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RELATIONSHIP BETWEEN LIQUEFACTION POTENTIAL INDEX AND LIQUEFACTION PROBABILITY

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ABSTRACT

Liquefaction potential index (LPI) is commonly used for liquefaction risk assessment. Although several LPI classifications have been proposed, those classifications are categorical and descriptive, which are not useful for quantitative risk assessments (risk = probability × consequence). In this study, a total of 135 soil liquefaction data points from Taiwan, Turkey, and Italy were collected and used for developing a new empirical relationship between LPI and liquefaction probability with logistic regression. In addition, we conducted 10 additional sensitivity analyses to evaluate the possible impact of random data arrangement on this study, showing that the impact was very insignificant given the similar results obtained from the analyses.

Key words: Liquefaction potential index (LPI), liquefaction probability, logistic regression.

1. INTRODUCTION

It is well known that big earthquakes can cause soil liquefaction, which could cause severe damage to infrastructures. Therefore, for mitigating soil liquefaction hazards, a variety of measures have been proposed, including liquefaction potential index (LPI). LPI was proposed by Iwasaki *et al.* (1982) as a measure to characterize liquefaction potentials at a site (the higher the LPI, the higher the soil liquefaction potential). Afterwards, a number of case studies were conducted for evaluating the liquefaction potentials at target sites based on LPI (Toprak and Holzer 2003; Papathanassiou *et al.* 2005; Sonmez *et al.* 2008; Heidari and Andrus 2010; Kang *et al.* 2014; Rahman *et al.* 2015).

The mathematical expression of LPI is as follows (Iwasaki *et al.* 1982):

$$\mathrm{LPI} = \int_0^{20} F_L \times (10 - 0.5z)\, dz \tag{1}$$

where z is depth (in meter) from the ground surface; F_L is the liquefaction resistance factor, governed by the liquefaction factor of safety (FS):

$$F_L = \begin{cases} 1 - FS & ; FS \le 1 \\ 0 & ; FS > 1 \end{cases} \tag{2}$$

In addition to the proposal of LPI computation, Iwasaki *et al.* (1982) also recommended a classification system for liquefaction risk/potential evaluation as follows: when LPI = 0, liquefaction risk/potential is very low; 0 < LPI ≤ 5, liquefaction risk/potential is low; 5 < LPI ≤ 15, liquefaction risk/potential is high; LPI > 15, liquefaction risk/potential is very high. Quoted from the paper of Iwasaki *et al.* (1982): "It is found from this figure that LPI for liquefied sites seems to be higher than that at non-liquefied sites, *i.e.,* for non-liquefied sites LPI is mostly less than 15 and the percentage that LPI is less than 5 is about 70%, and on the other hand for liquefied sites the percentage that LPI less than 5 is only 20% and for about 50% of the site LPI is more than 15. From these results, the following simplified procedure for assessing soil liquefaction based on LPI maybe proposed as a preliminary guideline." According to the statement (also the only statement present in that paper) addressing the rational of the simplified classification, it should be evident that subjective engineering judgments were somehow involved in determining this LPI classification, which was not entirely based on objective, mathematical calculations.

By definition, risk = probability × consequence (Fenton and Griffiths 2008). For example, if the liquefaction probability at a site within the next 10 years is 0.001 and the consequence caused by the soil liquefaction is $1,000,000, the risk is calculated as $1,000 (= 0.001 × 1000000) in terms of monetary loss. However, although LPI that was indeed calculated with quantitative measurements (*e.g.,* SPT and CPT) can be used for judging the liquefaction potential of the site in relative to others, the LPI value itself has nothing to do with liquefaction probability. In other words, based on LPI only, quantitative liquefaction risk assessments (risk = probability x consequence) cannot be conducted.

Since the first LPI classification system was proposed (Iwasaki *et al.* 1982), different LPI classifications were also proposed in following years. Based on the field data from the Darfield and Christchurch earthquakes, Maurer *et al.* (2014) recommended the LPI classification systems as follows: as 8.4 < LPI ≤ 13.1, liquefaction risk/potential is marginal; 13.1 < LPI ≤ 21, liquefaction risk/potential is moderate; LPI > 21, liquefaction risk/potential is severe. As those proposed by Iwasaki *et al.* (1982), these classifications are also categorical and descriptive.

Other LPI classification systems include those proposed by Lee *et al.* (2003), Ku and Ma (2017), and Papathanassiou (2008). Along with the previous classifications, Table 1 summarizes the five LPI classifications.

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Table 1 Five different LPI classifications

–	LPI and risk/potential classifications			
Iwasaki *et al.* (1982)[a]	0	0 ~ 5	5 ~ 15	> 15
	Very low	Low	High	Very high
Lee *et al.* (2003)[b]	< 8	8 ~ 16	> 16	–
	Low	High	Extremely high	–
Papathanassiou (2008)[c]	< 19	19 ~ 29	> 29	–
	No failure	Medium	High	–
Maurer (2014)[d]	< 8.4	8.4 ~ 13.1	13.1 ~ 21	> 21
	–	Marginal	Moderate	Severe
Ku and Ma (2017)[e]	< 5.6	5.6 ~ 12.5	12.5 ~ 21.7	> 21.7
	Minor	Medium	High	Extremely high

[a] The LPI datasets were calculated based on SPT along with the Japanese Highway Bridge Design Code calculation procedure (JSHE 1990); the datasets were from Japan, collected after the Nobi, Tonankai, Fukui, Niigata, Tokachi-oki, Miyagi-ken-oki earthquakes in Japan.

[b] The LPI datasets were calculated based on CPT along with the Olsen calculation procedure (Olsen 1988); the datasets were from Taiwan, collected after the Chi-Chi earthquakes in Taiwan.

[c] The LPI datasets were calculated based on SPT along with the Youd calculation procedure (Youd *et al.* 2001); the datasets were from Taiwan, Turkey and Greece, collected after the Chi-Chi (Taiwan), Kocaeli (Turkey), and Lefkada (Greece) earthquakes.

[d] The LPI datasets were calculated based on CPT along with the Robertson calculation procedure (Robertson and Wride 1998); the datasets were from New Zealand, collected after the Darfield and Christchurch earthquakes.

[e] The LPI datasets were calculated also based on CPT along with the Robertson method (Robertson 2009); the datasets were from Taiwan, Turkey and Italy, collected after the Chi-Chi earthquake (Taiwan), the Kocaeli (Turkey) earthquake, and the 2012 Northern Italy earthquakes.

Here, we use an example to demonstrate the ambiguity and uncertainty resulting from different LPI classifications. For a site with LPI = 10, Iwasaki *et al.* (1982) would consider the site has a high liquefaction risk/potential, Maurer *et al.* (2014) would consider a marginal liquefaction risk/potential, Lee *et al.* (2003) would consider high liquefaction risk/potential, Ku and Ma (2017) would consider a medium liquefaction risk/potential, and Papathanssiou (2008) would consider a no-failure (or non-liquefaction risk/potential). As a result, it is clear that the uncertainty exists when it comes to LPI classifications. Besides, all of the LPI classifications are descriptive and categorical, not in association with liquefaction probability which is one of the essential elements for quantitative risk assessments (risk = probability x consequence).

Here, we would like to elaborate the liquefaction research of Ku and Ma (2017) and Papathanssiou (2008) that served the key references for this study. Not only did Ku and Ma (2017) compile a new LPI database, they also proposed a new LPI classification (see Table 1). Basically, the boundaries of the LPI classification were determined with the mean values and standard deviations of both liquefaction and non-liquefaction datasets. Similarly, Papathanssiou (2008) also compiled another LPI database (*i.e.,* 79 data points) based on the SPT measurements and earthquake data from Taiwan, Greece, and Turkey, and proposed a new LPI classification. Essentially, the study used the box-whisker plot for determining the boundaries of the new LPI classification. It is worth noting that unlike other classifications, the thickness of the non-liquefiable cap layer was taken into account in this classification. More details about the two LPI classifications were given in the following.

Here, we would like to elaborate how the above researchers developed their own LPI classification systems (Table 1).

- Lee *et al.* (2003) considered the upper-bound LPI should be the one encompassing 70% of the liquefaction cases of their database (*i.e.,* 20 data points). By contrast, the lower-bound LPI should be the one encompassing 70% of the non-liquefaction cases.
- Based on the visual observation/judgment during site investigations, Papathanassiou (2008) separated their LPI database into three groups in the first place, namely High, Medium, Low severity. Next, they calculated the 25th and 75th percentiles (LPI) of each group, then using them as the lower- and upper-bound LPI for each group, respectively.
- Similarly, Maurer *et al.* (2014) grouped their data as No liquefaction, Marginal liquefaction, and so on so forth during site investigations. Next, they calculated the average LPI of the No-liquefaction group, and recommended it as the upper-bound LPI for this group (also as the lower-bound value for the next group, *i.e.,* Marginal liquefaction), and so on so forth.
- Ku and Ma (2017) determined the boundaries of their LPI classifications based on the mean values and standard deviations (SD) of both liquefaction and non-liquefaction datasets. Specifically, they used the value of {mean – SD} of the non-liquefaction dataset as the lower-bound LPI for Medium liquefaction potential, and used the value of {mean + SD} of the liquefaction dataset as the upper-bound LPI for High liquefaction potential. They also defined a so-called α value, which is a function of mean values and standard deviations of both liquefaction and non-liquefaction datasets, then using the value of {mean + $\alpha \times$ SD} of the non-liquefaction dataset as the upper-bound LPI for Medium liquefaction potential.

As mentioned previously, although the above LPI classification systems can be used for judging the liquefaction potential of a site in relative to others, they cannot be used for quantitative liquefaction risk assessments. As a result, the scope of this study is to establish an empirical relationship between LPI and liquefaction probability using logistic regression. Furthermore, motivated by artificial neural network (ANN), we divided the database into the "training-set" and "testing-set" groups with a ratio of 50%-to-50%, then using the former to develop a logistic model between LPI and liquefaction probability, while using the latter to evaluate the model's performance in terms of accuracy rate.

2. LOGISTIC REGRESSION

2.1 Methodology

Regression analysis is one of the methods commonly used for data analysis, especially the ordinary regression for characterizing the relationship between the dependent variable (Y) and independent variables (Xs). Note that in the ordinary regression, both variables are continuous.

By contrast, when the dependent variable is dichotomous or categorical (*e.g., Yes* and *No*; Success and Failure), the ordinary regression is not applicable, and another regression analysis will be used for establishing the relationship between the categorical variable (Y) and the continuous variables (Xs). In particular, such an analysis is known as logistic regression.

Given that the dependent variable Y is dichotomous or binary, the mathematical model of the logistic regression is as follows (Kleinbaum and Klein 2002):

$$\Pr(Y = Yes) = \frac{\exp(a + bX)}{1 + \exp(a + bX)} \tag{3}$$

where Pr( ) denotes probability; a and b are model parameters, which will be calculated based on X-Y datasets (or samples). It is noted that unlike ordinary regression that uses the least-square algorithm to calculate the model parameters, the maximum likelihood estimation is used for calculating the model parameters for logistic regression (Kleinbaum and Klein 2002). The procedure to resolve the parameters a and b of logistic regression is described in Appendix I.

2.2 Applications

A few soil liquefaction studies using logistic regression have been reported in the literature. For example, Lai *et al.* (2006) developing a logistic model for predicting liquefaction probability based on the following two factors: (1) the normalized cone-tip resistance from CPT, and (2) the so-called soil behavior index. Juang *et al.* (2002) also conducted logistic regression analyses on their soil liquefaction datasets, then proposing a logistic model between liquefaction factor of safety and liquefaction probability. Last but not least, Papathanassiou (2008) developed a logistic model between LPI and liquefaction probability based on the SPT measurements and earthquake data from Taiwan, Turkey, and Italy. Nevertheless, it is noted that the studies did not conduct further assessments on the model's performance.

3. DATA

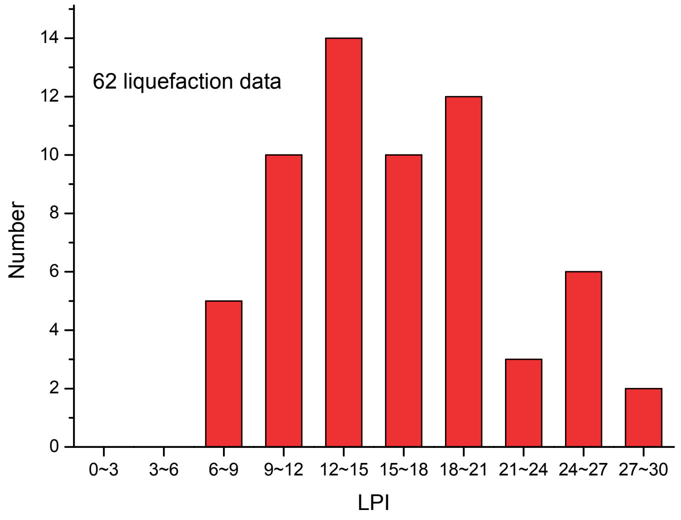
3.1 Introduction

A total of 135 data points were collected from the Yuanlin region of Taiwan (36 data points) during the M_w 7.6 Chi-Chi earthquake on September 21, 1999, from the Adapazari region of Turkey (39 data points) during the M_w 7.4 Kocaeli earthquake on August 17, 1999, and from the Emilia-Romagna region of Italy (60 data points) during the nearby M_w 6.1 earthquake on May 20, 2012 (Papathanassiou *et al.* 2015; Ku and Ma 2017). Figure 1 shows the histograms of the data used in this study. Among the 62 liquefaction data points, the maximum and minimum LPI are 29.3 and 6.9, respectively. By contrast, the maximum and minimum are 20 and 0 among the 73 non-liquefaction data points, respectively.

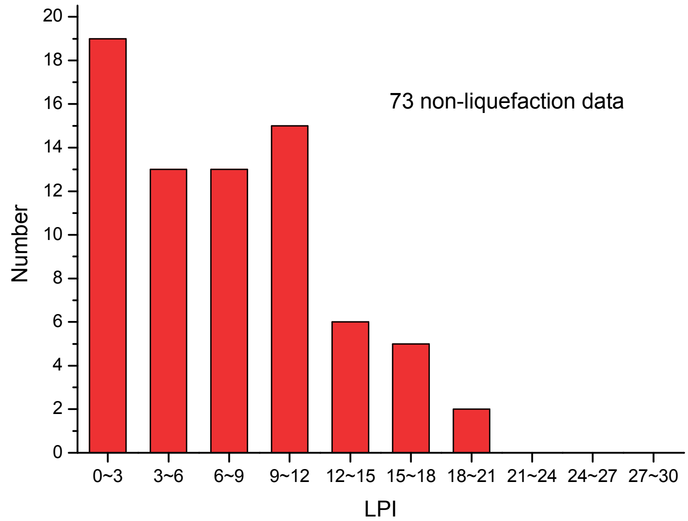
For conducting the analyses of this study, we labelled the 135 data points at first. As shown in Fig. 2, Data #1 to #62 are the LPI data points from liquefaction sites, and Data #63 to #135 are the LPI data points from non-liquefaction sites.

As shown in Figs. 1 and 2, the 135 LPI data points demonstrate certain trends/correlations between LPI and soil liquefaction. For LPI less than 6.9, not a site of the database had shown soil liquefaction. On the contrary, for LPI greater than 20, every site had shown soil liquefaction. However, for LPI between 6.9 and 20, the uncertainty was present, with both liquefaction and non-liquefaction data points.

Table 2 provides more information about the 135 LPI data points used in this study, including site locations and the causative earthquakes. Note that all of the 135 data points were obtained/calculated based on CPT measurements with the calculation procedure proposed by Robertson (2009). Nevertheless, the phenomenon (*e.g.*, soil boiling) accompanying soil liquefaction was not specified in the references, and therefore such information was not provided here, either.



(a) Liquefaction data



(b) Non-liquefaction data

Fig. 1 The 135 LPI data points used in this study

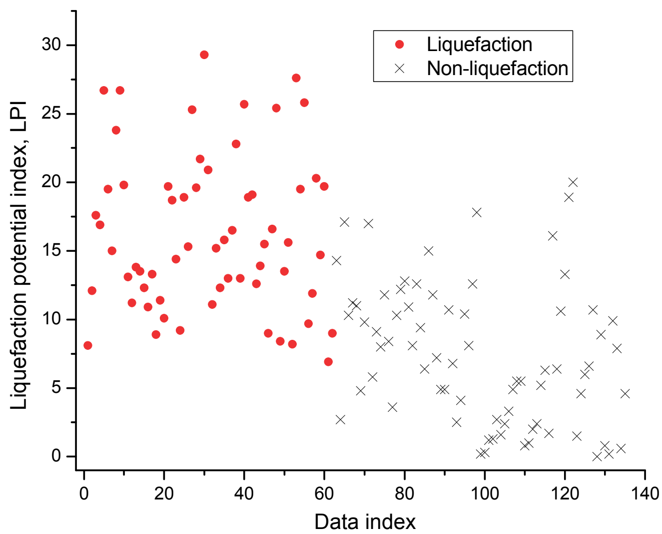


Fig. 2 The respective data index for each data point

Table 2 The 135 data points used in this study

Data index	LPI*	Liquefaction	Purpose	Note
1	8.1	Yes	Testing	(1)
2	12.1	Yes	Training	(1)
3	17.6	Yes	Testing	(1)
4	16.9	Yes	Testing	(1)
5	26.7	Yes	Testing	(1)
6	19.5	Yes	Training	(1)
7	15	Yes	Testing	(1)
8	23.8	Yes	Training	(1)
9	26.7	Yes	Testing	(1)
10	19.8	Yes	Testing	(1)
11	13.1	Yes	Training	(1)
12	11.2	Yes	Training	(1)
13	13.8	Yes	Training	(1)
14	13.5	Yes	Training	(1)
15	12.3	Yes	Training	(2)
16	10.9	Yes	Training	(2)
17	13.3	Yes	Training	(2)
18	8.9	Yes	Testing	(2)
19	11.4	Yes	Training	(2)
20	10.1	Yes	Testing	(2)
21	19.7	Yes	Training	(2)
22	18.7	Yes	Testing	(2)
23	14.4	Yes	Testing	(2)
24	9.2	Yes	Training	(2)
25	18.9	Yes	Training	(2)
26	15.3	Yes	Training	(2)
27	25.3	Yes	Testing	(2)
28	19.6	Yes	Testing	(2)
29	21.7	Yes	Testing	(2)
30	29.3	Yes	Testing	(2)
31	20.9	Yes	Testing	(2)
32	11.1	Yes	Testing	(2)
33	15.2	Yes	Testing	(2)
34	12.3	Yes	Training	(2)
35	15.8	Yes	Testing	(2)
36	13	Yes	Training	(2)
37	16.5	Yes	Training	(2)
38	22.8	Yes	Testing	(2)
39	13	Yes	Training	(2)
40	25.7	Yes	Testing	(3)
41	18.9	Yes	Testing	(3)
42	19.1	Yes	Training	(3)
43	12.6	Yes	Training	(3)
44	13.9	Yes	Testing	(3)
45	15.5	Yes	Testing	(3)
46	9	Yes	Testing	(3)
47	16.6	Yes	Training	(3)
48	25.4	Yes	Testing	(3)
49	8.4	Yes	Testing	(3)
50	13.5	Yes	Training	(3)
51	15.6	Yes	Testing	(3)
52	8.2	Yes	Training	(3)
53	27.6	Yes	Training	(3)
54	19.5	Yes	Testing	(3)
55	25.8	Yes	Training	(3)
56	9.7	Yes	Testing	(3)
57	11.9	Yes	Testing	(3)
58	20.3	Yes	Testing	(3)
59	14.7	Yes	Training	(3)
60	19.7	Yes	Training	(3)
61	6.9	Yes	Training	(3)
62	9	Yes	Training	(3)
63	14.3	No	Testing	(1)
64	2.7	No	Training	(1)
65	17.1	No	Training	(1)
66	10.3	No	Training	(1)
67	11.2	No	Training	(1)
68	11	No	Training	(1)
69	4.8	No	Training	(1)
70	9.8	No	Training	(1)
71	17	No	Training	(1)
72	5.8	No	Testing	(1)
73	9.1	No	Testing	(1)
74	8	No	Training	(1)
75	11.8	No	Training	(1)
76	8.4	No	Testing	(1)
77	3.6	No	Testing	(1)
78	10.3	No	Testing	(1)
79	12.2	No	Testing	(1)
80	12.8	No	Testing	(1)
81	10.9	No	Training	(1)
82	8.1	No	Training	(1)
83	12.6	No	Testing	(1)
84	9.4	No	Testing	(1)
85	6.4	No	Testing	(2)
86	15	No	Training	(2)
87	11.8	No	Testing	(2)
88	7.2	No	Testing	(2)
89	4.9	No	Testing	(2)
90	4.9	No	Testing	(2)
91	10.7	No	Testing	(2)
92	6.8	No	Training	(2)
93	2.5	No	Training	(2)
94	4.1	No	Training	(2)
95	10.4	No	Training	(2)
96	8.1	No	Testing	(2)
97	12.6	No	Training	(2)
98	17.8	No	Training	(2)
99	0.2	No	Testing	(3)
100	0.3	No	Training	(3)
101	1.2	No	Training	(3)
102	1.3	No	Training	(3)
103	2.7	No	Testing	(3)
104	1.6	No	Testing	(3)
105	2.4	No	Training	(3)
106	3.3	No	Training	(3)
107	4.9	No	Training	(3)
108	5.5	No	Testing	(3)
109	5.5	No	Testing	(3)
110	0.8	No	Testing	(3)
111	1	No	Testing	(3)
112	2	No	Training	(3)
113	2.4	No	Testing	(3)
114	5.2	No	Training	(3)
115	6.3	No	Training	(3)
116	1.7	No	Testing	(3)
117	16.1	No	Testing	(3)
118	6.4	No	Training	(3)
119	10.6	No	Training	(3)
120	13.3	No	Testing	(3)
121	18.9	No	Testing	(3)
122	20	No	Training	(3)
123	1.5	No	Testing	(3)
124	4.6	No	Training	(3)
125	6	No	Testing	(3)
126	6.6	No	Training	(3)
127	10.7	No	Training	(3)
128	0	No	Training	(3)
129	8.9	No	Training	(3)
130	0.8	No	Testing	(3)
131	0.2	No	Testing	(3)
132	9.9	No	Training	(3)
133	7.9	No	Testing	(3)
134	0.6	No	Testing	(3)
135	4.6	No	Training	(3)

* All based on CPT measurements using the Robertson and Wride calculation procedure

(1) From the Yuanlin region of Taiwan during the M_w 7.6 Chi-Chi earthquake on September 21, 1999 (36 data points)

(2) From the Adapazari region of Turkey during the M_w 7.4 Kocaeli earthquake on August 17, 1999 (39 data points)

(3) From the Emilia-Romagna region of Italy during the nearby M_w 6.1 earthquake on May 20, 2012 (60 data points)

3.2 Training-Set and Testing-Set Groups

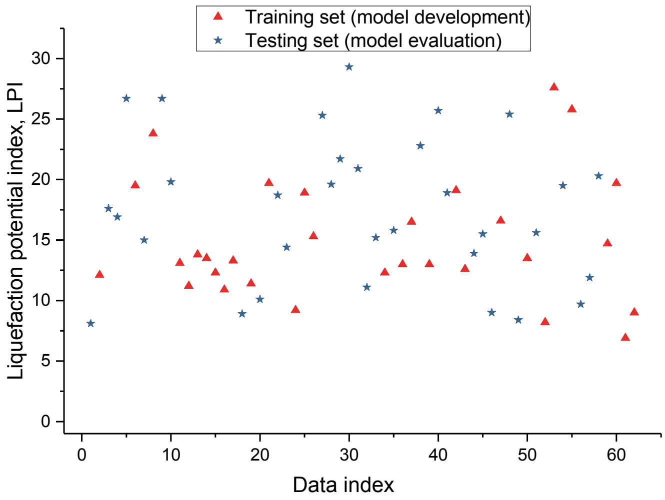
Artificial neural network (ANN) is an advanced approach in machining learning, which allows us to develop a complicated, mathematical model to capture the trends present in the data. For developing an ANN model, the first step is to separate the database into "training-set" and "testing-set" groups, then using the former to develop an ANN model, and using the latter to evaluate the model's performance. In ANN, normally the ratio of data points is 1 to 1, or 50% to 50% (Golafshani *et al.* 2020).

The motive of dividing data into two groups in ANN can be explained with this analogy: Given that we have 4 data points and want to develop a regression model; if we use a third degree polynomial to fit the data, the model will be perfect with a R^2 value = 1. However, without assessing the model's performance with additional data, the model is useless. As a result, no scientific study would adopt such an approach for data analysis.

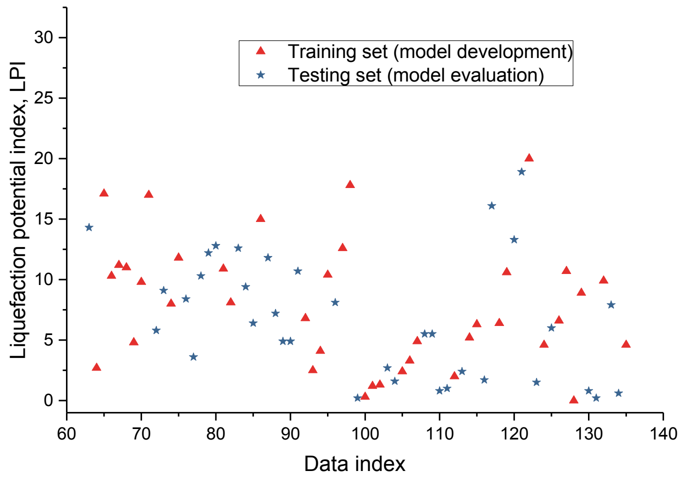
Therefore, motivated by ANN, we also divide our data into training-set (for model development) and testing-set (for model evaluation) groups. Following most ANN studies, the data-points ratio of the two groups is also 50% to 50%.

As to the criteria for data arrangement, it is randomly arranged by computer software (clearly, we have to compile a computer program for performing this task). Here we used an example to explain this process. Let's say we have 10 data points that need to be divided into two groups, and the following procedures can be used. Step 1: labelling the 10 data points as Data #1, #2, …, #10. Step 2: using a random number generator (available in computer software like Excel) to generate a series of 5 different numbers (within 1 to 10), such as {1, 2, 3, 5, 8}. Step 3: the 5 selected data (*i.e.,* #1, #2, #3, #5, #8) will be grouped as the training set for model development, while the remaining (*i.e.,* #4, #6, #7, #9, #10) as the testing set for model evaluation.

In this study, we utilized the random number generator available in Excel (function name: RANDBETWEEN) for such random data arrangement. Figure 3 shows the result of the random data arrangement for the following analyses.



(a) liquefaction



(b) non-liquefaction

Fig. 3 Respective data points for model development and model evaluation

4. LOGISTIC MODEL BETWEEN LPI AND LIQUEFACTION PROBABILITY

4.1 Model Development

Based on the training-set group (Data #2, #6, … in Fig. 3), the logistic model between LPI and liquefaction probability was developed as:

$$P_L = \frac{\exp\,(0.28 \times \mathrm{LPI} - 3.388)}{1 + \exp\,(0.28 \times \mathrm{LPI} - 3.388)} \tag{4}$$

where P_L denotes liquefaction probability; the Cox & Snell pseudo-R^2 value (denoted as $R^2_{C\&S}$) is equal to 0.32. More information about $R^2_{C\&S}$ is given in Appendix II.

Based on this model, Fig. 4 shows the relationship between liquefaction probability and LPI. For instance, given LPI = 10, the liquefaction probability is around 35.7% based on this empirical relationship.

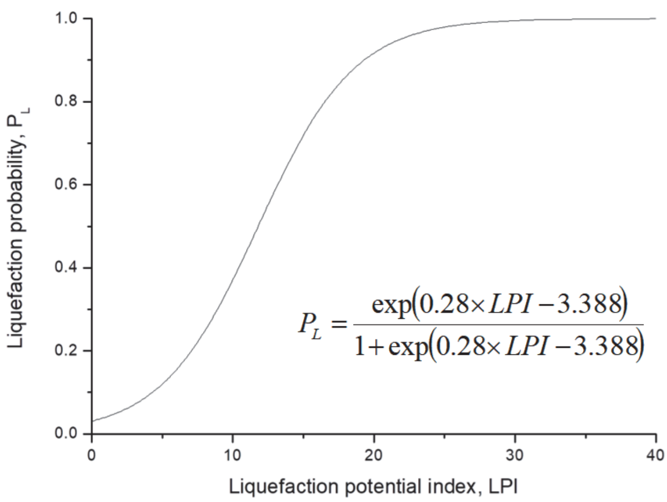


Fig. 4 The empirical relationship between LPI and liquefaction probability

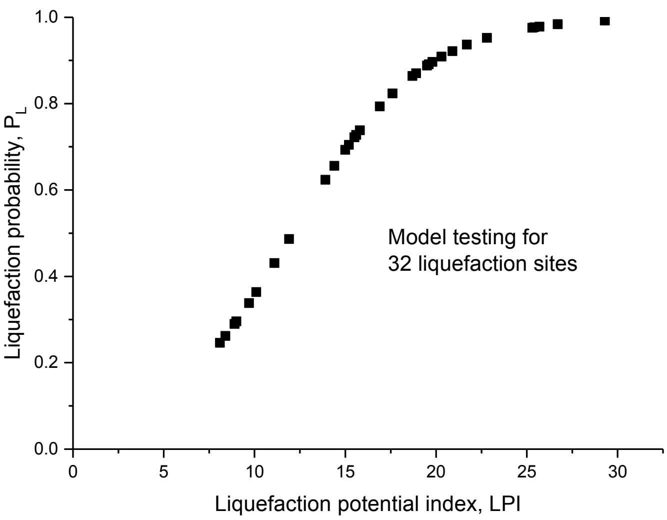
4.2 Model Evaluation: Accuracy Rate

Next, we used the testing-set group (Data #1, #3, … in Fig. 3) for model evaluation. Understandably, if the model is very robust, a large liquefaction probability (say 80% ~ 90%) should be returned for liquefaction sites, while a low liquefaction probability (say 10% ~ 20%) should be returned for non-liquefaction sites.

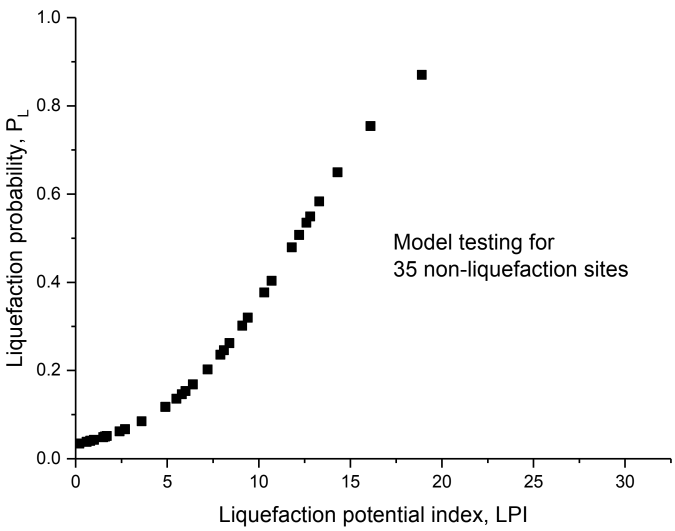
Figure 5 shows the result of model evaluation based on the 67 data points of the testing-set group. The estimated liquefaction probabilities are from 24% to 99%, which is a reflection to the LPI range from 8.1 to 29.3 of the 32 sites. By contrast, based on the 35 non-liquefaction data points, the estimated liquefaction probabilities are from 3% to 87%, reflecting the LPI range from 0.2 to 18.9 of the 35 non-liquefaction sites.

For integrating the two types of model evaluation on a comparable basis, the model's accuracy rate, A_R, was defined as follows:

$$A_R = \begin{cases} P_L & \text{; for liquefaction sites} \\ 1 - P_L & \text{; for non-liquefaction sites} \end{cases} \quad (5)$$



(a) based on the 16 liquefaction sites



(b) based on the 18 non-liquefaction sites

Fig. 5 The result of model evaluation

Understandably, if the model is perfect, its accuracy rate must be equal to 100%. That is, for a liquefaction site, the liquefaction probability estimated with the logistic model must be equal to 100%, for obtaining a 100% accuracy rate based on Eq. (5). On the contrary, for a non-liquefaction site, the liquefaction probability estimated with this (perfect) logistic model must be equal to 0, which will also lead to a 100% accuracy rate based on Eq. (5).

Here we used two examples to show how to calculate AR based on Eq. (5). According to random data arrangement, Data #5 and #72 belong to training-set and testing-set groups, respectively (see Table 2 or Fig. 3). Given Data #5 as {Liquefaction; LPI = 26.7}, a liquefaction probability of 98% was obtained by substituting LPI = 26.7 into the logistic model (*i.e.,* Eq. (4)). As a result, the model's accuracy rate is equal to 98% based on Eq. (5). By contrast, given Data #72 as {Non-liquefaction; LPI = 5.8}, a liquefaction probability of 15% was obtained by substituting LPI = 5.8 into the logistic model, which resulted in a non-liquefaction probability equal to 85% (= 1 – 15%). Therefore, the model's accuracy rate is equal to 85% based on Eq. (5) for this non-liquefaction data point.

Following the demonstration for the A_R calculation shown above, Fig. 6 shows the model's accuracy rate based on the 67 data points of the testing-set group. It shows that the average A_R is close to 73.7%.

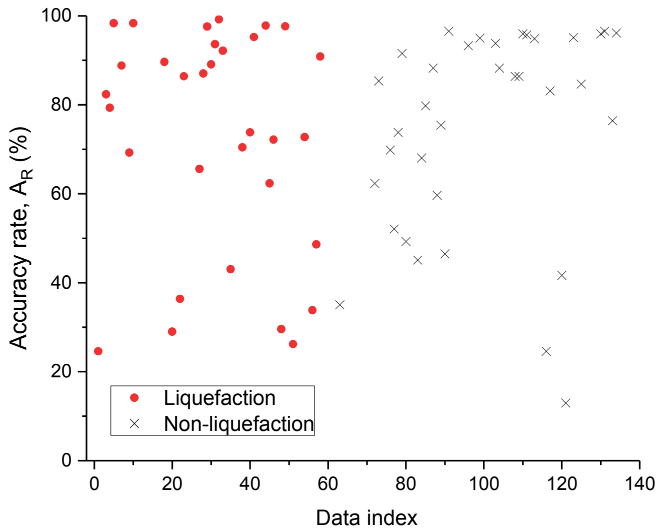


Fig. 6 Model evaluation in terms of accuracy rate (A_R)

5. SENSITIVITY ANALYSIS

Understandably, because the training-set (50%) and testing-set (50%) data arrangement is random, the result will be different if we conduct a new analysis from scratch. As a result, we conducted the so-called sensitivity analysis to examine the possible effect of the random data arrangement on the logistic model developed. Specifically, 10 additional analyses (the same as the previous one) were conducted with different random data arrangements. Given that the results of the 10 sensitivity analyses are very similar, the effect of the random data arrangement on this study is insignificant.

Table 3 summarizes the results of the 10 sensitivity analyses, including the model parameters (*i.e., a* and *b*) and accuracy rate. It shows that the results of the 10 analyses are similar to one another; taking the accuracy rate for example, it ranged from 70.9% to

Table 3 Summary of the 10 sensitivity analyses

Sensitivity test	*a*	*b*	Accuracy rate (%)
1	0.32	– 3.94	73.4
2	0.29	– 3.66	71.5
3	0.27	– 3.06	71.2
4	0.25	– 3.09	71.4
5	0.27	– 3.12	71.2
6	0.32	– 4.09	72.5
7	0.28	– 3.39	73.7
8	0.38	– 4.26	71.1
9	0.32	– 3.91	71.2
10	0.27	– 3.32	70.9

73.7% from the 10 sensitivity analyses. As a result, such a sensitivity study clarifies that the impact of the random data arrangement on this logistic regression is insignificant, which is similar to other studies also utilizing random data arrangement for model development and evaluation (Golafshani *et al.* 2020).

In addition, from the 10 sensitivity tests, we can also estimate the standard deviation of model parameters a and b, which are equal to 0.038 and 0.446, respectively.

6. DISCUSSION

6.1 Sample Size

It is understood that sample size is important to a statistical or empirical study (obviously, the more the better). Clearly, this new logistic model between LPI and liquefaction probability can be further refined and evaluated with additional data. However, it must be noted that it is difficult to collect liquefaction data from the field, mainly owing to the long return period of destructive earthquakes that are capable of inducing soil liquefactions. Just imagine that although a soil column is highly liquefiable, soil liquefaction will not occur at the site if no big earthquakes strike the region.

Nevertheless, the 135 field data used in this study is definitely not a small sample size. Based on the rule of thumb in probability and statistics, it is considered that a sample size greater than 30 should be adequately representative of a population (Tang *et al.* 2017).

To further discuss this issue, we looked up the sample size of other empirical studies. For instance, an empirical relationship between the soil's plasticity index (PI) and flow index (IF) was established with 50 samples (Sridharan *et al.* 1999); an empirical relationship between the soil's liquid limit (LL) obtained using the ASTM D4318 and the British Standard BS1377 procedures was developed with 75 samples (Feng 2001). As to the classical study by Wells and Coppersmith (1994) that has been cited for more than 6,000 times (which means those empirical relationships between earthquake magnitude and causative parameters were commonly used in earthquake studies), the sample sizes are in a range from 15 to 167. Therefore, in comparison to others, the empirical relationship between LPI and liquefaction probability developed with 135 data points in this study should be robust from the perspective of sample size.

Another reason we did not mix LPI datasets for the sake of increasing the sample size of this study is as follows: As one should have understood, LPI is dependent on test methods (*e.g.,* CPT and SPT) and calculation procedures. In other words, for the same site, LPI can be different depending on the test method and calculation procedures. As a result, in order to make this study and the results more robust and applicable, we only collected one specific type of LPI, which are those based on CPT measurements using the calculation procedure proposed by Robertson (2009). As a result, although Iwasaki *et al.* (1982) also presented LPI datasets in their paper, they were based on SPT measurements using the calculation procedure of the Japanese Highway Bridge Design Code (JSHE 1990), which is a different type of LPI than those presented in this study.

6.2 LPI Calculation Based on CPT Soundings

CPT is a common in-situ test for site characterizations, and the measurements (*i.e.,* cone-tip resistance and friction ratio) can also be used for LPI calculation. However, to the best of our knowledge, three different methods have been proposed for LPI calculation with CPT measurements. In this study, they are referred to as the O method, the R&W method, and the J method, which were proposed by Olsen (1997), Robertson and Wride (1998), and Juang *et al.* (2000), respectively. Nevertheless, the R&W method is very similar to the approach proposed by Robertson (2009), and thus we considered the two are the same method in this study.

The most distinct features of the three calculation procedures are summarized as follows: for the O method, the CRR (cyclic resistance ratio) is calculated with cone-tip resistance and friction ratio (from CPT), and the fines content and average grain size are not taken into account in this procedure. For the R&W method, the CRR is a function of the normalized cone-tip resistance for clean sand, which is governed by the cone-tip resistance and the CSR (cyclic stress ratio) based on PGA, earthquake magnitude, etc. For the J method, unlike the previous two procedures in which the liquefaction threshold is somehow based on empirical relationships, the J method used the concept of the limit-state equilibrium with probability theories to determine the liquefaction threshold.

As a result, the three approaches cast another source of (epistemic) uncertainty when it comes to calculating LPI with CPT measurements. It is noted that the 135 LPI data points used in this study were based on the same calculation procedure, *i.e.,* the R&W method.

Therefore, it must be noted that the logistic model proposed by this study for estimating liquefaction probability is exclusive for the R&W method. In other words, it is improper to use this logistic method for estimating liquefaction probability when the LPI is calculated using the O method or the J method (although based on CPT measurements), unless the LPI from the O method or the J method can be somehow converted to the LPI based on the R&W method.

To investigate this uncertainty, Chen (2001) compared the three LPI values for the same sites based on CPT measurements. Figure 7 shows the LPI based on the O method and the R&W method at different 20 sites. The relationship between the two based on the ordinary regression is as follows:

$$\mathrm{LPI}_{\mathrm{R\&W}} = 7.43 + 0.62\,(\mathrm{LPI}_{\mathrm{O}}) \pm 1.37 \tag{6}$$

where $\mathrm{LPI}_{\mathrm{R\&W}}$ and $\mathrm{LPI}_{\mathrm{O}}$ denote the LPI value based on the R&W method and the O method, respectively. Note that 1.37 is the standard deviation of the model error, and the coefficient of determination (R^2) of this ordinary regression is equal to 0.82. More statistical indices of this regression model are summarized in Table 4.

Table 4 Summary of the statistical indices of the regression model given in Eq. (6) and Fig. 7

	Coefficients	Standard error	T statistics	P-value	Lower 95%	Upper 95%
Intercept	8.027	1.358	5.911	0.000	5.162	10.892
X variable (LPI from O*)	0.599	0.063	9.454	0.000	0.465	0.733

* LPI from the O method

By contrast, Fig. 8 shows the LPI based on the J method and the R&W method at the same 20 sites. The relationship between the two based on the ordinary regression is as follows:

$$\mathrm{LPI}_{\mathrm{R\&W}} = 6.01 + 0.99\ (\mathrm{LPI}_{\mathrm{J}}) \pm 0.67 \tag{7}$$

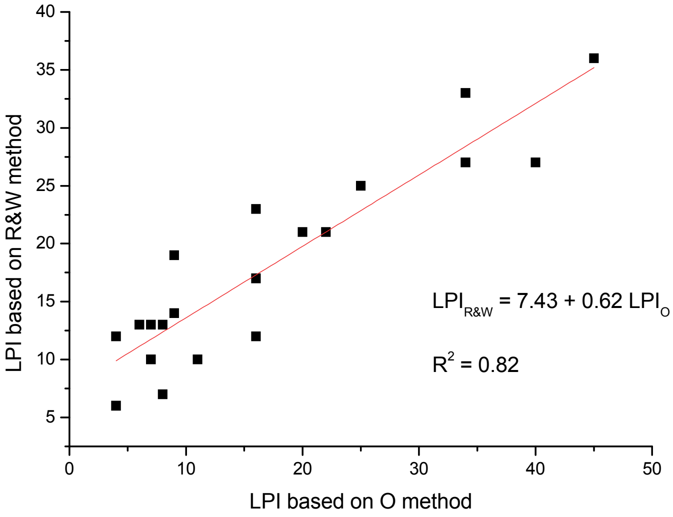


Fig. 7 The relationship between $\mathrm{LPI}_{\mathrm{O}}$ and $\mathrm{LPI}_{\mathrm{R\&W}}$

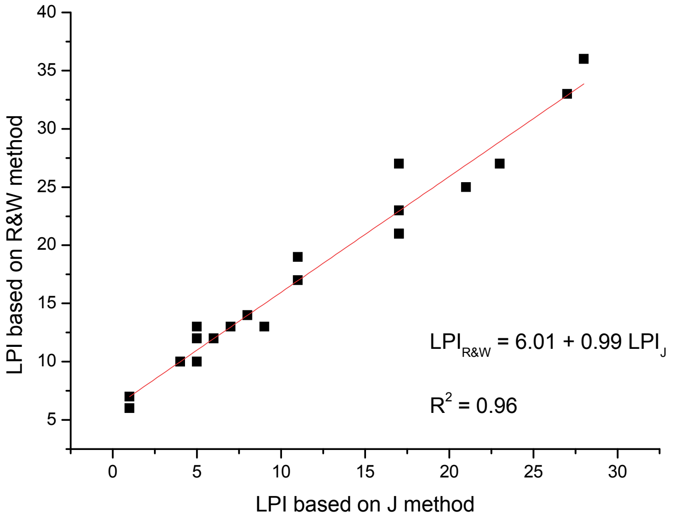


Fig. 8 The relationship between $\mathrm{LPI}_{\mathrm{J}}$ and $\mathrm{LPI}_{\mathrm{R\&W}}$

where $\mathrm{LPI}_{\mathrm{J}}$ denote the LPI value based on the J method. Also note that 0.67 is the standard deviation of the model error, and the coefficient of determination (R^2) of this ordinary regression is equal to 0.96. More statistical indices of this regression model are summarized in Table 5.

It is worth noting that the two regression models were developed by this study, with the raw data from the technical report compiled by Chen (2000).

With the two empirical relationships, we can estimate liquefaction probability at a site with the LPI value calculated with the O method or the J method. For example, given that the $\mathrm{LPI}_{\mathrm{J}}$ is equal to 15 at a site, the $\mathrm{LPI}_{\mathrm{R\&W}}$ should be close to 20.9 based on Eq. (7). Then using the logistic model developed with this study, the liquefaction probability at the site will be estimated at 93% with $\mathrm{LPI}_{\mathrm{J}}$ equal to 15.

6.3 Logistic Regression Model and Accuracy Rate

As mentioned previously, a logistic regression is a relationship between a continuous variable and a discrete variable, and like other indices (like $R^2_{C\&S}$), the accuracy rate is an index to portray the level of correlation between the two variables; the higher the correlation, the larger the index. For this logistic model, its accuracy rate was found between 70.9% to 73.7% based on 11 analyses (1 + 10 sensitivity analyses), inferring a moderate correlation between LPI and liquefaction should be present.

Understandably, some might consider this logistic model with 70.9% ~ 73.7% accuracy rate could be of limited application values, while some might consider the model is satisfactory for engineering practices. However, regardless of the accuracy rate, the merit of a liquefaction logistic model is that it allows us to calculate liquefaction probabilities for further conducting quantitative risk assessments (risk = probability × consequence).

That is, assuming no obvious correlation is found in the data (we expect a lower accuracy rate, say 50%, would be obtained), such a logistic model is still valuable for quantitative risk assessments, because it can be used for estimating liquefaction probabilities regardless. As a matter of fact, this is the fundamental motivation of this study, aiming to develop a logistic model between LPI and soil liquefaction that can be used for estimating liquefaction probabilities with a given LPI, then using the liquefaction probability for risk assessments along with expected consequences. In other words, the correlation (and its indices) between LPI and liquefaction is not as important as the logistic model itself. After all, we cannot conduct liquefaction risk assessments with LPI directly, until we have a logistic model between LPI and liquefaction probability.

Table 5 Summary of the statistical indices of the regression model given in Eq. (7) and Fig. 8

	Coefficients	Standard error	T statistics	P-value	Lower 95%	Upper 95%
Intercept	6.013	0.670	8.974	0.000	4.606	7.421
X variable (LPI from J*)	0.995	0.046	21.521	0.000	0.898	1.092

* LPI from the J method

7. CONCLUSIONS

Liquefaction potential index, LPI, is a common measure for liquefaction assessments. Although several LPI classifications have been proposed, those are categorical and descriptive, which is not applicable for quantitative (liquefaction) risk assessments (risk = probability × consequence). In this study, we developed a new logistic model for characterizing the relationship between LPI and liquefaction probability based on the field data from three different areas (*i.e.,* the Yuanlin region of Taiwan, the Adaparazi region of Turkey, and the Emilia-Romagna region of Italy). This (empirical) logistic relationship can help estimate liquefaction probability based on LPI for conducting qualitative liquefaction risk assessments along with expected consequences. As to the model's performance, the accuracy rate should be close to 70.9% ~ 73.7% based on 11 analyses (1 + 10 sensitivity analyses), inferring a moderate correlation between LPI and liquefaction should be presented.

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DATA AVAILABILITY

All data and/or computer codes used/generated in this study are included in this paper.

CONFLICT OF INTEREST STATEMENT

The authors certify that they have no affiliation or involvement in any organization with any financial interest or non-financial interest in the materials discussed in this paper.

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APPENDIX I: ALGORITHM FOR SOLVING *a* AND *b* OF A LOGISTIC MODEL

A logistic model can be expressed as $\Pr(Y = Yes) = \exp(a + bX)/1 + \exp(a + bX)$. Here we use a simple example to show how to solve a and b analytically. Given that the 4 data points are available ($X = 1$, $Y = Yes$), ($X = 2$, $Y = Yes$), ($X = 3$, $Y = Yes$), and ($X = 5$, $Y = No$), we then substitute each one into the model, and we will get the four probabilities: $\Pr(Y = Yes) = \exp(a + b \times 1)/1 + \exp(a + b \times 1)$, $\Pr(Y = Yes) = \exp(a + b \times 2)/1 + \exp(a + b \times 2)$, $\Pr(Y = Yes) = \exp(a + b \times 3)/1 + \exp(a + b \times 3)$, and $\Pr(Y = No) = 1 - (\exp(a + b \times 5)/1 + \exp(a + b \times 5))$. Specifically, the product of the probabilities is referred to as the likelihood function, and the set of a and b that can maximize the likelihood function is the parameters of the logistic model.

APPENDIX II: COX & SNELL PESUDO-R^2 OF LOGISTIC MODEL

The Cox & Snell Pesudo-R^2 of logistic models, denoted as $R^2_{C\&S}$, was defined as follows (Cox and Snell 1989):

$$R^2_{C\&S} = 1 - \left(\frac{L_0}{L_m}\right)^{2/n} \tag{E.1}$$

where L_0 is the likelihood function with no relation to the independent variable (X), L_m is the likelihood function governed by the independent variable, and n is the sample size. As a result, $R^2_{C\&S}$ is an index showing the level of model improvement with the consideration of the correlation between independent variables (X) and dependent variable (Y). That is, the higher the index, the stronger the correlation between X and Y. In this study, the $R^2_{C\&S}$ of the liquefaction logistic model is 0.32, inferring a moderate correlation between LPI and soil liquefaction is present, given that $R^2_{C\&S}$ is usually between 0.1 ~ 0.4 (Hu *et al.* 2006).