

# BEHAVIOR OF UNSATURATED SAND-SILT MIXTURE THROUGH EQUIVALENT INTERGRANULAR VOID RATIO CONCEPT

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## ABSTRACT

The effect of silt content on the undrained shear strength of sand-silt mixture has been experimentally investigated in the past three decades. These studies showed that, at the given confining pressure and global void ratio, the undrained shear strength decreases with increasing fines content. Between a threshold fines content and a limiting fines content, further addition of fines would lead to an increase of the shear resistance, where the behavior is controlled by both fine and coarse parts. Beyond the limiting fines content, the shear resistance will increase, and the behavior of soil is controlled by the fines content. On this basis, the state parameter of the equivalent intergranular void ratio is replaced by the global void ratio to describe the mechanical behavior of sand-silt mixture. In this study, a series of triaxial tests were performed on unsaturated sand-silt mixture with constant water (CW) conditions. Three levels of confining pressure and initial suction were considered. All specimens were prepared with the same equivalent intergranular void ratio to study its performance in the unsaturated state. The results of these tests showed that the suction effects on the soil strength are dependent on the fines content. For specimens with fines contents less than the threshold fines content or higher than the limiting fines content, the soil strength increased with increasing initial suction. However, for specimens with fines content between the threshold and limiting fines contents, the strength decreased with increasing initial suction. It was observed that the changes in volume did not have a definite relationship with the initial suction. The results also indicated that the application of the equivalent intergranular void ratio relations in the unsaturated state requires further considerations about the effect of matric suction.

**Key words:** Matric suction, silty sand, unsaturated triaxial test, equivalent intergranular void ratio, fines content.

## 1. INTRODUCTION

It is well known that the presence of non-plastic fines in the voids of granular soils affects their mechanical behavior. Previous studies showed that the increase of fines content has a significant effect on the soil liquefaction and static undrained strength of silty sand (Vaid 1994; Polito 1999; Frost and Park 2003; Stamatopoulos 2010; Nguyen *et al.* 2015; Porcino and Diano 2017). In this regard, the global void ratio is not a suitable index for characterizing the mechanical behavior of sand-silt mixtures (Thevanayagam 2002). Even with the same void ratio, different relative participations and grain sizes lead to distinct behaviors for different silt contents (*e.g.*, Koester 1994). On this basis, the concepts such as the equivalent intergranular void ratio and threshold fines content are usually used to express the behavior of sand-silt mixtures. To investigate the behavior of the sand-non-plastic-fines mixtures, among many variations, Thevanayagam (2002) categorized three extreme limiting cases of microstructure. Two categories of these extreme limiting cases are shown in Fig 1. For the category (a), the sand grains are in contact with each other (Cases (i) to (iii) in Fig. 1(a)). For the category (b), silt grains are in contact with each other (Case (iv) in

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Fig. 1(b)). At the same global void ratio, with an increase in the fines content ( $f_c$ ), a transition in microstructure from Cases (i) through (iv) can occur. For fines content less than the threshold value ( $f_{c_{th}}$ ) (*i.e.*, Cases (i) to (iii)), the sand-grain contacts play a primary role in the soil behavior, and the fines present a secondary contribution. For the fines content greater than the threshold value, the fine-grain content begins to play a major role in the soil behavior, while sand grains present a secondary effect. The sand-grain contribution depends on the degree of their dispersity. On this basis, Thevanayagam (2002) defined a limiting fines content ( $f_{c_l}$ ) beyond which soil behavior will be controlled by the non-plastic fines (*i.e.*, Case iv-2 in Fig. 1(b)). This creates a transition zone between  $f_{c_{th}}$  and  $f_{c_l}$  before the soil behavior is entirely controlled by the fine grains (Case iv-1).

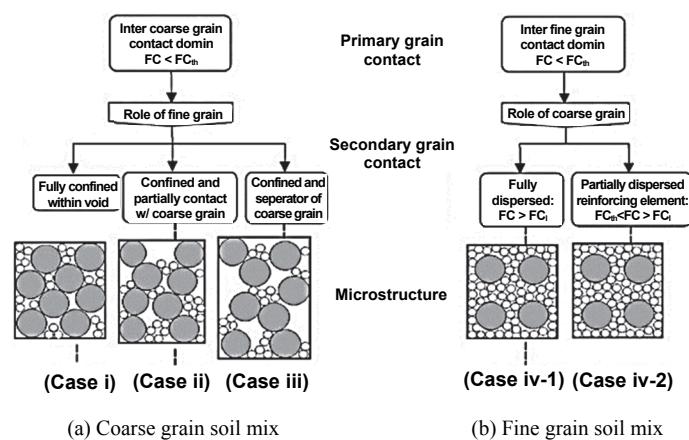


Fig. 1 Intergranular mix classification (Thevanayagam 2002)

Using the experimental data, Rahman and Lo (2008) presented the following equation to determine the threshold fines content.

$$fc_{th} = A \left( \frac{1}{1 + e^{\alpha - \beta \chi}} + \frac{1}{\chi} \right) \quad (1)$$

where  $A$  is a dimensionless parameter;  $e$  is the void ratio;  $\alpha$  and  $\beta$  are the parameters dependent on the soil type;  $\chi = D_{10}/d_{50}$  ( $D_{10}$  is for coarse grains and  $d_{50}$  is for fine grains). Since  $e$  is not capable of accurately characterizing soils with significant fines contents, the equivalent intergranular void ratio concepts have been used by various researchers. In fact, the equivalent intergranular void ratio is the void ratio of the coarse-grain texture without the fines (Mohammadi and Qadimi 2014). The equivalent intergranular void ratio was first used by Mitchell (1976) to determine the inactive fines of a soil.

For the fines content less than the threshold value, Thevaniagam (1998) presented the following relation for the intergranular void ratio:

$$(e_g)_{eq} = \frac{e + fc}{1 - fc} \quad (2)$$

where  $(e_g)_{eq}$  is the intergranular void ratio considered as a first-order index of active coarser-granular frictional contacts. By adding silt grains up to a certain threshold value, the influence of fines becomes secondary, and the shear strength of the soil decreases. However, beyond the threshold value, a further increase of silt leads to the increase of the soil shear strength. The experimental results for cases with  $fc < fc_{th}$  indicated that this effect cannot be neglected even for a low contact of the fine grains. On this basis, Thevaniagam (2000) introduced the following equivalent intergranular void ratio:

$$(e_g)_{eq} = \frac{e + (1-b)fc}{1 - (1-b)fc} \quad (3)$$

The other investigation considering the effect of silt and clay contents has been presented by Ni *et al.* (2006), who introduced the following equation:

$$(e_g)_{eq} = \frac{e + (1-a)cc + (1-b)sc}{1 - (1-a)cc - (1-b)sc} \quad (4)$$

where  $cc$  and  $sc$  are clay and silt contents, respectively;  $a$  and  $b$  are the proportions of clay and silt, respectively.

The general form of Eq. (3) as a basic state parameter has been used by different authors (e.g., Rahman *et al.* 2012; Mohammadi, and Qadimi 2014). However, novel definitions for  $b$  were proposed. Rahman *et al.* (2008) proposed the following relation for  $b$ :

$$b = \left[ 1 - \exp(-2.5 \frac{(fc)^2}{k}) \right] \cdot \left( r \frac{fc}{fc_{th}} \right)^r \quad (5)$$

where  $r = \chi - 1 = d_{50}/D_{10}$  ( $d_{50}$  is for fines and  $D_{10}$  is for sand);  $k = (1 - r^{0.25})$ . Another expression for parameter  $b$  has been presented by Rahman and Lo (2008):

$$b = \left[ 1 - \exp(-m \frac{(fc / fc_{th})^n}{k}) \right] \cdot \left( r \frac{fc}{fc_{th}} \right)^r \quad (6)$$

where  $m$  and  $n$  are empirical constants. Rahman and Lo (2008) suggested that  $m = 0.3$  and  $n = 1$ .

In a total void ratio and constant stress level, the soil strength decreases with increasing fines content up to the threshold fines content. However, when the threshold fines content is increased, the soil strength increases (Polito *et al.* 2001; Thevanayagam *et al.* 2002; Frost and Park 2003; Yang *et al.* 2006; Papadopoulou *et al.* 2008).

The undrained static behavior of sand mixed with non-plastic fines was experimentally evaluated in terms of equivalent granular state parameter by Porcino *et al.* (2019). They showed that for sand-silt mixture, the position of the steady state line in the  $e - \ln(p)$  plane depends on the percentage of silt. However, in terms of the equivalent intergranular void ratio, the position of the steady state line in the  $(e_g)_{eq} - \ln(p)$  plane does not depend on the fines content.

For unsaturated silty sand soils, relevant studies are scarce in the literature so far (e.g., Rahardjo *et al.* 2004; Maleki and Bayat 2012; Leal-Vaca *et al.* 2012; Schnellmann *et al.* 2013; Zhou *et al.* 2016; Patil *et al.* 2017), and the contribution of the fines content and the performance of the equivalent intergranular void ratio concepts have not been thoroughly investigated.

This research studies the effect of matric suction on the mechanical behavior of sand-silt mixtures considering different silt contents and various confining pressures. For this purpose, a series of triaxial tests for unsaturated soils in constant water (CW) conditions and a series of consolidated undrained (CU) triaxial tests for saturated soils were conducted. Since the equivalent intergranular void ratio parameter has been of interest to many researchers for homogenizing the behavior of the saturated silty sands. Therefore, in this research, to evaluate the performance and the reliability of the application of this parameter for unsaturated silty sands, all specimens were prepared with the same equivalent intergranular void ratio, and then the results were analyzed.

## 2. TESTING APPARATUS

In this research, an unsaturated triaxial apparatus capable of the axis translation technique at Bu-Ali Sina University was used for testing. Using double-wall cells, the unsaturated triaxial apparatus can measure the total volume change and the pore water pressure during the test. Using this cell, the confining loading is possible in all directions up to 10 atmospheres. A view of the double-wall cell of the triaxial apparatus and its base plate are shown in Figs. 2(a) and 2(b), respectively. Two ceramic disks with air entry value of 500 kPa (for measuring water pressure and movement) and also two porous stones (for controlling pore air parameters) are placed in the top cap and base pedestal.

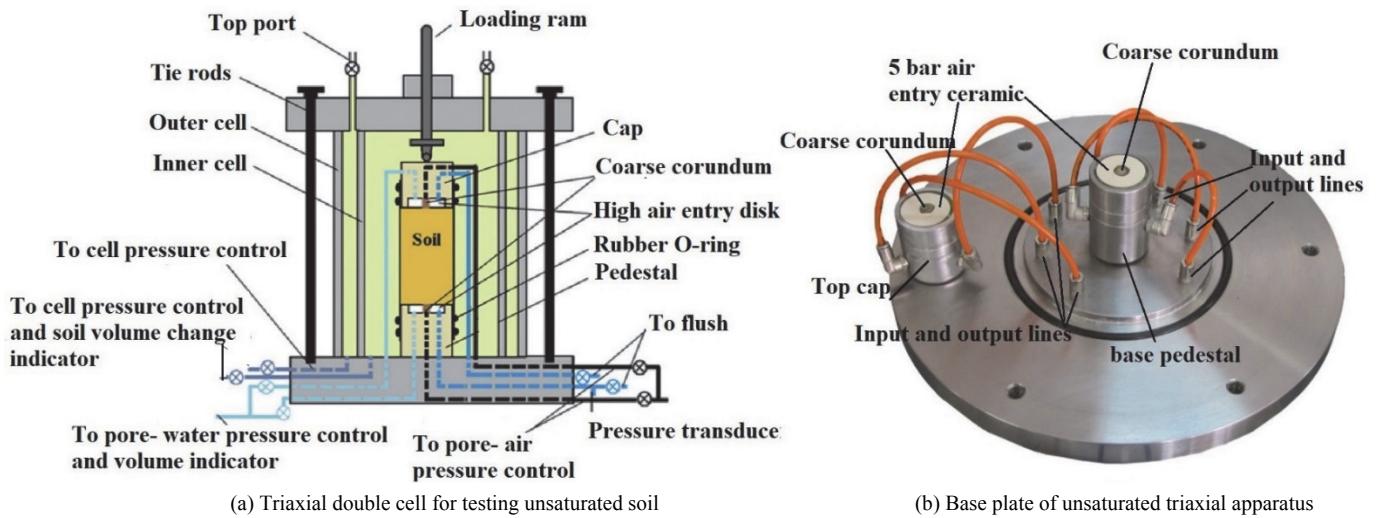


Fig 2 Details of unsaturated triaxial test (Maleki and Bayat 2012)

### 3. TESTS PROGRAM

#### 3.1 Soil Material

The soil used in this study is composed of sand and silt. For the coarse-grain part of the soil, the grading test was carried out in accordance with ASTM D-422-63 (2007) standard, and for the fine-grain part of the soil, a hydrometer test was carried out in accordance with ASTM-D-422-63 (2007) standard. In Fig. 3, soil grading diagrams are presented, and Table 1 shows the classification of the soil used. The Atterberg limit tests also revealed that the silt was non-plastic.

To identify the constituent minerals of the silt particles, an XRD test was carried out (Fig. 4). The results of this experiment showed that the minerals with the highest content in the powdered silt were quartz, feldspar, mica, and carbonate, respectively.

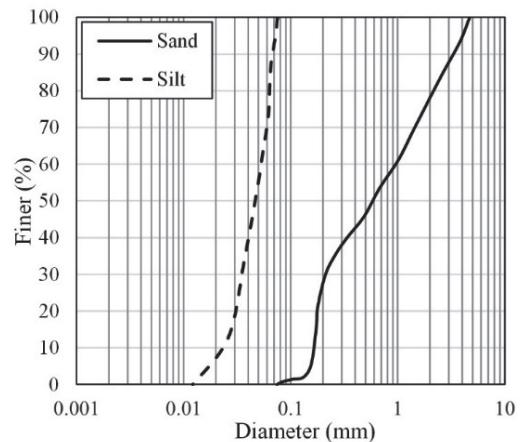


Fig. 3 Grain size distribution curves

Table 1 Specifications of the soil used

Fines content (%)	USCS	$G_s$	$D_{10}$ (mm)	$D_{60}$ (mm)	$C_u$	$C_c$
10	SP-SM	2.73	0.046	0.753	16.73	0.566
15	SM	2.72	0.041	0.670	16.30	0.591
25	SM	2.71	0.029	0.518	17.86	0.563

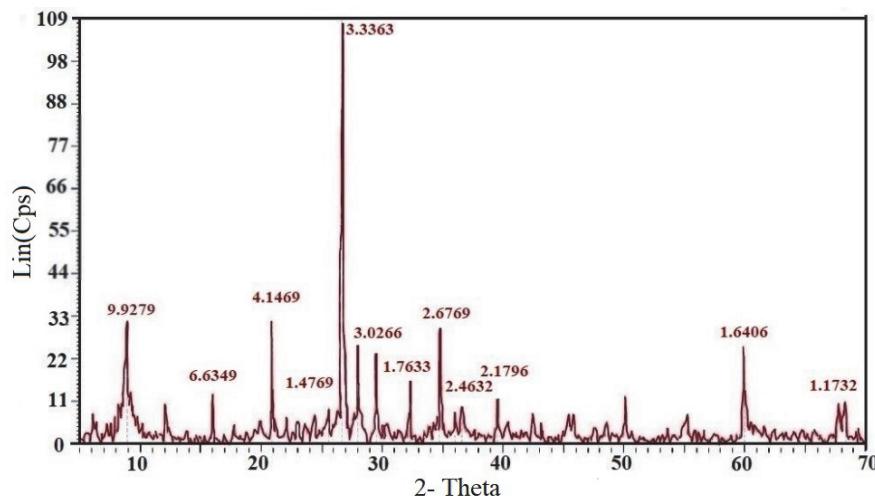


Fig 4 XRD test result

### 3.2 Sample Preparation

Mulilis *et al.* (1977) showed that the method of preparing specimens in the triaxial test affects their behavior. According to Frost and Park (2003), for easy control of density and also its applicability for a wide range of void ratio, the moist tamping method was used in this study to prepare saturated and unsaturated specimens.

### 3.3 Experimental Details

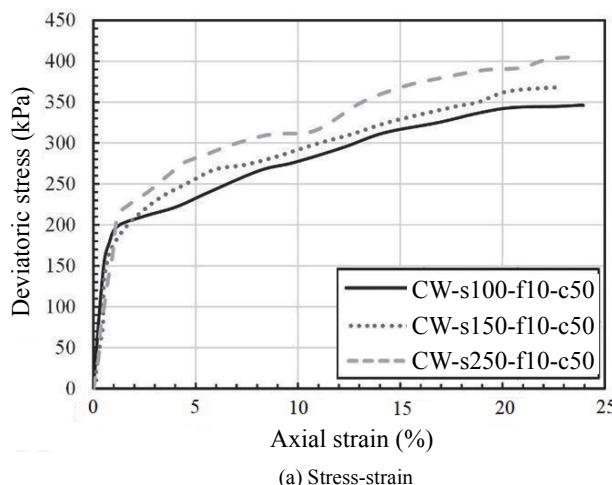
#### 3.3.1 Unsaturated state

To study the effect of the silt content on the shear strength behavior of the silty sand, some tests were carried out on the specimens with silt contents of 10%, 15%, and 25%. These experiments were carried out at three initial suctions of 100, 150, and 250 kPa and the confining pressures of 50, 100, and 150 kPa. In these specimens, the equivalent intergranular void ratio was considered as equal to 0.85. Then, further experiments were carried out on the specimens with the fines contents of 30% and 40% in constant void ratio. The water content of the specimens is considered to be such that the specimens reaching the balance in the drying path.

#### 3.3.2 Saturated state

In this research, for silt contents of 10%, 15%, 20%, and 25%, a consolidated undrained (CU) triaxial test was performed with a confining pressure of 150 kPa. This test was aimed to compare the soil strength behavior in saturated and unsaturated states as well as to determine the experimental value of the threshold fines content. It should be noted that in this case, the equivalent intergranular void ratio was also considered as equal to 0.85. Using the equation presented by Rahman and Lo (2008), the threshold fines content was calculated as 30%. In Table 2, the specifications of the specimens used for saturated and unsaturated tests in this study are presented. In this table, the values of  $b$  are calculated using Eqs. (5) and (6), but Eq. (5) is used to prepare samples.

It should be noted that the name of each test indicates the conditions of that test. For example, the CW-s100-f15-c50 test indicates a test carried out under CW conditions with an initial suction of 100 kPa, silt content of 15%, and confining pressure of 50 kPa.



**Table 2** Specifications of saturated and unsaturated samples

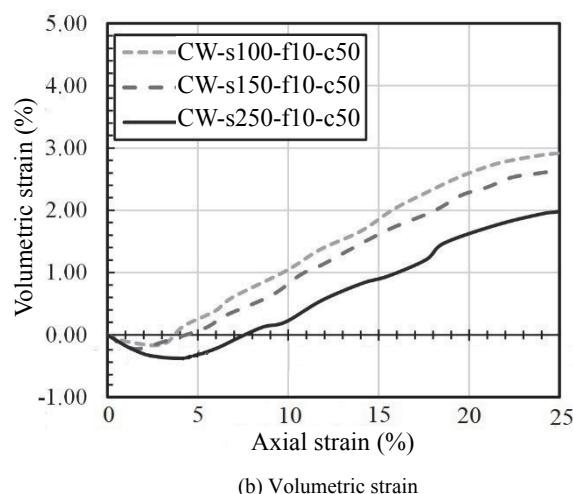
Fines content (%)	$(e_g)_{eq}$	$b$ (Eq. (5))	$b$ (Eq. (6))	$e_{total}$ (Eq. (5))	$e_{total}$ (Eq. (6))	$\gamma_d$ (gr/cm <sup>3</sup> )
10	0.85	0.027	0.158	0.69	0.67	1.58
15	0.85	0.135	0.244	0.67	0.61	1.64
20	0.85	0.216	0.325	0.60	0.56	1.70
25	0.85	0.265	0.399	0.57	0.51	1.75
30	—	—	—	0.55	0.55	1.74
40	—	—	—	0.55	0.55	1.73

## 4. RESULTS AND DISCUSSIONS

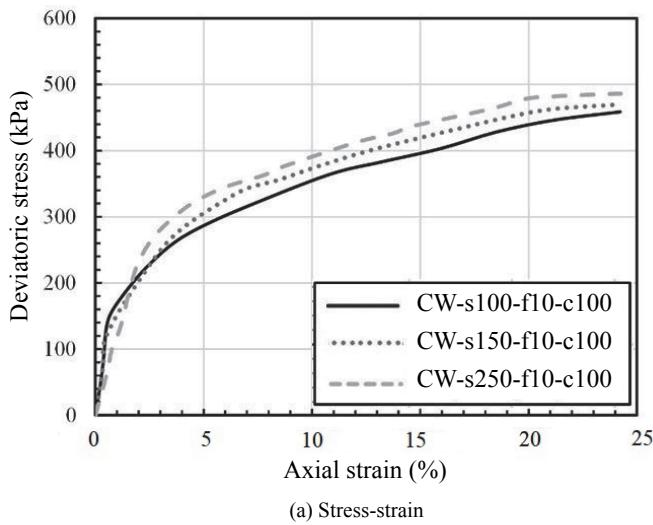
In the current study, all specimens were sheared in the constant water conditions which are among the most realistic paths for the short-term stress-strain behavior of unsaturated soils. In these conditions, matric suction and soil density are varied during shearing and due to the high compressibility of air phase, maximum shear strength of all specimens occurred at a large deformation (20% to 25%). In the classical soil mechanics, for such levels of deformation, the soil usually has reached the critical state. Therefore, in the present study, the shear strength can be considered as the critical-state shear strength.

### 4.1 Effect of Matric Suction on Stress-Strain Curves for $fc < fc_{th}$

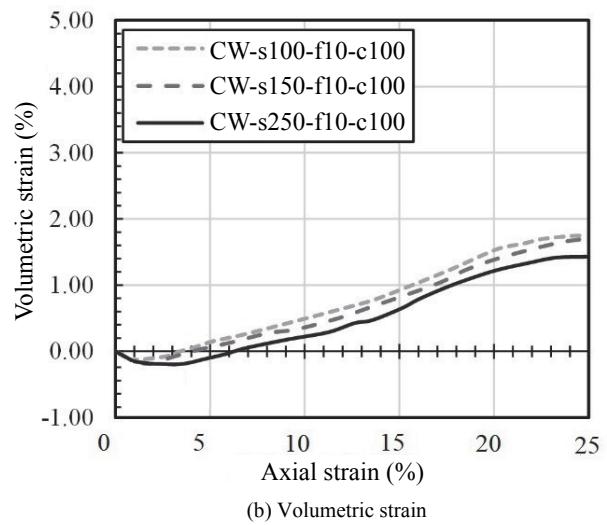
In this section, the test results are analyzed under the conditions where the fines content and confining pressure remain constant and only the matric suction changes. Figures 5(a) ~ 10(a) show the stress-strain curves for the specimens with fines contents of 10% and 15%. As shown, in these figures, where the fines content is less than the threshold fines content, increasing the initial suction will increase the strength nonlinearly (according to Rahardjo *et al.* 2004; 2013). Given the higher interaction of the coarse-grain aggregates in the presence of higher suction, this increased strength due to the increased suction is justifiable. Also, since all of the specimens were on the drying path, according to Leal-Vaca *et al.* (2012), the increased number of menisci can also be considered as a factor that increases the strength.



**Fig. 5** Results of unsaturated tests on specimens with silt content of 10% at initial suctions of 100, 150, and 250 kPa and confining pressure of 50 kPa

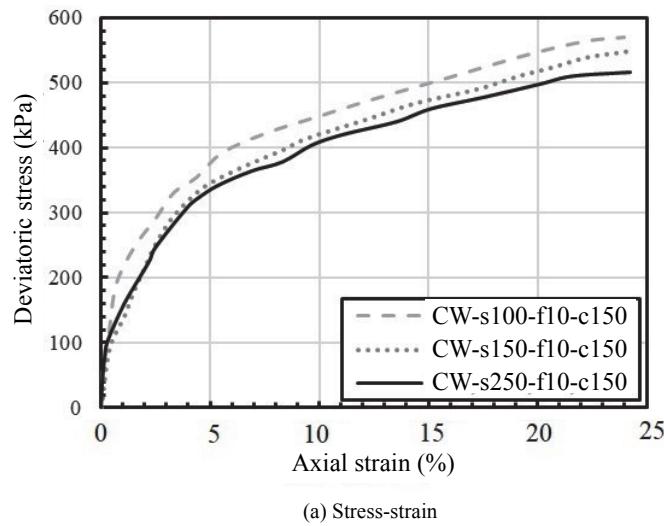


(a) Stress-strain

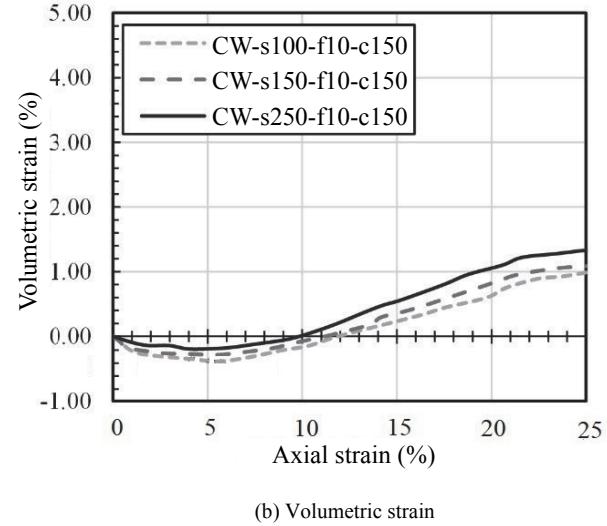


(b) Volumetric strain

**Fig. 6 Results of unsaturated tests on specimens with silt content of 10% at initial suctions of 100, 150, and 250 kPa and confining pressure of 100 kPa**

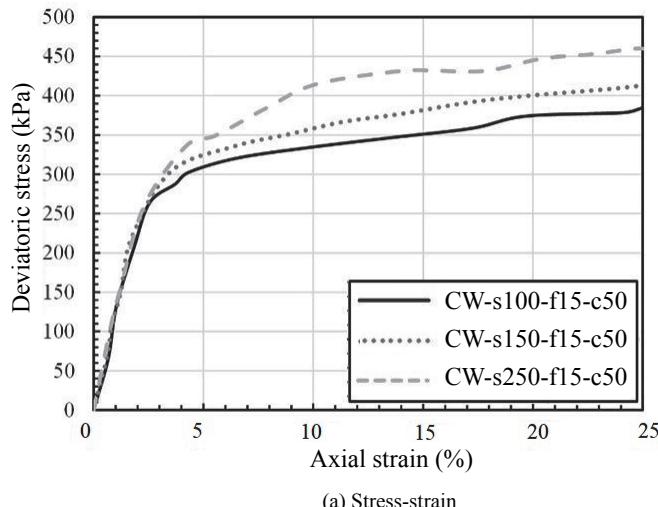


(a) Stress-strain

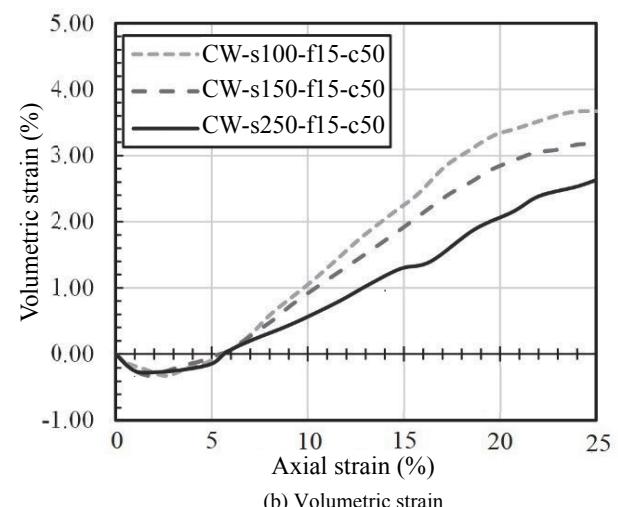


(b) Volumetric strain

**Fig. 7 Results of unsaturated tests on specimens with silt content of 10% at initial suctions of 100, 150, and 250 kPa and confining pressure of 150 kPa**

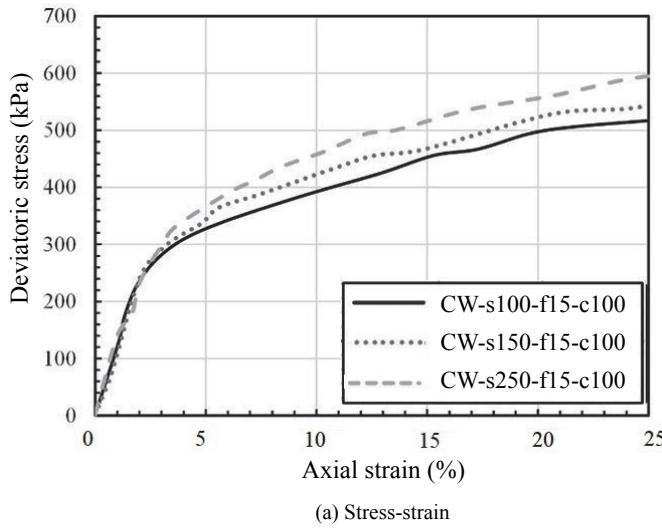


(a) Stress-strain

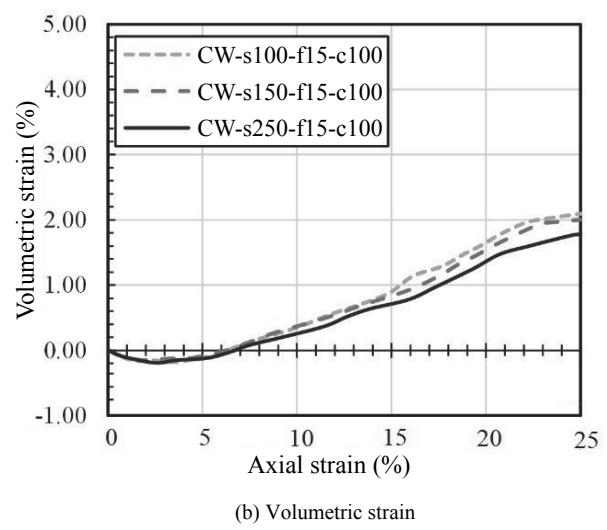


(b) Volumetric strain

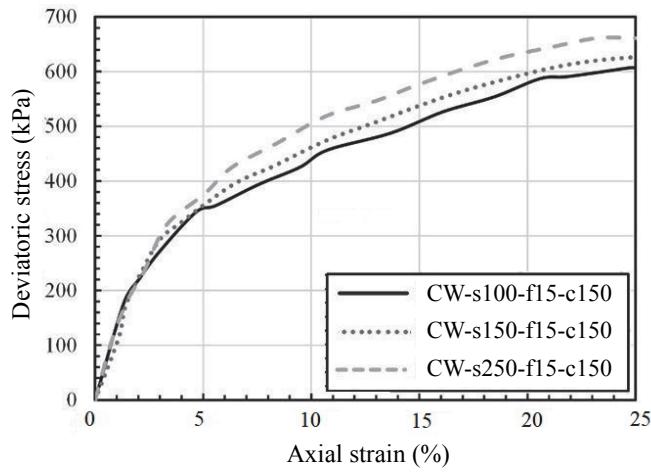
**Fig. 8 Results of unsaturated tests on specimens with silt content of 15% at initial suctions of 100, 150, and 250 kPa and confining pressure of 50 kPa**



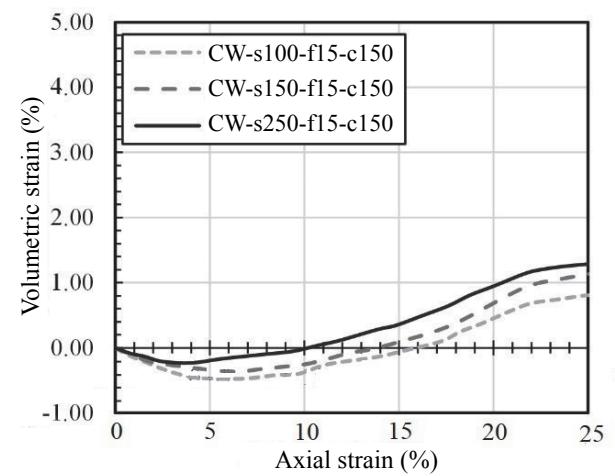
(a) Stress-strain



(b) Volumetric strain

**Fig. 9 Results of unsaturated tests on specimens with silt content of 15% at initial suctions of 100, 150, and 250 kPa and confining pressure of 100 kPa**

(a) Stress-strain



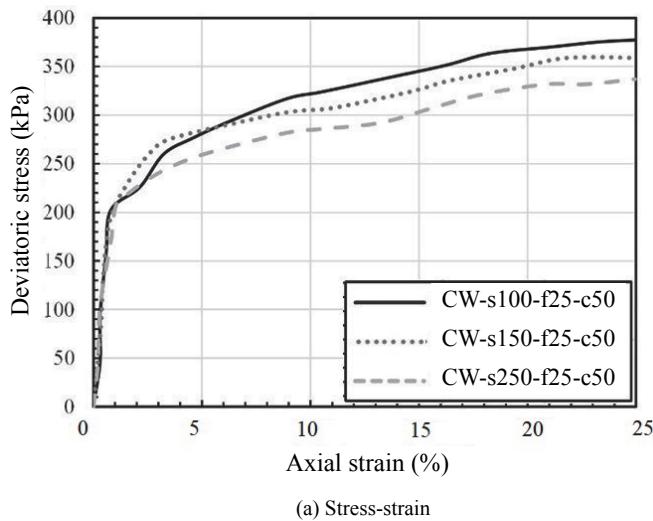
(b) Volumetric strain

**Fig. 10 Results of unsaturated tests on specimens with silt content of 15% at initial suctions of 100, 150, and 250 kPa and confining pressure of 150 kPa**

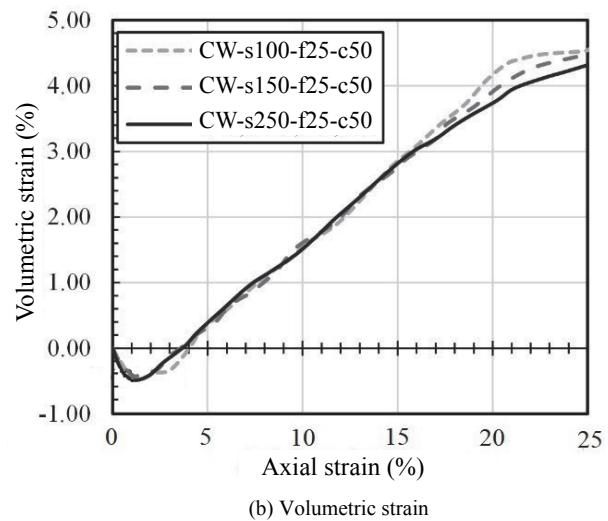
Figures 11(a) ~ 13(a) show the stress-strain curves for the specimens containing 25% of silt. However, according to this figure, despite the fact that the fines content of the specimens is less than the threshold value, increasing the suction will reduce the strength, which is consistent with those observed by Pereira *et al.* (2006) and by Leal-Vaca *et al.* (2012). Given that all the conditions for these tests were the same as the previous two series of tests ( $f_c = 10\%$  and  $15\%$ ), it seemed that the actual threshold of fines content was different from that calculated from Eq. (1). Therefore, to determine the exact amount of the threshold fines content, saturation tests in CU conditions were carried out for silt contents of 10%, 15%, 20%, and 25% under a constant confining pressure of 150 kPa, and with an equivalent intergranular void ratio of 0.85.

As shown in Fig. 14, as expected, with the same equivalent intergranular void ratio, the stress-strain diagrams behave the same way, especially in the critical state, but this behavioral similarity does not exist in silt content of 25%. This confirms the fact that

the threshold of the silt content was lower than 25%, which is acceptable for non-plastic fines according to Thevanayagam (2000). A remarkable point in this section is the significant difference between the shear strengths of the soils in saturated and unsaturated conditions. This remarkable difference can be due to the nature of the saturated undrained test and the constituent minerals that form the silt particles (quartz, feldspar, mica, and carbonate). Given that silica silt was used in the test, it is expected that the silt shows a non-cohesive behavior in the unsaturated test results, but due to the relatively high carbonate content (25% ~ 30%), in unsaturated state, it has been created certain bonding agent. This bonding agent acts as an agent reinforcing the bonding between sand grains. For the behavior of these materials in saturated conditions, it should be noted that carbonate is partly soluble in water, and this characteristic washes off the bindings leading to reduced soil strength, and the non-cohesive nature of the silt also exacerbates this behavior (loss of strength).

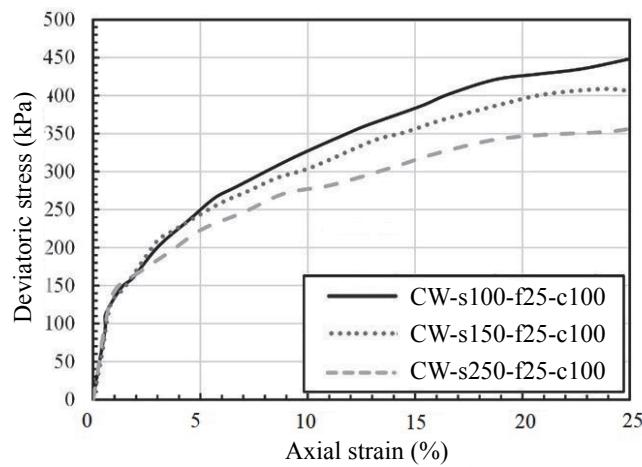


(a) Stress-strain

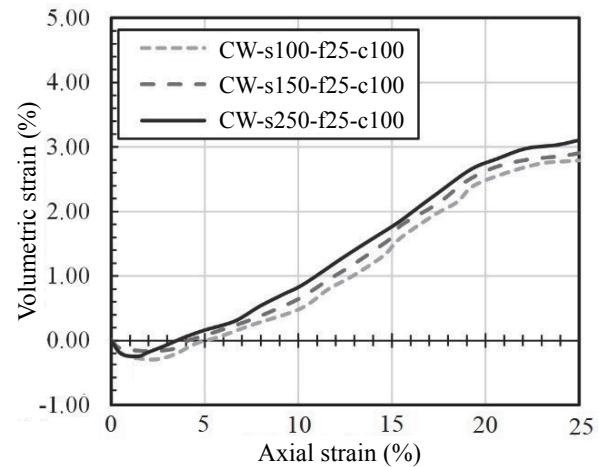


(b) Volumetric strain

**Fig. 11 Results of unsaturated tests on specimens with silt content of 25% at initial suctions of 100, 150, and 250 kPa and confining pressure of 50 kPa**

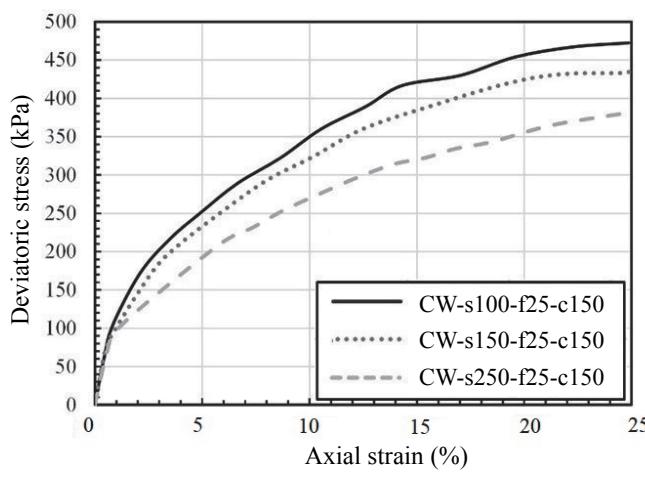


(a) Stress-strain

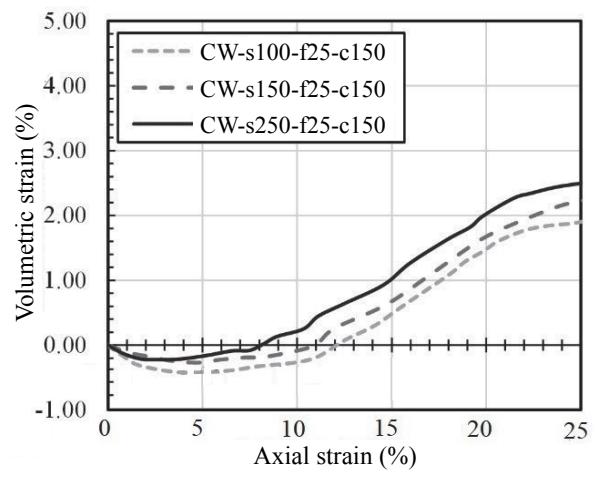


(b) Volumetric strain

**Fig. 12 Results of unsaturated tests on specimens with silt content of 25% at initial suctions of 100, 150, and 250 kPa and confining pressure of 100 kPa**

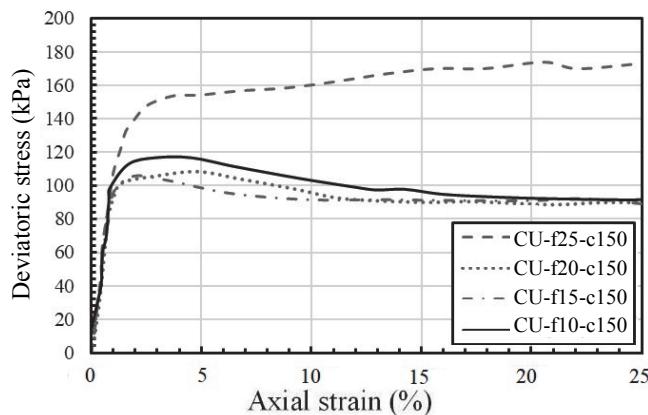


(a) Stress-strain



(b) Volumetric strain

**Fig. 13 Results of unsaturated tests on specimens with silt content of 25% at initial suctions of 100, 150, and 250 kPa and confining pressure of 150 kPa**



**Fig. 14 Stress-strain curves for four soil specimens under a confining pressure of 150 kPa**

#### 4.2 Evaluation of Performance of Equivalent Intergranular Void Ratio Parameter in Unsaturated State

Figure 14 shows that for silt contents less than the threshold value, the equivalent intergranular void ratio has had an appropriate function. Especially, at high deformations that represent a critical state, the stress-strain curves overlap well. However, in unsaturated state, this behavior is not observed. In specimen CW-s100-f10-c100, the final value of the deviatoric stress is 460 kPa. By increasing the fines content from 10% to 15%, the deviatoric stress is increased by 13% and reaches 518 kPa. This is also evident in other specimens (Figs. 4 ~ 9), which indicates the inefficiency of the concept of equivalent intergranular void ratio in homogenizing of the behavior in the unsaturated state compared to the saturated state.

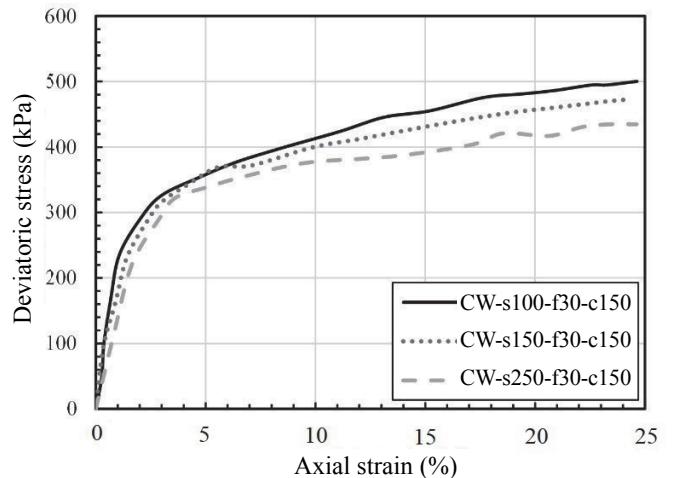
#### 4.3 Effect of Matric Suction on Stress-Strain Curves for $fc > fc_{th}$

In this case, the specimens were prepared with the same global void ratio. As shown in Figs. 15 and 16, for silt contents of 25% and 30%, the suction has the same effect on the soil strength behavior. That is, when the suction is increased, the soil strength is decreased, but for silt content of 40% (Fig. 16), by increasing the suction, the strength is increased nonlinearly, again.

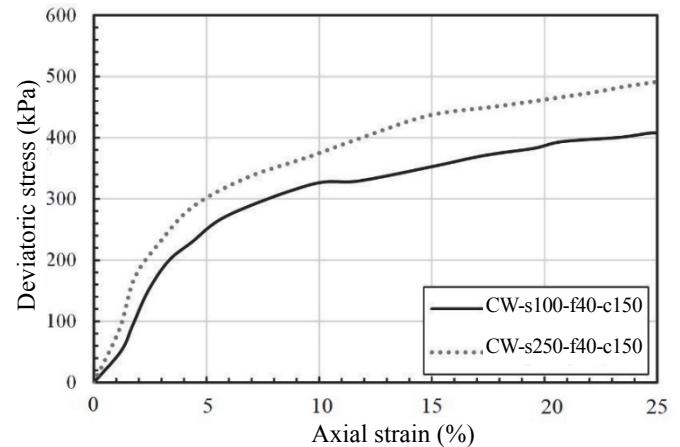
According to previous studies, using the two boundary values, *i.e.*, the threshold fines content ( $fc_{th}$ ) and the limiting fines content ( $fc_l$ ), Thevanayagam (2002) categorizes soils into several categories. First, soils with fines content less than the threshold content, where the soil behavior is controlled by the coarse-grain part. In this case, by increasing the suction, the coarse contacts, or, in other words, the effective internal friction angle of the soil  $\varphi'$  is increased (Rahardjo *et al.* 2013). This is accompanied by increased secondary cohesion that finally leads to the increased shear strength of the soil. Now consider the conditions where the fines content is greater than the limiting fines content. According to Thevanayagam (2002), under these conditions, the soil behavior is controlled solely by the fine-grain part. Here, similar to the soils in  $fc < fc_{th}$  conditions, if the same mechanism holds, increasing suction will increase the strength. However, in the third category where the fines content is between the threshold and limiting fines contents, the

soil behavior is controlled both by fine-grain (primary effect) and coarse-grain parts (secondary effect), and neither of these two effects can be ignored. In this situation, in fact, by changing the soil structure, the suction increase leads to reduced contact between the fine-grain and coarse-grain parts, and in some cases, it leads to the loss of the layered structure of the soil. As a result of this change, the association between the grains changes from coarse grained-coarse grained to coarse grained-fine grained which reduces the secondary effect (the strengthening role of the coarse-grain part) and ultimately, it leads to reduced soil strength.

According to the classification of soils based on Thevanayagam (2000, 2002), for silt contents of 10% and 15%, when the strength is increased by increasing the suction, the soil strength behavior is consistent with  $fc < fc_{th}$ , and for silt contents of 25% and 30%, when the strength is decreased by increasing the suction, the soil strength behavior is consistent with  $fc_{th} < fc < fc_l$ , and finally, by increasing the fines content and exceeding the fines limiting fines content, in the case of increasing the suction, we will observe an increase in strength again at silt content of 40%.



**Fig. 15 Stress-strain curves with 30% silt at confining pressure of 150 kPa**



**Fig. 16 Stress-strain curves with 40% silt at confining pressure of 150 kPa**

It should be noted that the limiting fines content in this research was about 40%, which is consistent with Thevanayagam's (2000) results that considered  $f_{cl}$  of 45%~60% for the studied soil. Additionally, according to the AASHTO classification, which considers the fines content of 35% as the separation boundary between coarse-grain and fine-grain soils, it seems that the 40% percentage obtained in this study is an acceptable limit to determine the limiting fines content.

#### 4.4 Effect of Matric Suction on Volume Change Behavior of Specimens

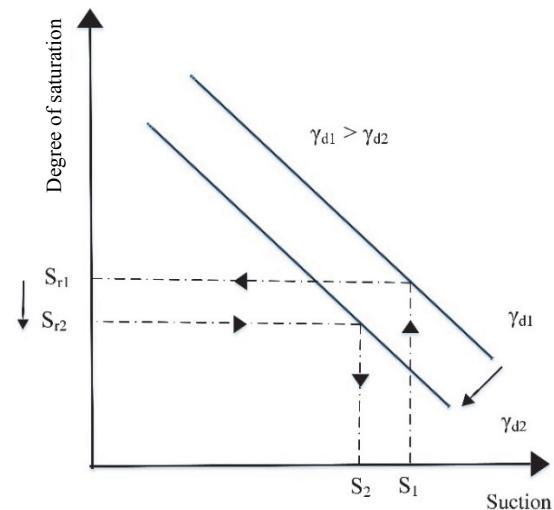
Figures 5(b)~13(b) illustrate volumetric strain variations for fines content (in percent) and various suction values. As shown in the figures, the volume change behavior due to shear is in a dilation state for all specimens; but the quantity of the dilation and how it is related to the amount of suction are different. As shown in Figs. 5(b), 6(b), 8(b), and 9(b), by increasing the suction, the volume changes will be decreased; this is not consistent with Kim's (2009) result. However, it can be seen in Figs. 7(b), 10(b), 12(b), and 13(b), by increasing the suction, the volume changes have also increased, and the results are consistent with Kim (2009). In Fig. 11(b), the behavior of the volume change appears to be very similar. With respect to the figures, it can be concluded that the volume change behavior and initial suction value do not have a definite relationship with each other, and sometimes they are inconsistently reported (Rahardjo 2004; Maleki and Bayat 2012). This can be due to the influence of other factors, such as soil texture.

#### 4.5 Variation of Matric Suction in Course of Shearing

Table 3 shows the final suction values of different specimens. As shown in all specimens, the suction decreases during loading. According to the theory of soil mechanics in a saturated state, in the case of specimen dilation, the pore water pressure decreases. However, this series of tests carried out in an unsaturated state shows the opposite behavior. Therefore, with the assumption of a saturated state as a reference for studying the soil behavior, one cannot achieve the true knowledge of unsaturated soils.

Many researchers have reported the suction change behavior of unsaturated soils in triaxial tests with constant water (CW) as a complex phenomenon, and sometimes, contradictory interpretations have been presented to describe this behavior. However, it seems that to explain the decreased suction (increased pore water pressure) during shearing, one can concentrate on the changes in the total specimen volume and make the density changes as a reference to answer the question. In fact, given the physical nature of water, solid grains, and the form of the free space between the grains, one can answer the question from a physical viewpoint.

According to previous studies (Frost and Park 2003; Jeong and *et al.* 2008; Dash and Sitharam 2011; Maleki and Bayat 2012), for the same suction, a specimen with higher density will require more moisture. This result is considered with the assumption of the same equilibrium trend, texture, and structure for a soil. Zhou *et al.* (2011) studied the effect of density changes on the soil-water characteristic curve of a soil. According to their results, the reduction in density moves the soil-water characteristic curve downward. Also, during shearing in triaxial tests, volume changes are occasionally occurred as dilation and sometimes as contraction. With the assumption of the homogeneity of deformation in the whole specimen (this assumption appears to be more correct for the contraction phenomenon), one can conclude that in the case of contraction, the soil density is increased, and in the case of dilation, the soil density is decreased. On the other hand, in an unsaturated triaxial test with a constant water content, due to the nature of the test, the water content within the specimen is constant in the course of shearing. However, the degree of saturation will be changed due to the change in soil density. On this basis, the two physical aspects simultaneously govern the behavior of the soil concerning suction changes. In the first aspect, the change in soil density will affect the parameters of the soil-water characteristic curve of the soil due to the change in its position (this issue has generally been neglected in the practical application of unsaturated soil mechanics). In the second aspect, the degree of saturation or the ratio of water volume to pore volume varies with the change in the size of the soil specimen. These aspects affect naturally the matric suction during shearing. In fact, in dilative specimens, regarding to Fig. 17, firstly, due to the decrease in density, the water-soil characteristic curve



**Fig. 17** Soil-water characteristic curve and the changing process of the specimen state for dilation along the shear

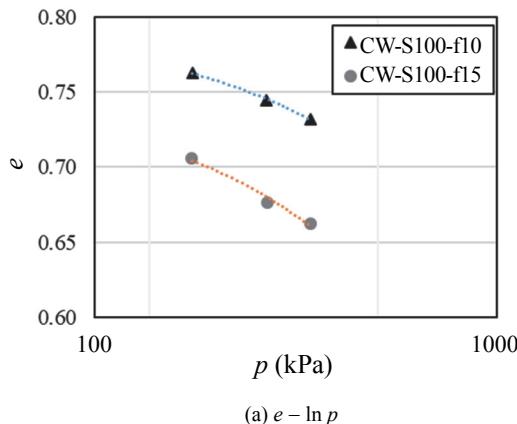
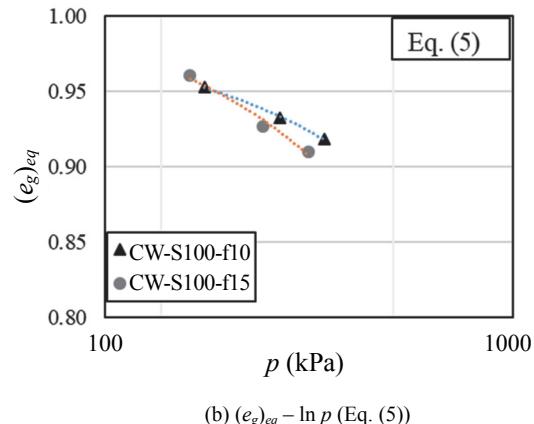
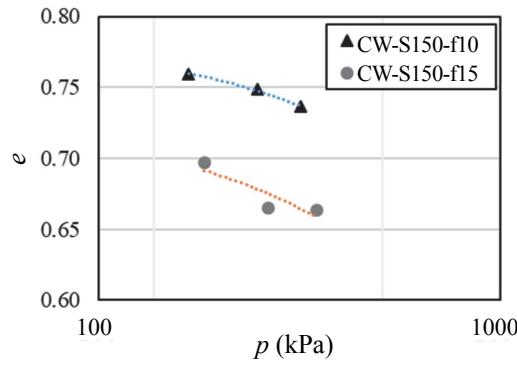
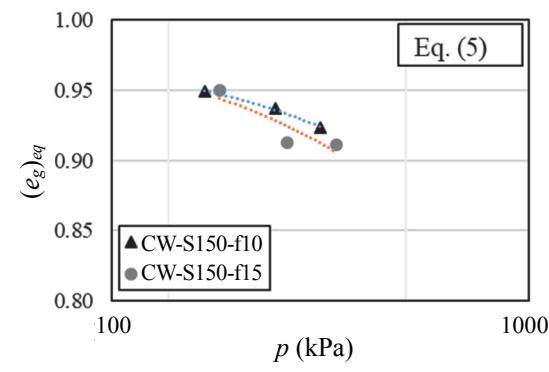
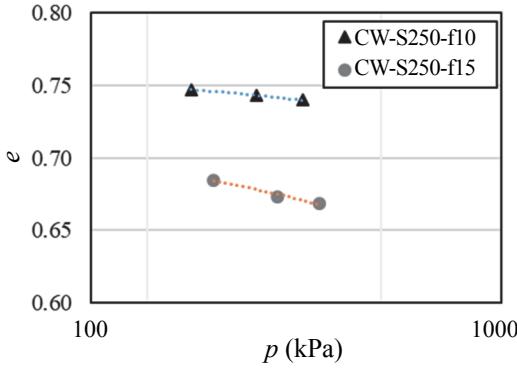
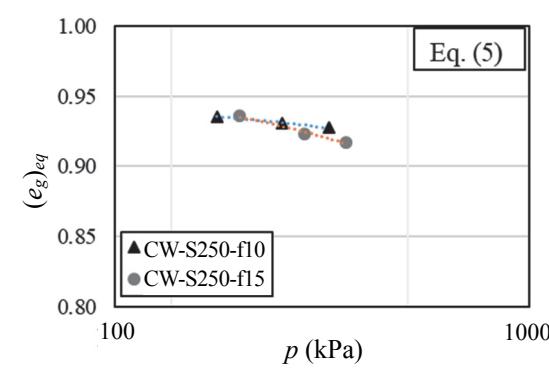
**Table 3** Final suction values of different specimens

Test	Initial suction (kPa)	Ultimate suction (kPa)	Test	Initial suction (kPa)	Ultimate suction (kPa)	Test	Initial suction (kPa)	Ultimate suction (kPa)
f10-c50	100	78	f15-c50	100	84	f25-c50	100	86
	150	88		150	120		150	134
	250	162		250	177		250	226
f10-c100	100	75	f15-c100	100	74	f25-c100	100	81
	150	116		150	123		150	123
	250	208		250	214		250	205
f10-c150	100	77	f15-c150	100	61	f25-c150	100	78
	150	134		150	120		150	115
	250	242		250	241		250	196

moves downward from an initial position corresponding to  $\gamma_{d1}$  to the new position related to  $\gamma_{d2}$ . For a point on the initial position on SWCC, suction  $S_1$  is corresponding to  $S_{r1}$ . In the course of shearing in conditions of CW, due to the increase in volume, the degree of saturation decreases to  $S_2$ . Corresponding suction of  $S_{r2}$  regarding to the new position of SWCC is  $S_2$ , which is less than  $S_1$ .

#### 4.6 Critical State Line

The test results at the ultimate state including net mean stress ( $p$ ), deviatoric stress ( $q$ ), intergranular void ratio ( $(e_g)_{eq}$ ), and global void ratio ( $e$ ) for two fines contents of 10 and 15% are presented in the  $e - \ln p$  and  $(e_g)_{eq} - \ln p$  planes, respectively (Figs. 18–20).

(a)  $e - \ln p$ (b)  $(e_g)_{eq} - \ln p$  (Eq. (5))**Fig. 18 Critical state lines with silt contents of 10% and 15% and initial suction of 100 kPa**(a)  $e - \ln p$ (b)  $(e_g)_{eq} - \ln p$  (Eq. (5))**Fig. 19 Critical state lines with silt contents of 10% and 15% and initial suction of 150 kPa**(a)  $e - \ln p$ (b)  $(e_g)_{eq} - \ln p$  (Eq. (5))**Fig. 20 Critical state lines with silt contents of 10% and 15% and initial suction of 250 kPa**

$\sim 20$ ). The equivalent intergranular void ratio was calculated using Eq. (5). As shown in Figs. 18–20, for all cases the application of the equivalent intergranular void ratio, in comparison with the global void ratio, makes the critical state lines closer to each other. It should be noted, in the case of saturated silty sand, based on the existing results (Dash and Sitharam 2011; Mohammadi and Qadimi 2014), critical state lines in  $(e_g)_{eq} - \ln p$  planes can be replaced by a unique line independent of fine contents. However, in the current study, there are certain gaps between the critical state lines. It appears in an unsaturated state, the matric suction affects the position of the critical state line.

**Table 4** Ultimate stress and initial tangent stiffness modulus ( $E$ ), values of different specimens

Test	Ultimate stress (kPa)	Initial $E$ (kPa)	Test	Ultimate stress (kPa)	Initial $E$ (kPa)	Test	Ultimate stress (kPa)	Initial $E$ (kPa)
f10-c50-s100	347	15591	f15-c50-s100	383	10192	f25-c50-s100	381	15462
f10-c50-s150	371	15818	f15-c50-s150	411	9059	f25-c50-s150	361	16077
f10-c50-s250	408	15328	f15-c50-s250	459	9143	f25-c50-s250	339	16071
f10-c100-s100	458	17778	f15-c100-s100	515	11068	f25-c100-s100	450	12276
f10-c100-s150	472	17200	f15-c100-s150	545	11000	f25-c100-s150	411	10438
f10-c100-s250	489	16800	f15-c100-s250	595	10533	f25-c100-s250	357	11890
f10-c150-s100	514	19028	f15-c150-s100	607	12259	f25-c150-s100	478	8389
f10-c150-s150	546	18200	f15-c150-s150	627	11692	f25-c150-s150	437	7480
f10-c150-s250	575	18750	f15-c150-s250	664	11552	f25-c150-s250	384	8281

#### 4.7 Effect of Matric Suction on Stress-Strain Behavior

In order to quantify the effect of matric suction on the stress-strain behavior of silty sand with constant equivalent intergranular void ratio, ultimate shear strength ( $q_u$ ), and tangent stiffness modulus calculated from the initial part of the stress-strain curves. The obtained results for different values of confining pressure, initial suction and fine contents were collected and listed in Table 4. As seen from Table 4, the ratio of ultimate strength to initial matric suction for both silt contents of 10% and 15% decreases when the confining pressure increases from 50 to 150 kPa. This means that an increase in matric suction has a more important effect for low confining pressure. Besides, for a given initial matric suction, an increase in confining pressure leads to reduction in the matric suction effect on the ultimate shear strength. For a given confining pressure and for silt contents of 10%, 15%, and 25%, the variation of initial matric suction does not influence the initial stiffness modulus. However, with an increase in the axial strain level, the effect of matric suction becomes more important when the confining pressure increases.

#### 5. SUMMARY AND CONCLUSIONS

In the present work, a series of triaxial tests were conducted on soils composed of sand and silt under unsaturated (constant water conditions) and undrained saturated states considering various silt contents, confining pressures, and initial suctions, and the results obtained by the tests were then analyzed. All specimens were made in the same equivalent intergranular void ratio with the aim of verifying its performance in an unsaturated state. The most important results of this research are as follows:

- With fines contents lower than the threshold value and higher than the limiting value, increasing the matric suction increases the strength of the silty sand nonlinearly.
- For fines contents between the threshold and limiting values, increasing the suction results in a nonlinear reduction in the shear strength of the silty sand.
- The suction variation process in unsaturated tests with constant water content depends on the volume change behavior of the specimen during the shearing.
- Although in silty sands, the concept of the equivalent intergranular void ratio has an appropriate effect on the homogenization of soil behavior particularly in the critical state, the use of this concept in unsaturated soil conditions requires more considerations about the effect of matric suction.

#### NOTATIONS

$A$	Dimensionless parameter for calculation threshold fines content
$b$	Active fraction of fines in force structure
$C_c$	Coefficient of curvature
$C_u$	Coefficient of uniformity
$cc$	Clay content (%)
$D_{10}$	The diameter in the particle-size distribution curve corresponding to 10% finer (for coarse soil) (mm)
$D_{60}$	The diameter in the particle-size distribution curve corresponding to 60% finer (for coarse soil) (mm)
$d_{50}$	The diameter in the particle-size distribution curve corresponding to 50% finer (for fine soil) (mm)
$E$	Stiffness modulus (kPa)
$e$	Void ratio
$e_{total}$	Total void ratio
$(e_g)_{eq}$	Equivalent intergranular void ratio
$fc$	Fine content (%)
$fc_l$	Limiting fines content (%)
$fc_{th}$	Threshold fines content (%)
$G_s$	Specific gravity
$k$	$1 - r^{0.25}$
$m$	Fitting constant
$n$	Fitting constant
$p'$	Effective mean stress (kPa)
$r$	Particle size ratio = $1/\chi$
$sc$	Silt content (%)
$\alpha$	The parameter dependent on the soil type for calculation threshold fines content
$\beta$	The parameter dependent on the soil type for calculation threshold fines content
$\gamma_d$	Dry density (gr/cm <sup>3</sup> )
$\chi$	Particle size ratio = $D_{10}/d_{50}$

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#### DATA AVAILABILITY

This study does not generate new data and/or new computer codes.

## CONFLICT OF INTEREST STATEMENT

The authors certify that they have no affiliations with any organizations or with any financial interest.

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