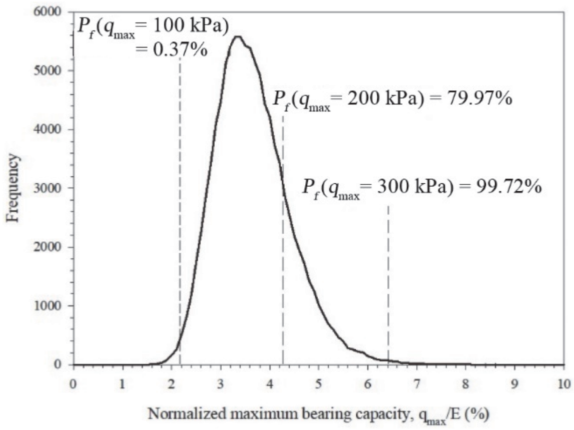
(a) Case 1



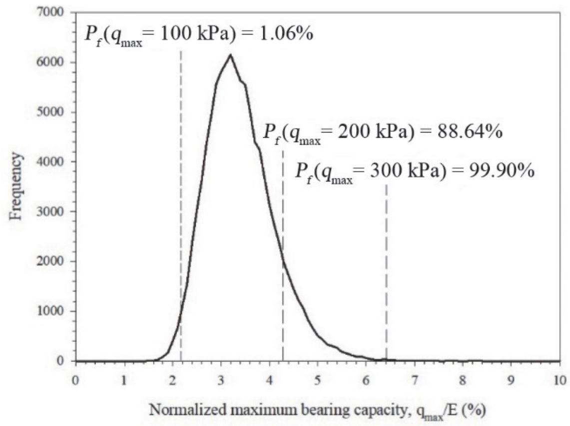
(b) Case 2



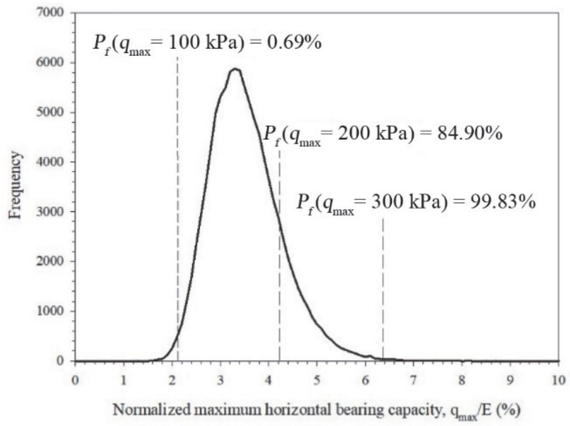
(c) Case 3



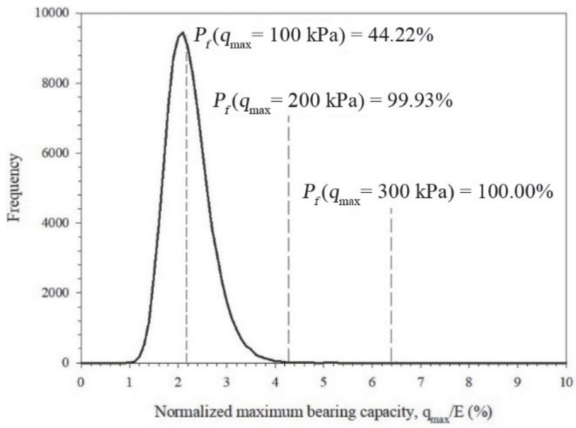
(d) Case 4



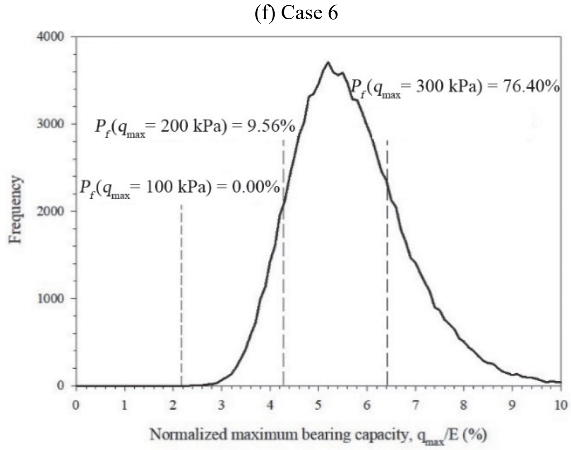
(e) Case 5



(f) Case 6



(g) Case 7



(h) Case 8

Fig. 11 Frequency based on SOM equation prediction and 100,000 MC simulations using different parameters Case 1 to Case 8

Table 5 Parameters impact on probability of failure under different surcharges

Case	Variable changed	Probability of failure		
		q_{applied} = 100 kPa	q_{applied} = 200 kPa	q_{applied} = 300 kPa
Base-Case	–	0.76%	86.18%	99.85%
Case 1	E = 2.66 MPa	2.91%	90.39%	99.91%
Case 2	E = 6.64 MPa	0.14%	78.60%	99.75%
Case 3	ϕ = 28.2°	1.49%	91.01%	99.94%
Case 4	ϕ = 30°	0.37%	79.97%	99.72%
Case 5	E_t = 625 kN/m	1.06%	88.64%	99.90%
Case 6	E_t = 2500 kN/m	0.69%	84.90%	99.83%
Case 7	IN = 0.6	44.22%	99.93%	100.00%
Case 8	IN = 1	0.00%	9.56%	76.40%

interaction coefficient may range between 0.6 and 1.0 for different geosynthetic material against different soil (Liu *et al.* 2009). The selection of appropriate geosynthetic material by conducting proper interaction test is very important for the bearing capacity of soft ground.

These findings are reiterated in Fig. 12. It shows the failure probability does not change much corresponding to wide range of E and E_t, while it changes obviously with surcharges. The failure probability can increased from negligible to more than 80% as surcharge increases from 100 to 200 kPa. Another merit of Fig. 12 is that the analysis results described a performance-based design framework. For example, when q_{applied} set on 200 kPa and E (elastic modulus of soil) is ranged between 2,000 kPa and 7,000 kPa. The probability of failure under different stiffness of geosynthetics can be easily checked on the plot. The reliability based design for bearing capacity of geosynthetics reinforced soft soil based on SOM equation is feasible.

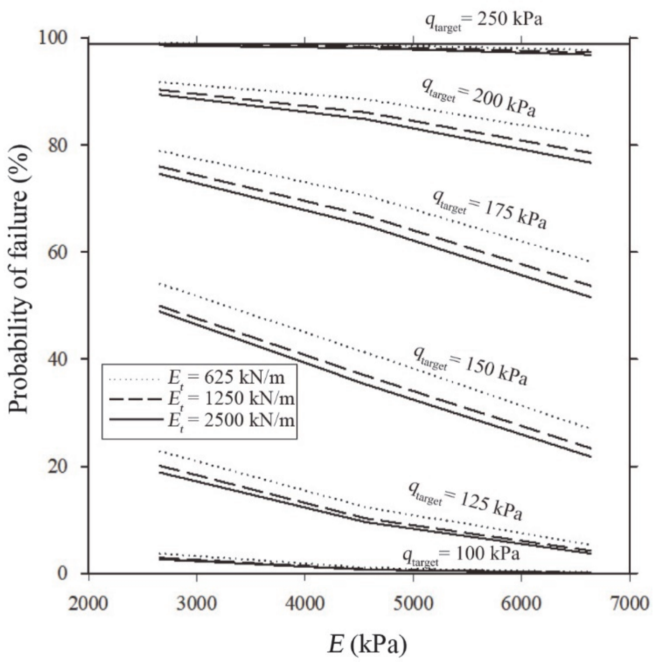


Fig. 12 Probability of failure under different combination of tensile stiffness of geosynthetics, elastic modulus of soil, and surcharges

6. CONCLUSIONS

The application of geosynthetics to reinforce geotechnical structures has been progressively improved by developments in material manufacture, design methods, and construction methods. The design and analysis of geosynthetic reinforced soft soil are mostly based on deterministic values of material properties. However, the recent trend is to conduct probabilistic design and analysis. Meanwhile, the consideration on performance of constructed facilities has increased in parallel with the advent of technology.

This research exemplifies how the probabilistic analysis of serviceability criteria may be achieved in geosynthetic reinforced soft soil. In this study, a numerical model for simulating bearing capacity of reinforced soft soil is developed and validated by simulating the behavior of reinforced foundation. A set of numerical experiments is conducted to generate a comprehensive database of bearing capacity using different combination of design parameters.

The experiment results were analyzed by using Second-Order Model (SOM), a non-linear multiple regression function. Compared with numerical analysis, the suggested SOM equation is an easy and accurate tool to estimate the probabilistic characteristic of a geosynthetic reinforced foundation. With the application of Monte Carlo simulation technique, the probabilistic distribution of bearing capacity was achieved. The obtained probabilistic distribution can be well fitted as a lognormal distribution. The exceeding probability of geosynthetic reinforced foundation under defined serviceability criterion can be estimated from the histogram. This study also performed parametric analysis to identify the important parameters affecting the failure probability of geosynthetic reinforced soft ground under surcharge. It is found that the bearing capacity is benefited from using a stronger soil and geosynthetics while the very significant factor is the interaction between soil and geosynthetic materials. Moreover, the reliability based design of geosynthetic reinforced foundation for any other applied of custom surcharge and material property such as tensile stiffness of reinforcement, bearing capacity or Young's modulus of soil on the criterion of bearing capacity is also feasible using the methodology presented here.

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DATA AVAILABILITY

The data and/or computer codes used/generated in this study are available from the corresponding author on reasonable request.

CONFLICT OF INTEREST STATEMENT

The authors declare that there is no conflict of interest.

NOTATIONS

Basic SI units are given in parenthesis.

$[A]$	The independent parameter matrix
$\{B\}$	The regression coefficient
B	Footing width (m)

$\{C\}$	The dimensionless independent matrix
c	Cohesion of soil (Pa)
COV	Coefficient of variation
d	Depth of reinforcement placement (m)
E	Young's modulus of soil (Pa)
E_t	Tensile stiffness of reinforcement (kN/m)
H	Model height (m)
h	Thickness between layers (m)
IN	Interface between soil and geosynthetics
L	Length of reinforcement (m)
M	Model uncertainty
n	Layers of reinforcement
P_f	Failure probabilities
q	Bearing capacity
s	Footing settlement
u	Thickness of top layer
$\beta_0 \ldots \beta_{27}$	Statistical parameters coefficients
Δ	Settlement (m)
ε	Model uncertainty of PLAXIS and Equation
ϕ	Friction angle of soil (°)
Φ	The standard normal cumulative distribution
γ_s	Unit weight of soil (kN/m^3)
λ	Shape parameters
ν	Poisson's ratio
ζ	Scale parameters

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