

# SHEAR STRENGTH AND VOLUME CHANGE BEHAVIOR OF BINARY GRANULAR MIXTURES

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## ABSTRACT

This study analyzes the compression and shear behavior of binary mixtures through one-dimensional consolidation and direct shear tests. Two particle size ratios of dry binary mixtures, 6.375 (Type A) and 12.070 (Type B), were adopted in the experiments. The relationships between the void ratio and the fines content (FC) of both mixture types were determined. Two deposition methods were applied during the remolding process on specimens with a relative density of 30% to reduce the separation of the coarse and fine particles. The results show that the two deposition methods have no significant effect on the results. Specimens with FC ranging from 15% to 35%, and an FC of 85%, have relatively unstable initial microstructure, and the compression index at FC 25% is higher than that of FC 35% for the same initial void ratio. Fine grains have a dominant effect on the swelling index within the range of FC 50% to 70%. Particle size ratio has a significant effect on volume change but has less effect on shear strength. Similar volume changes were found from the results of both tests. The findings of this study demonstrate that FC 85% exhibits more compressible behavior than FC 70% and 100%, which is unexpected.

*Key words:* Binary mixture, compression index, volume change, shear strength.

## 1. INTRODUCTION

Binary mixtures containing fines or silts, which influence the mechanical properties of the mixtures, have been of interest to scientists and engineers for decades (Aberg 1992; Pitman *et al.* 1994; Miura *et al.* 1997; Lade *et al.* 1998; Cubrinovski and Ishihara, 2002; Thevanayagam 2007; Bobei *et al.* 2009; Fuggle *et al.* 2014). The V-shaped curve on the plot of maximum and minimum void ratio against fines content (FC) depends on the inherent properties of soils, such as FC, particle shape, particle size distribution, and deposition method (McGeary 1961; Cho *et al.* 2006; Yilmaz *et al.* 2008; Chang *et al.* 2015). The trough of the graph usually falls between FCs of 20% ~ 40%, which is referred to as the transition zone. The corresponding FC is called transitional fines content (TFC), which is the demarcation between the coarse-dominated and fines-dominated areas. When the FC exceeds TFC, the sand-dominated behavior exhibited by the aggregate transforms to fines-dominated behavior (Yang *et al.* 2006). However, in binary mixtures, the void ratio cannot be used to interpret the mechanical behavior accurately, indicating the need for further research.

The chain network of internal interparticle forces in a particular space is formed by particle contacts. Characteristics such as stress-strain response, compressibility, and modulus are influenced by the network. Specimens of the same void ratio with different mixture types may display different mechanical behaviors.

When the particle size gap of binary mixtures is sufficiently large, the pores between the coarse grains can be larger than those of the fine grains. Some of the fine grains remain inactive and may move among pores without significantly contributing to the force chain (Thevanayagam *et al.* 2007). Based on the activities of fine grains (in coarse-grain-dominated regions) and coarse grains (in fine-grain-dominated regions) in the force chain, a set of contact density indices, including the intergranular, equivalent intergranular, inter-fine, and equivalent inter-fine void ratios are developed (Mitchell 1993; Vaid 1994; Thevanayagam 1998; Thevanayagam and Mohan 2000; Thevanayagam *et al.* 2007). These indices are believed to reflect the microstructure of binary mixtures, and can be used to interpret the behavior of gap-graded mixtures. A classification of binary mixture types with spherical particles has been presented. As the FC increases, the microstructures can be classified into coarse-grained and fine-grained soil mixture based on the threshold FCs.

In this study, one-dimensional consolidation tests and direct shear tests were performed to analyze the effects of FC on the shear strength and volume change characteristics of soil grains. The effect of FC on binary mixtures is emphasized in this study, while other factors, such as the particle size ratio and relative density, are also considered in assessing the characteristics of binary mixtures.

## 2. EXPERIMENT PROCEDURES

### 2.1 Materials

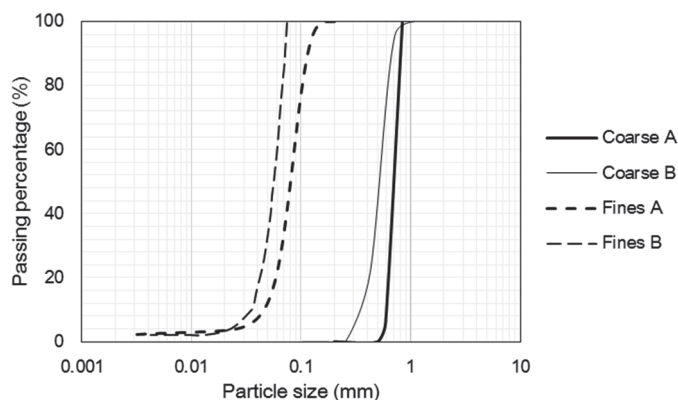
Both coarse-grained and fine-grained silica sands with a sub-angular shape and specific gravity of 2.65 were used in this study. Two types of gap-graded binary mixtures, labeled "Type A" and "Type B" mixtures, were prepared for comparing the effect of particle size ratio. Figure 1 shows the particle size distribution curve

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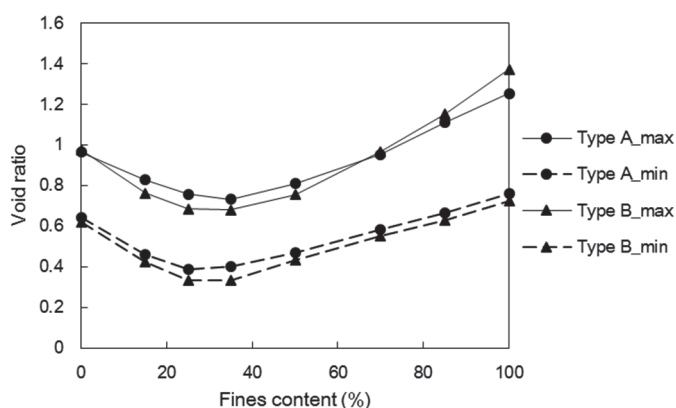
**Fig. 1 Particle size distribution curves of Type A and Type B mixtures**

of four kinds of prepared grain mixtures. The sizes of  $D_{50}$  from the coarse and fine particles are 0.70 (Coarse A), 0.51 (Coarse B), 0.08 (Fines A), and 0.058 (Fines B) mm. For Type A mixtures with a particle size ratio of 6.375 (0.51/0.08), sand grains passing through a 0.59 mm (No. 30) sieve and retained on a 0.5 mm (No. 35) sieve were selected as the coarse particles, and mixed with the fines that passed through a 0.088 mm sieve (No. 170) and retained on 0.075 mm (No. 200) sieve. For Type B with a larger particle size ratio of 12.07 (0.70/0.058), the mixtures were prepared by mixing particles between No. 20 ~ 30 sieves and particles passing through No. 200 sieve.

The Japanese testing standards for maximum and minimum densities of sands were applied on two types of binary mixtures with 0%, 15%, 25%, 35%, 50%, 70%, 85%, and 100% FC by weight were produced. Their maximum and minimum void ratios determined by applying the Japanese testing standard (JGS 0161) are displayed in Fig. 2. The transition zone in the plot varies between 25% and 35% FC. The V-shape of the curve becomes increasingly apparent when the particle size ratio increases. Further experiments were performed with FC within the transition zone (*i.e.*, close to 30%).

## 2.2 One-dimensional Consolidation Test

The correlation between the FC and compression characteristics, including compression index ( $C_c$ ) and swelling index ( $C_s$ ),



**Fig. 2 Maximum and minimum void ratios of two Type A and Type B binary mixtures at different fines contents**

was obtained from a type of one-dimensional consolidation tests conducted under dry conditions. Two deposition methods were applied to produce a relatively loose condition, and to prevent the gravity-caused separation of binary mixtures. The first method of specimen preparation involved depositing a pre-weighed amount of mixtures through a funnel. The funnel spout was initially placed on a porous bottom stone. Filling the mold by circling the funnel and maintaining a minimum falling distance can effectively yield a loose condition. However, some fine particles may get attached to the wall of the funnel due to static electricity. The second method involved slowly filling the mold using a spatula, where coarse and fine particles can stay together; however, the remolding process takes more time. Two level-arm consolidometers were used simultaneously to determine the effect of the specimen preparation methods.

The specimens were prepared in a consolidation ring of 5 cm diameter and 1.9 cm height, and their relative densities were controlled at approximately 30%. The leverage ratio of the consolidation apparatus was 9.82. The loading increments were 0.05, 0.15, 0.25, 0.5, 0.5, 1, 1, 2, and 4 kg, and loading was successively released until the contact force was attained. After the unloading steps were completed, the last loading stage was reloaded, and two extra loading increments of 8 kg were added and unloaded until the contact force was reached. The displacement-time history was recorded using Humboldt HM-2432.3F data logger. In contrast with cohesive soils, vertical settlement occurred immediately after the force on the specimen was applied. The precision of the recording time was 0.1 sec, and 0.001 mm for the displacement.

## 2.3 Direct Shear Test

The specimen initial conditions were the same as those for the one-dimensional consolidation test (the same relative density and FCs). For each mixing configuration, four specimens were tested at 20, 40, 80, and 160 kgf axial loading to determine their shear strength parameters. The test was performed according to ASTM D-3080 specifications, where data point was recorded every 3 seconds.

## 3. RESULTS

The  $C_c$  results of Type A specimens from both preparation methods are presented in Fig. 3. There seems to be a slight difference between the two methods, indicating that both methods are feasible and reliable. Figure 4 presents the average values of the consolidation test results. The spoon method was adopted for the following consolidation and direct shear tests because it was easier for producing the specimens.

### 3.1 Compression and Swelling Indexes

The compressibility of Type B (particle size ratio of 12.07) is higher than that of Type A (particle size ratio of 6.375). Figure 4(a) shows that the trend of the two particle-size ratios are similar. The values of  $C_c$  increase and attain the peak value at approximately FC 25%, and then gradually reduce when the FC gets to approximately FC 50%.  $C_c$  remains at a similar value till at FC 70%, followed by a slight increase at FC 100% for Type A, and a significant increase at FC 100% for Type B, where a second peak is found at approximately FC 85%. It was expected that compressibility should be correlated with void ratio; however, the results show irregular patterns.

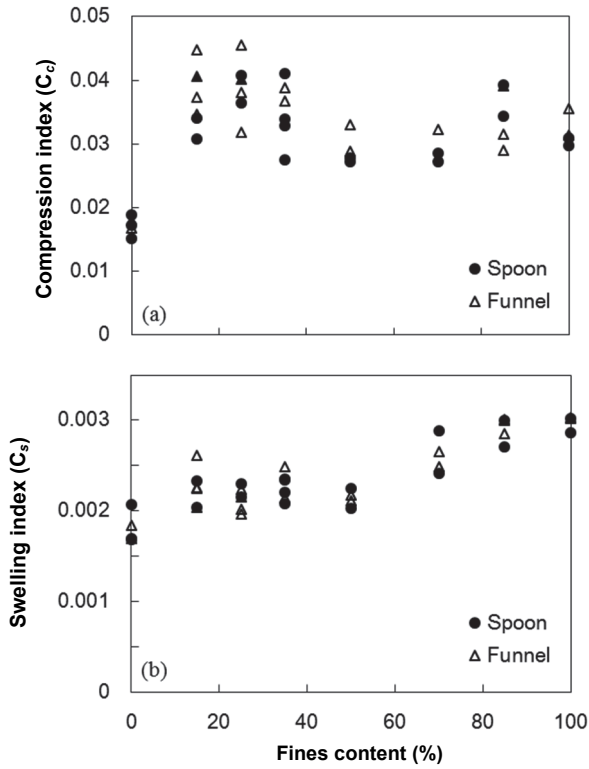


Fig. 3 Consolidation test results of Type A using different deposition methods (a) compression index; (b) swelling index

For the  $C_s$ , both mixture types display a similar trend, as shown in Fig. 4(b). The values of Type A mixtures are higher than those of Type B at FC below 50% but lower than Type B values from FC 70% to 100%. The intersection of the graph for the two types is located at an FC between 50% and 60%. The results indicate that cohesionless soils have slightly swelling characteristics, which becomes significant with the increase in FC. However, the  $C_s$  values are not proportional to FC;  $C_s$  barely changes at low FC up to 50% until the FC increases to 70%. Corresponding to  $C_c$ , it might be considered that the FC between 50% to 70% is the demarcation of coarse-dominated to fine-dominated sand.

### 3.2 Shear Strength and Deformation

For the binary mixtures with varying FC, the shear behaviors were determined by conducting direct shear test under dry condition. The residual strength concerning the normal stress for Types A and B form a linear regression line. The regression  $R^2$  for all the specimens exceeds 0.99, and it can be seen that the results are reliable.

Figure 5 presents the peak or residual strength of both types of mixtures. The vertical loading increments are 2, 4, 8, and 16 kg. Both types have no significant effect on shear strength with varying FC when the normal stress is less than 263.80 kPa (which had been calculated). The effect of FC on shear strength can be observed at high normal stress 512.89 kPa.

Figure 6 shows the relationship between average friction angles of Types A and B and varying FC. The lowest friction angle for both types occurs at FC 15%. The peak values of Types A and B are located at FC 35% and 50%, respectively. It appears that particle size ratio may determine the FC that corresponds to the peak value of friction angle. Figure 7 depicts the relationship

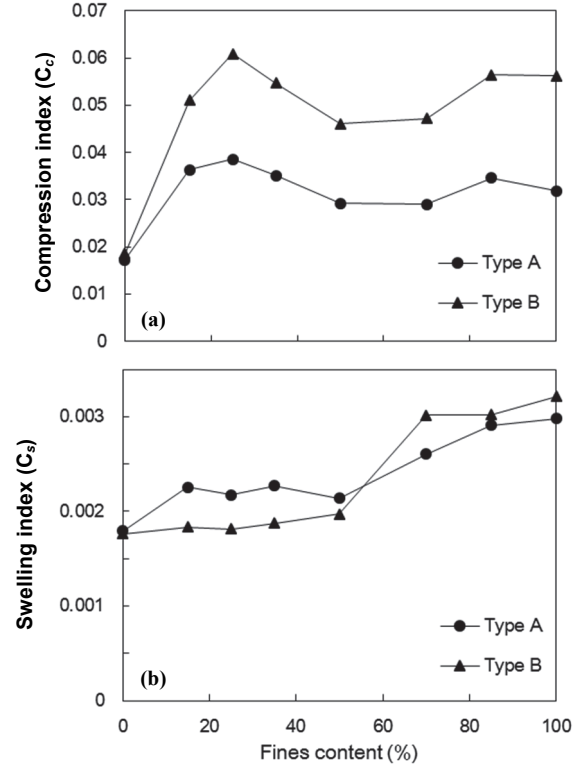


Fig. 4 Results of consolidation test: (a) compression index; (b) swelling index

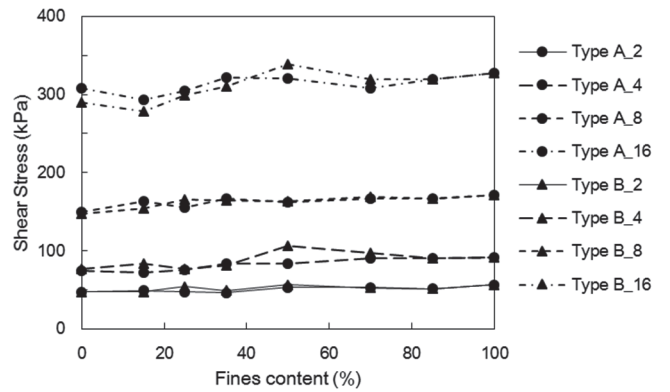


Fig. 5 Shear stress of Types A and B mixtures

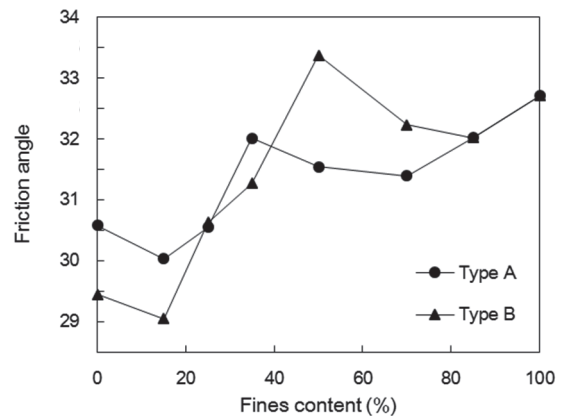
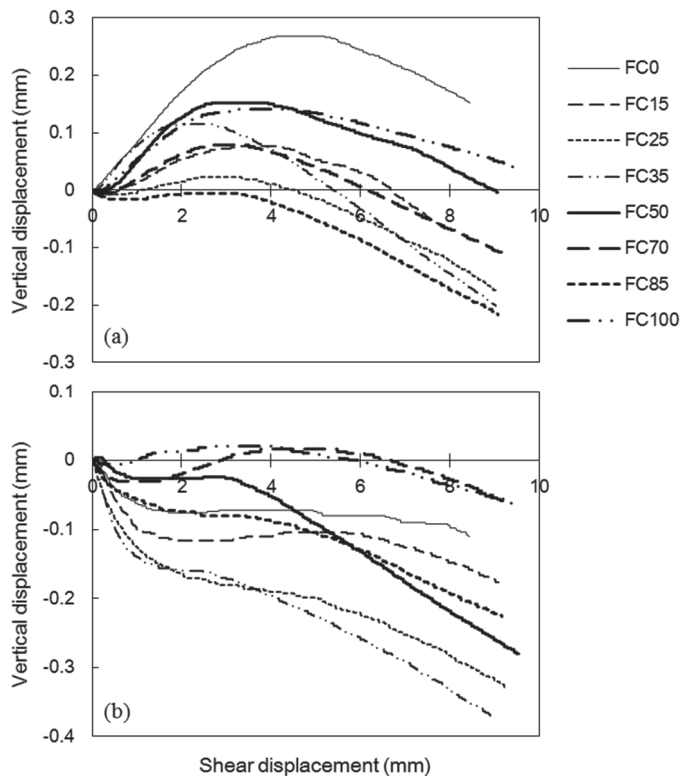


Fig. 6 Friction angles of Types A and B with varying fines content



**Fig. 7** Shear displacement versus vertical displacement with varying fines content at low normal stress: (a) Type A; (b) Type B

between the shear displacement and vertical displacement at low normal stress (77.77 kPa). Most of the Type A mixtures display dilative behavior during shearing, whereas Type B mixtures exhibit contractive behavior. The results correspond to the effect of  $C_c$  in one-dimensional consolidation test. The particle size ratio is higher than a specific value, and the mixtures display apparent contractive behavior.

## 4. DISCUSSION

### 4.1 Effect of Particle Size Ratio

In general, the skeleton of soil particles reassemble during consolidation and shearing, which increases the vertical displacement. Based on previous results, mixtures with higher particle size ratio show higher compressibility and larger volume change. The pores existing between coarse grains are large enough to contain more fines. During consolidation and shearing, fine grains easily filled pores after the initial structure was reassembled, which deformed the specimen more. Particle size ratio also influences the maximum and minimum void ratio, and the values of Type B are lower than that of Type A. The limiting void ratio is controlled by the range of particle sizes (Lade *et al.* 1998). The decrease in the void ratio becomes much gentle when the size ratio exceeds 7. A higher difference in the particle size of the mixtures results in a more significant reduction in bulk volume (Bodman and Constantin 1965), which means that the mechanical properties of binary mixtures might have significant differences when particle size ratio is high enough.

### 4.2 Ratio of $C_c$ and $C_s$

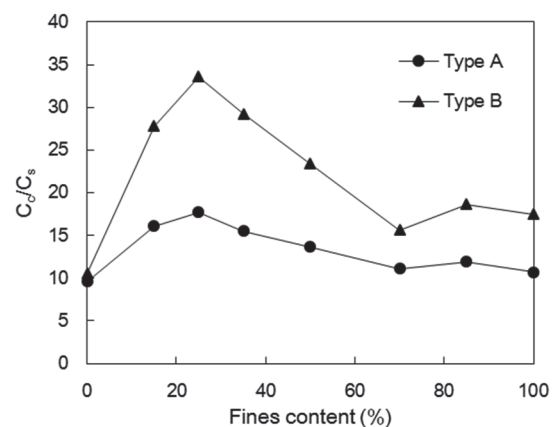
Figure 8 presents the plot of the  $C_c$ -to- $C_s$  ratio against FC, and depicts an upside-down V-shape for FC below 70%. The highest point is located at FC 25%. When FC exceeds 70%, the ratio slightly increases and reaches a peak value at approximately FC 85%. Generally, the ratio of  $C_c$  to  $C_s$  mostly ranges from 5 to 10. The maximum value of Type B mixtures is 33.59, which is almost twice that of Type A. Type A curve is much smoother than the Type B curve. The lowest point of both types of mixtures corresponds to FC 70%, as it has a lower  $C_c$  and higher  $C_s$  values. The  $C_c$  results obtained for both Types A and B mixtures at FC 85% are unexpected because the specimens with FC 85% have higher values compared with FC ranging from 70% to 100%; the reason herein is unknown. From the above results on compressibility, not only are FC 25% to 35% important, FC at approximately 85% should also be of concern.

### 4.3 Effect of FC 25% and 35%

For Type B mixtures, FC 25% and 35% have the same minimum void ratio (0.58), which has the same initial void ratio and volume. Both mixtures have unstable initial microstructures, with FC 25% having a higher compressibility than FC 35%. When the FC increases from 25% to 35%, part of the coarse grains are replaced by fine grains, and the pores between coarse grains contain more fines. Compared to FC 25%, the mixtures containing FC 35% provide less space for fine grains to move freely between the coarse grains, which indicates that volume compressibility is reduced. Equation (1) is an approximate means of estimating the threshold fines content ( $C_{Fth}$ ), which is presented by Thevanayagam (2007) for changing the state of fine grains from inactive to active in the force chain. Where  $e$  is global void ratio, and  $e_{max,HF}$  is the maximum void ratio of host fines.

$$C_{Fth} = \frac{100 e}{e_{max,HF}} (\%) \quad (1)$$

When the particle size ratio exceeds 6.5 and the FC is less than  $C_{Fth}$ , the fine grains do not fill up the pores existing between the inter-coarse grains entirely. Here, the contact force chain does not occur between the fine particles. When the mixtures contain a sufficient amount of fine grains, they reach the threshold state such that the force chain is formed by direct contact between the fine



**Fig. 8** Ratio of  $C_c$  and  $C_s$  against fines content



grains. The  $C_{Fth}$  of both FC is 42.26%. From the results of  $C_c$ , the concept mentioned above is verified. When FC is smaller than  $C_{Fth}$ , the closer it is to  $C_{Fth}$ , the greater the effect of fine particles on the contact force chain. This increased force explains why specimens with FC of approximately 35% have lower compressibility than those with FC of 25%.

#### 4.4 Volume Change of Binary Mixtures

Figure 7(b) presents the results of the shearing deformation of Type B mixtures. The trend of the vertical displacements with varying FCs is similar to the compression index trend. FC 15%, 25%, and 35% have higher vertical displacement during shearing. As the FC increases, the behaviors of the mixtures, except FC 85%, change from contractive to dilative. The same phenomenon is observed in Type A: beyond FC 35%, dilative behavior occurs with increasing FC, and FC 85% specimen also shows contractive behavior. FC 85% exhibits higher value, corresponding to the results of  $C_c$ . From the above results, the reason causes the unexpected results of FC 85% and needs to be studied.

The consolidation test results obtained at a relative density of 30% shows an uncorrelated trend with the void ratio, which means that the global void ratio is not a suitable parameter for interpreting the compression behavior of coarse-fine particle mixtures. The effect of particle contact on microstructure becomes significant. The internal force chain network is formed by interparticle contact, which determines the stress-strain response, strength, compressibility, and modulus of the binary mixtures. When the mixtures have lower FC than the transition zone in the FC-e plot, the microstructure of the initial state of the remolded specimens shows that part of fine grains are constrained in the voids within the coarse-grain skeleton. Here, the fine grains are inactive in the force chain, and part of them as separation role between coarse grains. This separation leads to an unstable microstructure. When loading is applied to the mixtures, the fine grains move freely from one pore of the coarse grains to another, and then change the position of coarse grains. This process induces smaller pores, higher volume compression, and denser structure. The phenomenon is also reflected in Fig. 3(a), where the data points are significantly dispersed from FC 15% to 35%, which indicates that the system of particle arrangement influences compressibility. However, the mechanism of volume change at FC 85% still needs to be investigated further.

#### 5. CONCLUSIONS

A binary mixture is a material that contains a blend of two different particle sizes, which are both gap distributed. Its mechanical behaviors are different from those of single mixtures or other uniformly distributed mixtures. A V-like trend is found in the relationship between the maximum and minimum void ratios and FC, and the value of Type B mixtures (particle size ratio of 12.07) is lower than Type A value (particle size ratio of 6.375). At the same FC, a higher particle size ratio mixture has larger inter-coarse-grain spaces for fine grains to fill up, increasing the mixture density.

From the results of the one-dimensional consolidation test, FC 25% has the highest compression index in both Types A and B, although its initial void ratio is the same as that of FC 35%. It might be considered that the FC 35% mixtures provide less space

than FC 25% mixtures for fine grains to move freely between the coarse grains. The mixtures containing FC 50% and 70% have lower volume compression, which indicates that their microstructures are relatively stable in the initial state. FC 85% gives another peak value of  $C_c$  among specimens containing between 70% to 100% FC. The data at FC 85% on the graph of  $C_c$  is dispersed, which is the same from FC 15% to 35%, irrespective of the applied deposition method. The unstable initial microstructure reassembles the particles in the mixtures and facilitates larger volume change, which can be also be observed from the direct shear test results.

For the direct shear, both mixture types have no significant effect on shear strength till the normal stress increases to a certain value. The friction angle seems to be influenced by the particle size ratio. Type A mixtures exhibit dilative behavior, and most of the Type B mixtures show contractive behavior. The volume change results have a similar trend as the compression index results, which have less dilative or larger contractive effects of mixtures from FC 15% to 35% and also at FC 85%. The relationships between the volume change and FC from the consolidation and direct shear tests are inconsistent with the void ratio from FC 50% to 100%. The volume change at FC 85% are unexpected, and the mechanism needs to be investigated further.

The correlation of volume change with FC is inconsistent with the strength results. Furthermore, the volume change and strength of binary mixtures have no direct connection to the global void ratio. Although a certain FC has the lowest minimum void ratio, this FC was not associated with the largest deformation and highest strength in the consolidation test and the direct shear test, respectively.

#### FUNDING

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#### DATA AVAILABILITY

The data used in this study are available from the corresponding author on reasonable request.

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