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RELATIONSHIP BETWEEN EARTHQUAKE MAGNITUDE AND FAULT LENGTH FOR TAIWAN: BAYESIAN APPROACH

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ABSTRACT

This paper presents a relationship between fault length (L) and earthquake moment magnitude (M) for Taiwan, using the Bayesian approach integrating two local M-L datasets as observation with 77 non-local datasets as prior information. Like other Bayesian studies, the purpose of using such Bayesian updating is to compensate limited local data with relevant prior information. In addition to the detailed Bayesian calculations, this paper also shows that the new Bayesian model should be more suitable for Taiwan than the prior model.

Key words: Bayesian approach, fault length, earthquake magnitude, Taiwan.

1. INTRODUCTION

Estimating a proper earthquake magnitude is important to geotechnical earthquake engineering and seismic hazard analysis. However, owing to natural randomness and our imperfect understandings of earthquakes, it is difficult to estimate earthquake magnitudes with analytical approaches. As a result, it is common to use empirical relationships, developed with historical data, to estimate earthquake magnitudes associated with active faults. In this paper, a Bayesian algorithm is introduced to estimate earthquake magnitudes for Taiwan areas. In essence, the Bayesian algorithm uses limited local data as "observation" to update a "prior" global database for obtaining the Bayesian estimate from the "posterior" information.

In this paper, literature review on earthquake magnitude estimation and Bayesian analyses are first summarized. Next, we elaborate the proposed Bayesian calculation to integrate the prior information with the local datasets. Finally, the results of this study are presented and discussed.

2. LITERATURE REVIEW

Without constant, successful forecasts on major earthquake occurrences, earthquake prediction is still beyond our capability (Geller *et al.* 1997). One of the reasons making earthquake prediction so difficult is because we cannot monitor the stress and strain conditions of rocks a few kilometers below the ground surface where earthquakes are initiated (Geller *et al.* 1997). Under the circumstances, a number of earthquake empirical relationships were developed with historical data. For instance, ground motion prediction equations (GMPEs) are empirical relationships developed with strong ground-motion records, and they become the underlying performance function of seismic hazard analysis for earthquake risk assessment (*e.g.,* Wang *et al.* 2013; Farhadi and Mousavi 2016; Ayele 2017).

In addition to GMPEs, empirical relationships between earthquake magnitudes and the characteristics of active faults were also developed. For example, Wells and Coppersmith (1994) developed a series of empirical relationships to predict earthquake magnitudes, which have been commonly applied to relevant studies (Basili *et al.* 2008; Campbell and Bozorgnia 2008, 2014; Leonard 2010; Ide *et al.* 2011; Horton 2012; Stirling *et al.* 2012; Akkar *et al.* 2014; Field *et al.* 2014; Keranen *et al.* 2014). Note that those empirical relationships (Wells and Coppersmith 1994) were based on a global database containing major earthquakes since the 1850s, in an attempt to enrich the sample size for making the empirical relationship more representative.

Generally speaking, a statistical model developed with more than 30 samples is considered statistically representative (Tang *et al.* 2017). However, owing to the long return period of major earthquakes, the sample size is inadequate for developing local empirical relationships concerning major earthquakes. For instance, it is difficult to develop a local empirical relationship between earthquake magnitude and fault length for Taiwan areas, because very few instrumented data from the past decades are available.

In order to develop a more representative model with limited "project-specific" data, the Bayesian approach is commonly employed. The essential of the Bayesian approach aims to integrate limited project-specific data with prior information, which could be from similar projects, from past experiences, and even from the engineer's best judgments. For example, Wang and Xu (2015) utilized the Bayesian approach to estimate the standard deviation of the friction angle of the soil at a given site, using only one project-specific sample to update the recommended variability range (Phoon and Kulhawy 1999); similarly, Wang *et al.* (2010) proposed a Bayesian method to determine the probabilistic distribution of the friction angle of the soil at the site, based on project-specific cone penetration tests along with relevant prior information from the literature (Akkaya and Vanmarcke 2003). Other Bayesian analyses and applications to geotechnical engineering and engineering geology include those for liquefaction evaluation

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(*e.g.,* Juang *et al.* 1999), slope stability assessment (*e.g.,* Cheung and Tang 2005), and model uncertainty characterizations (Gilbert and Tang 1995; Ching *et al.* 2006; Zhang *et al.* 2009).

In addition to geotechnical engineering and engineering geology, the Bayesian approach was also commonly used in earthquake engineering and applied seismology. For instance, Arnold and Townend (2007) used the Bayesian approach to estimate tectonic stress with several kinds of seismological data; Mignan *et al.* (2011) proposed a Bayesian method to examine the completeness of earthquake catalogs; Imoto and Ishiguro (1986) used Bayesian algorithms to monitor the sudden change of earthquake magnitude-frequency relationships; Wang and Brant (2015) proposed a new Bayesian method to characterize earthquake source-to-site distance probability functions for probabilistic seismic hazard assessment; such examples can go on and on (*e.g.,* Esteva 1969; Cua and Heaton 2007).

3. MODEL DEVELOPMENT

3.1 Local Data

As mentioned previously, the scope of the study is to develop an empirical relationship between fault length (L) and earthquake magnitude (M) for Taiwan areas, so that the first task is to collect local M-L datasets. Note that earthquake magnitude used in this paper is moment magnitude, unless it is stated otherwise.

Two local datasets were collected from the literature. The first is associated with the 1999 Chi-Chi earthquake, which registered a moment magnitude of 7.6 with a rupture length of 90 km (Wang *et al.* 2017). The second is associated with the 1946 Tainan earthquake (on 4th Dec., 1946), which registered a moment magnitude of 6.7 with a rupture length of 12 km (Wells and Coppersmith 1994). More discussion concerning the limited local datasets is given in Section 4.1.

3.2 Prior Information

With the limited local datasets, the second task is to search for relevant prior information. From the literature, we found the following relationship (Fig. 1) developed with 77 M-L datasets outside Taiwan (Wells and Coppersmith 1994):

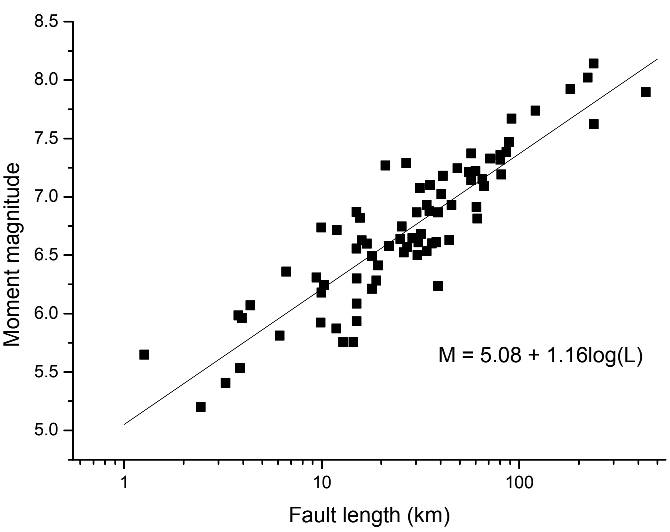


Fig. 1 The prior information for this Bayesian calculation, which is the non-local M-L dataset and the regression model

$$M = 5.08 + 1.16\ \log(L) \pm .0.28 \quad (1)$$

where 0.28 is the standard deviation of the model error (e) that is a random variable following the normal distribution with mean value = 0. Also note that the standard deviations of the two model parameters were also reported in the reference. For the intercept (denoted as a) with mean = 5.08 (see Eq. (1)), its standard deviation was reported as 0.1; for the slope (denoted as b) with mean = 1.16 (see Eq. (1)), its standard deviation was equal to 0.07 (Wells and Coppersmith 1994).

3.3 Local *M-L* Relationship for Taiwan from Bayesian Approach

With the prior information (Eq. (1)) and the local datasets, this section will elaborate the Bayesian calculation for developing a new M-L relationship for Taiwan areas. Following the functional form of the prior model, the local M-L relationship for Taiwan is expressed as follows:

$$M = a + b\log(L) \pm 0.28 \quad (2)$$

Therefore, the purpose of this Bayesian calculation is to update a (intercept) and b (slope) based on the prior information and the local sample, while presuming the local model has the same error characteristics (e) as the global model.

The first step of the calculation is to establish the prior, joint probability mass function for a and b, or the function that calculates the joint probability given $a = a^*$ and $b = b^*$ (*i.e.,* a^* and b^* are two given constants). Since a and b both follow the normal distribution with their respective mean value and standard deviation as 5.08 ± 0.1 and 1.16 ± 0.07, their joint probability distribution can be developed (Fig. 2) as the product of the two marginal distributions, disregarding whatever degree of correlation that the two variables might have. More discussion on this joint probability function is given in Section 4.4.

The second step of the Bayesian updating is to compute the likelihood function. For example, given one local dataset as $L = 90$ km and $M = 7.6$, the likelihood function can be written as:

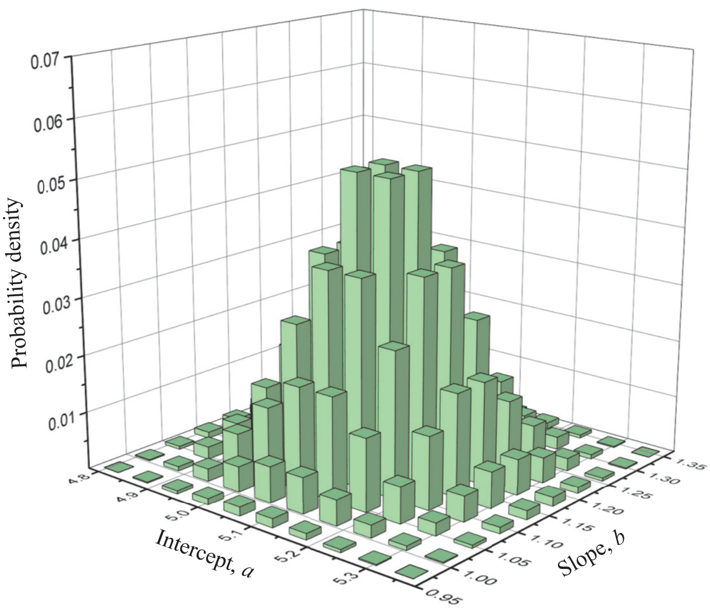


Fig. 2 The prior joint probability mass function for intercept (*a*) and slope (*b*)

$$\Pr(\varepsilon \mid a = a^* \text{ and } b = b^*) = \Pr(L = 90, M = 7.6 \mid a = a^* \text{ and } b = b^*) \quad (3)$$

where ε denote local observation. Next, we substituted a^*, b^*, and L = 90 km into the local model (*i.e.,* Eq. (2)) and obtained the mean magnitude as $a^* + b^* \times \log(90)$. Since the standard deviation of the error (e) is equal to 0.28, the likelihood function can be written as:

$$\Pr(\varepsilon \mid a = a^* \text{ and } b = b^*) = \Pr(M = 7.6; \mu = a^* + b^* \times \log(90), \sigma = 0.28) \quad (4)$$

where μ and σ denote mean and standard deviation, respectively. Furthermore, because the *M-L* relationship is a regression model in essence, the dependent variable *M* will follow the normal distribution based on regression theory (Ang and Tang 2007). Therefore, the likelihood function is calculated as follows:

$$\Pr(\varepsilon \mid a = a^* \text{ and } b = b^*) = \frac{1}{0.28\sqrt{2\pi}} e^{-(7.6-(a^*+b^*\times\log 90)^2/(2\times 0.28^2)} \quad (5)$$

Note that Eq. (5) is in the form of the probability density function of the normal distribution.

Based on the calculation shown in Eq. (5), we can go on computing the likelihood function for each *a-b* combination in Fig. 2. Accordingly, Fig. 3 shows the likelihood function for each *a-b* combination.

With the prior probability (Fig. 2) and likelihood function (Fig. 3), the final step of the Bayesian calculation is to use them to calculate the posterior probability with the governing equation of the Bayesian approach as follows:

$$\Pr''(\theta_i) = \frac{\Pr'(\theta_i) \times \Pr(\varepsilon \mid \theta_i)}{\sum_{i=1}^{n} \Pr'(\theta_i) \times \Pr(\varepsilon \mid \theta_i)} \quad (6)$$

where $\Pr'(\theta_i)$ and $\Pr''(\theta_i)$ denote prior probability and posterior probability for parameter θ_i (here is the combination of *a* and *b*); $\Pr(\varepsilon|\theta_i)$ is the probability for the observation (ε) to occur given θ_i, also known as the likelihood function of the Bayesian approach.

By substituting the prior (Fig. 2) and the likelihood function (Fig. 3) into the governing equation, we obtained the posterior distribution (Fig. 4) for each combination of a^* and b^*. Next, we established the two marginal distributions (Fig. 5) for *a* (intercept) and *b* (slope) based on the posterior distribution. Accordingly, the mean value of intercept is equal to 5.1, and the mean value of slope is equal to 1.18. As a result, the *M-L* relationship for application in Taiwan areas becomes:

$$M = 5.1 + 1.18 \log(L) \pm 0.28 \quad (7)$$

Once again, it is noted that the local (Bayesian) model is presumed to have the same standard deviation of 0.28 as the global model, or the prior information of this Bayesian computation.

3.4 Model Evaluation

To examine the suitability of this Bayesian model (*i.e.,* Eq. (7)) for local applications, another local *M-L* dataset from the 1946 Tainan earthquake was used. As mentioned previously, this earthquake in southern Taiwan registered a moment magnitude of 6.7 with a rupture length of 12 km. Nevertheless, it has to be noted that the data were considered not as reliable as the pool of data used for developing the global model (*i.e.,* Eq. (1)), and hence this particular dataset was only used for the model validation so far.

On the use of the Bayesian model (Eq. (7)), the expected moment magnitude is 6.4 given rupture distance equal to 12 km. By contrast, on the use of the global model (Eq. (1)), the expected moment magnitude is equal to 6.3. Accordingly, the estimate/prediction (*M* 6.4) from the new Bayesian "local model" is closer to the observation (*M* 6.7) than the estimate/prediction (*M* 6.3) from the global model, which somehow validates that the Bayesian model is more suitable than the global model for local applications in Taiwan.

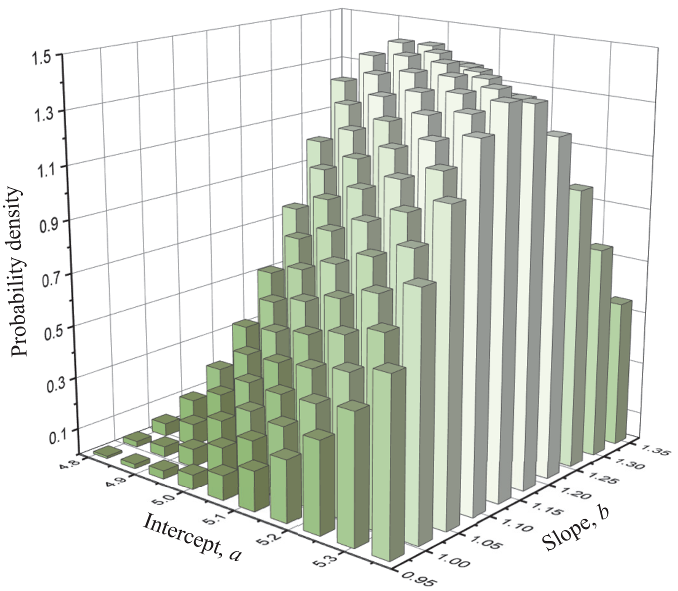


Fig. 3 The likelihood function for intercept (*a*) and slope (*b*)

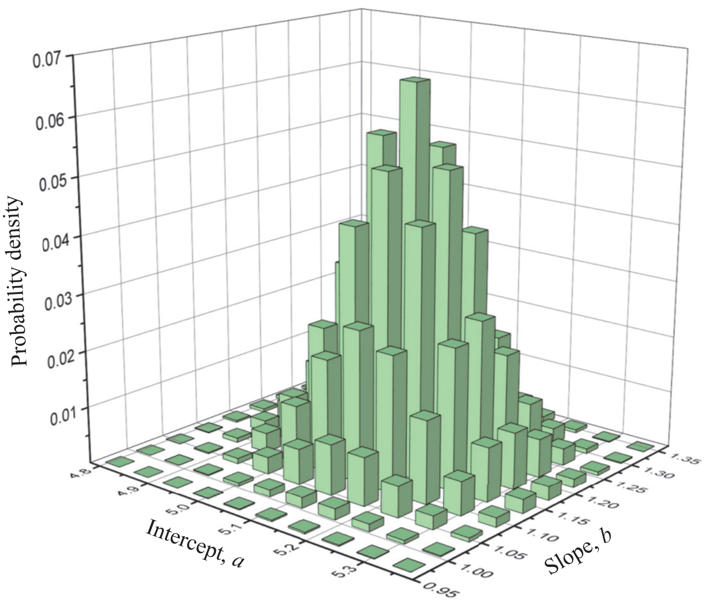
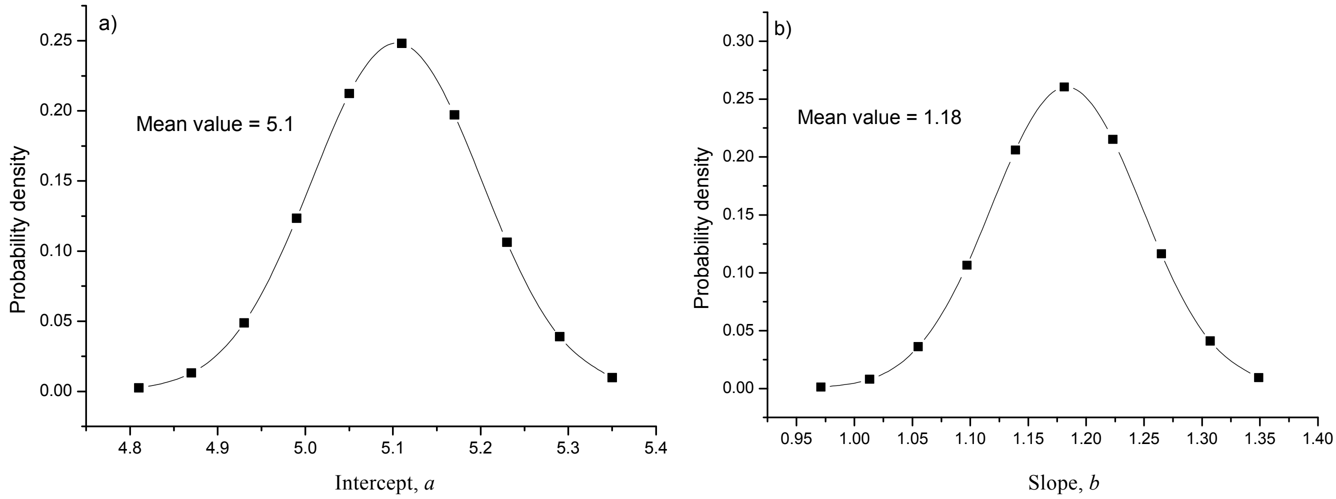


Fig. 4 The posterior probability function for intercept (*a*) and slope (*b*)



(a) For intercept (b) For slope

Fig. 5 The marginal distributions based on the posterior distribution

3.5 Sensitivity Analysis

The Bayesian calculations shown previously were using the data from the 1999 Chi-Chi earthquake as project-specific observation to update the prior information, then using the data from the 1946 Tainan earthquake to evaluate/validate that the newly developed Bayesian model is more suitable for local applications than the global model. However, we wonder if a similar inference could be obtained given that the data from the Tainan earthquake is used for model updating instead, while the data from the Chi-Chi earthquake is used for model evaluation.

The "sensitivity" calculation shows that when the data from the Tainan earthquake was used for Bayesian calculation, the updated *M*-*L* relationship became $M = 5.12 + 1.18\log(L)$. Next, we used the data from the Chi-Chi earthquake to evaluate the model. Given the rupture distance equal to 90 km, the estimated magnitude from this Bayesian model is equal to 7.4. By contrast, given the same rupture distance, the estimated magnitude from the global model is equal to 7.3, which is less close to the observation in magnitude 7.6 in comparison to the prediction (7.4) from the Bayesian model. According to this "sensitivity" analysis, it also shows that the Bayesian model should be more suitable for local applications than the global model.

3.6 The Bayesian Model Updated with Two Local Datasets

To maximize the "value" of the limited local datasets, it is rational to use them altogether as project-specific observation in such a Bayesian calculation, which could possibly make the updated model even more oriented to local situations. In this case, it is understood that the likelihood function would change as the two local datasets simultaneously used as project-specific observation (the prior remains the same). Specifically, the likelihood function can be expressed as:

$$\Pr(\varepsilon \mid a^*, b^*) = \Pr(M = 7.6;\ \mu = a^* + b^* \times \log(90),\ \sigma = 0.28 \text{ and } M = 6.7;\ \mu = a^* + b^* \times \log(12),\ \sigma = 0.28) \tag{8}$$

or it can be fully written as:

$$\Pr(\varepsilon \mid a^*, b^*) = \left\{ \frac{1}{0.28\sqrt{2\pi}} e^{-(7.6-(a^*+b^*\times\log 90))^2/(2\times 0.28^2)} \right\} \times \left\{ \frac{1}{0.28\sqrt{2\pi}} e^{-(6.7-(a^*+b^*\times\log 12))^2/(2\times 0.28^2)} \right\} \tag{9}$$

Based on this formulation, the likelihood functions for each pair of a^* and b^* can be calculated and shown in Fig. 6. With the prior shown in Fig. 2, the posterior distribution for each pair of a^* and b^* can be obtained as Fig. 7, or with its analytical formulation as:

$$\Pr''(a_i^*, b_j^*) = \frac{\Pr(a_i^*, b_j^*) \times \Pr(\varepsilon \mid a_i^*, b_j^*)}{\sum_{i=1}^{n}\sum_{j=1}^{m} \Pr(a_i^*, b_j^*) \times \Pr(\varepsilon \mid a_i^*, b_j^*)} \tag{10}$$

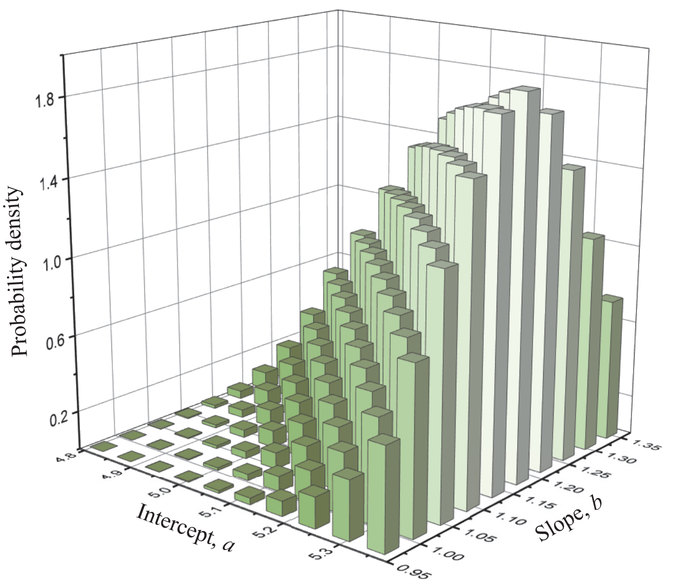


Fig. 6 The likelihood function for intercept (*a*) and slope (*b*) with two local datasets used as observation altogether

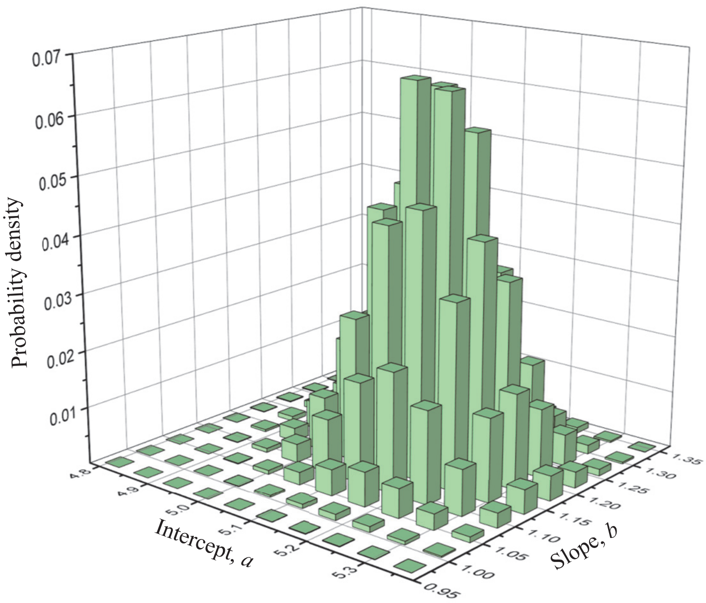


Fig. 7 The posterior distribution for intercept (*a*) and slope (*b*) with two local datasets used as observation altogether

where the likelihood function Pr ($\varepsilon|a^*$, b^*) is as expressed in Eq. (10). In particular, the values of n and m are both equal to 10 in this calculation.

Based on the posterior distribution, we can calculate the mean value of a and b as 5.13 and 1.2 based on the respective marginal distributions. As a result, the *M*-*L* relationship for local applications in Taiwan is suggested as Eq. (11), according to this Bayesian calculation with 77 non-local datasets as prior information, and with 2 local datasets as project-specific observation.

$$M = 5.13 + 1.2\log(L) \pm 0.28 \tag{11}$$

where 0.28 is the standard deviation of the model error.

3.7 Comparison

This section shows the comparison among the three *M*-*L* relationships: (1) the *M*-*L* relationship based on 77 non-local data through regression analysis; (2) the *M*-*L* relationship based on 2 local and 77 non-local data also through regression analysis; 3) the *M*-*L* relationship based on 2 local and 77 non-local data through the Bayesian updating. As shown in Fig. 8, with the 2 local data added to the non-local data pool, the *M*-*L* relationship developed with regression analysis is very close to the one (also developed with regression analysis) sorely based on the non-local data. However, we can see that the *M*-*L* relationship developed with the Bayesian updating is quite different than that based on the non-local data. In other words, when using the Bayesian calculation to integrate the local data with non-local data, the "role" of the non-local data in the *M*-*L* relationship can be intensified.

3.8 Application

Evidence has shown that several major earthquakes had occurred in Taipei in the past 10000 years (*e.g.,* Huang *et al.* 2007). As a result, a number of studies focusing on characterizing the potentials of major earthquake recurrence in Taipei were reported (*e.g.,* Wang *et al.* 2013; Wang and Kuo-Chen 2015; Xu and Wang 2017). It is generally agreed that if a major earthquake occurs in Taipei, it should be associated with the Shanchiao fault in northern Taiwan. Reportedly, the best-estimate fault length or rupture length is 35 km (Xu and Wang 2017).

Given the best-estimate fault length as 35 km, Fig. 9 shows the probability density function for the earthquake magnitude based on the Bayesian model updated with the two local datasets, and based on the global model without considering the local data whatsoever. The result shows that the magnitude prediction from the Bayesian model is stronger or more conservative than that from the global model. Specifically, the Bayesian model estimates a 48% probability that the fault could induce an earthquake with magnitude exceeding 7.0, in relative to 32% predicted by the global model.

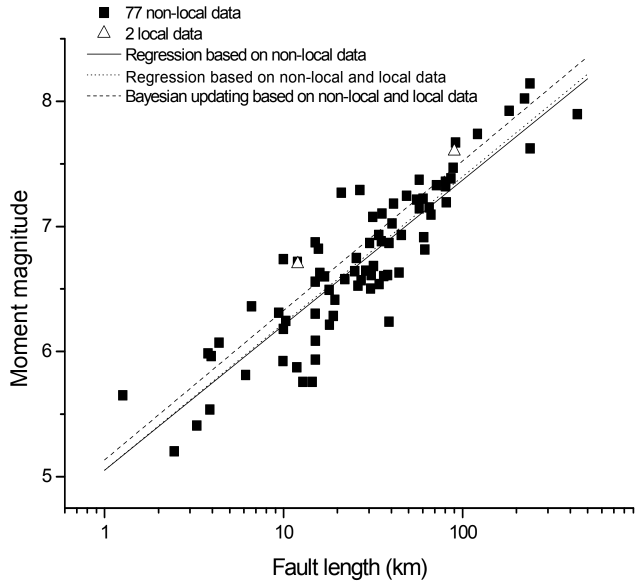


Fig. 8 The comparison between the three *M*-*L* relationships

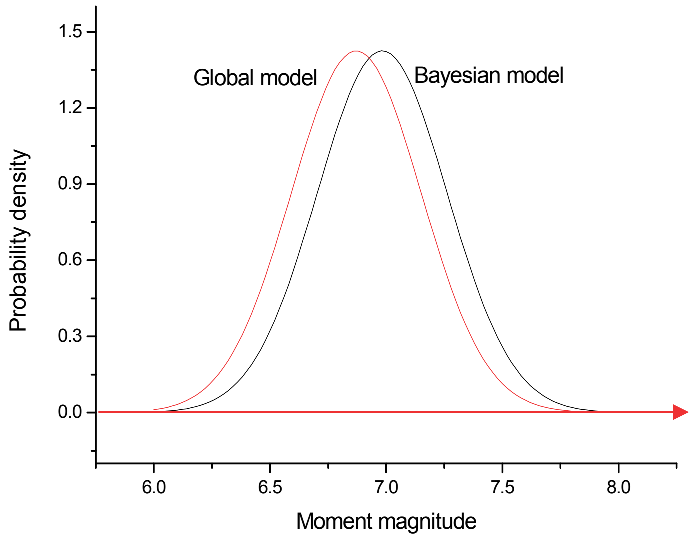


Fig. 9 Comparisons of the probability estimates with the prior (global) model and the posterior (local) model

4. DISCUSSION

4.1 Weight of Local Sample

The Bayesian model presented herein is by all means mathematically robust, and the result is completely transparent and repeatable without using any subjective judgments, which is otherwise needed for a weighted regression analysis, say, 9:1 weighting ratio assigned to local and non-local data. Nevertheless, with the model developed as $M = 5.13 + 1.2\log(L)$ from the Bayesian approach, the weight on the local sample can be back calculated with the fundamentals of weighted regression analysis. In short, the weight can be determined by the following equation in an objective manner,

$$w = -\frac{\sum_{i=1}^{n}\left[(a + bx_i - y_i)x_i\right]}{\sum_{j=1}^{m}\left[(a + bx_j^L - y_j^L)x_j^L\right]} \tag{12}$$

where w is the weight on the local sample x^L (length) and y^L (magnitude); x_i (length) and y_i (magnitude) are the non-local samples; n and m are the sample size of non-local and local data, respectively ($n = 77$ and $m = 2$ in this study); a (= 5.13) and b (= 1.2) are the model parameters that have been determined with the Bayesian calculation. For more details of the derivation, see the Appendix.

According to Eq. (12), we found the weight on the local sample is equal to 33 in relative to each of the 77 non-local ones. In other words, the Bayesian model (*i.e.,* Eq. (11)) is equivalent to the weighted regression model with the local sample weighted 33 times as much as the non-local ones. It is noted that this back-calculated weight is specific to this study with the specific prior information and local sample. In other words, it does not infer that the weight on project-specific data must be equal to 33 for other cases.

On the other hand, based on Eq. (12), a general rule for the back-calculated weights on local samples should be as follows: (1) the sample sizes of local and non-local data should affect the back-calculated weights on the local samples, *i.e.,* the bigger the difference between two sample sizes, the larger the weight (on local sample) should be obtained; (2) when the local samples are more deviated from the global (non-local) trends, the back-calculated weights on the local samples should also be increased.

4.2 Bayesian Method and Classical Approach

Obviously, the inferences from Bayesian methods and classical (statistical) approaches are equally robust, given that the two have been well established. Under the circumstances, a question is raised as follows: which estimate should be more statistically sound? To resolve the question mathematically, one can compare the two estimators' biasedness, efficiency, and sufficiency, which are the parameters for prioritizing different estimators in statistics. For instance, an unbiased estimator is considered superior to a biased one. However, since a Bayesian estimator is very complex in terms of mathematical formulation, it is very difficult to derive and calculate those statistical parameters for a Bayesian estimator (Wang 2016).

Nevertheless, when it comes to statistical estimation, the following perspectives should be agreed on: (1) local observation is more reliable than prior information, so when local data are adequate, the inferences from classical (statistical) methods based on only local data should be preferred, without the need of adding prior information, which is less reliable, into the estimation using the Bayesian approach; (2) if project-specific data are too limited (like this study having only one local sample) to obtain a representative statistical inference, the Bayesian inference that considers both (limited) local data and prior information should be preferred, on the consideration that less reliable data are better than no data.

4.3 *a-b* Joint Probability Function

As mentioned previously, the *a-b* joint probability distribution (Fig. 2) was developed disregarding whatever degree of correlation that the two variables might have. To verify this, we used the bootstrap method to obtain 100 intercepts and slopes from population A and population B, with both being a random variable following the standard normal distribution (mean = 0 and standard deviation = 1). For A and B that are strongly correlated with correlation coefficient = 0.8, the 100 intercepts and slopes obtained from the bootstrap method was found nearly independent, with their correlation coefficient = – 0.08 (Fig. 10(a)). Similarly, for A and B being moderately correlated (correlation coefficient = 0.5) and weakly correlated (correlation coefficient = 0.2), the 100 intercepts and slopes obtained from the bootstrap method were also independent (Figs. 10(b) and 10(c)). Therefore, from the numerical experiments, the presumption used in the study – no matter how A and B might be correlated, their joint probability function is the product of the two marginal distributions – is robust based on the verification using the bootstrap method.

5. SUMMARY

The Bayesian approach has been increasingly employed in the projects of geotechnical engineering and earthquake engineering, aiming to develop a more representative estimate based on limited local observation and relevant prior information. In this paper, we used the Bayesian method to develop an empirical relationship between fault length and earthquake magnitude for Taiwan areas. Specifically, the local *M-L* datasets associated with the 1946 Tainan earthquake and 1999 Chi-Chi earthquake were used as observation, and the 77 non-local datasets were used as prior information. Based on the data, we obtained the *M-L* relationship for Taiwan areas as $M = 5.13 + 1.2\log(L)$, which should be more suitable for local applications in Taiwan than the global (prior) model.

FUNDING

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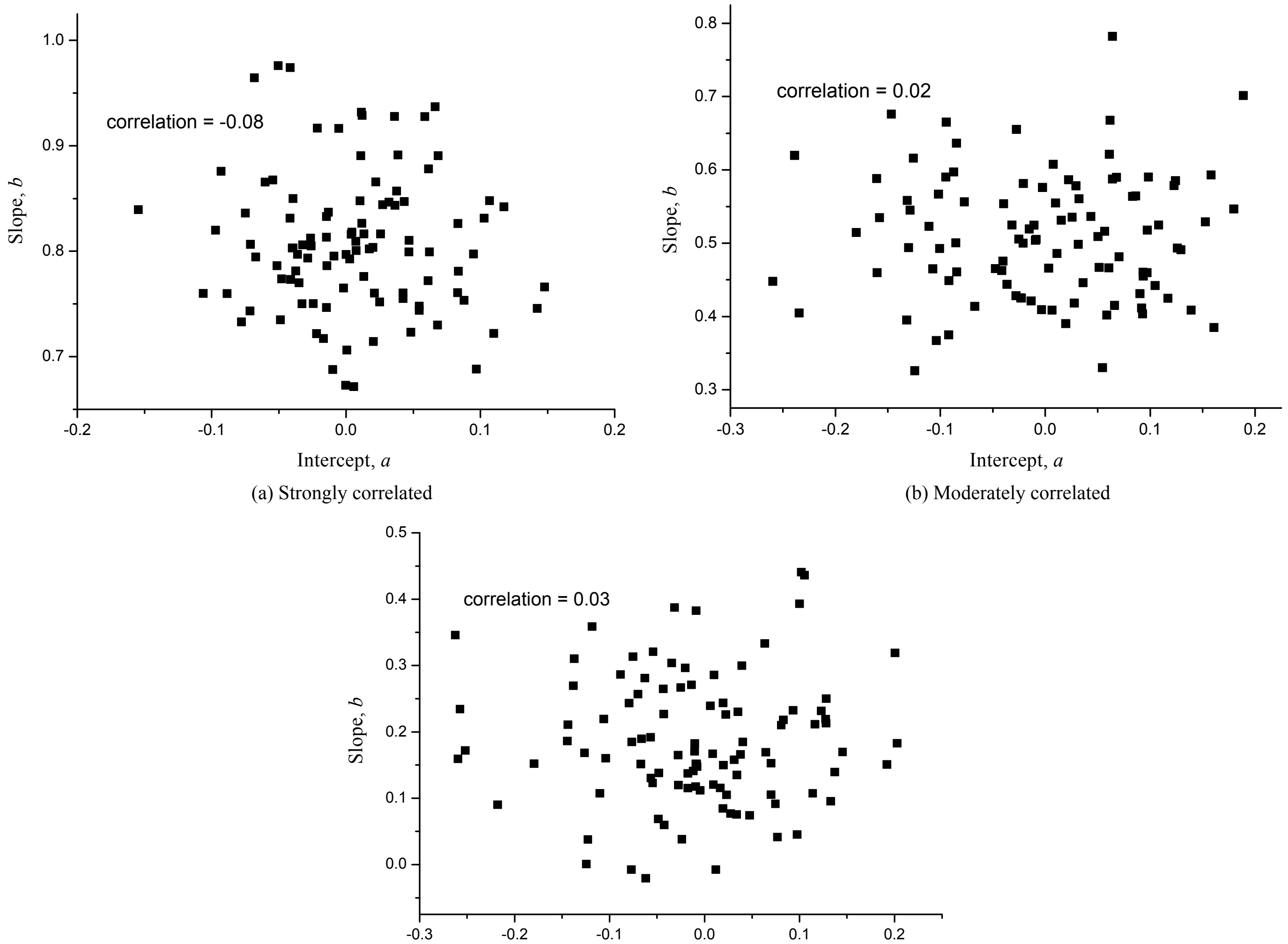
DATA AVAILABILITY

All data and/or computer codes used/generated in this study are included in this paper.



(a) Strongly correlated

(b) Moderately correlated

(c) Weakly correlated

Fig. 10 The correlation between slope and intercept on three different conditions between variables A and B

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APPENDIX: THE WEIGHT CALCULATION OF A WEIGHTED REGRESSION ANALYSIS

The sum square error (SSE) of a weighted regression analysis can be expressed as follow:

$$\mathrm{SSE} = \sum_{i=1}^{n}(a+bx_i-y_i)^2 + \sum_{j=1}^{m}(a+bx_j^L-y_j^L)^2 w \tag{A-1}$$

where w is the weight on the local data x^L (length) and y^L (magnitude); x_i (length) and y_i (magnitude) are the non-local data; n and m are the sample size of non-local and local data, respectively; a and b are the model parameters that have been determined with Bayesian calculations. As a result, by minimizing SSE we can obtain w equal to Eq. (9), or as follows:

$$w = -\frac{\sum_{i=1}^{n}\left\{(a+bx_i-y_i)x_i\right\}}{\sum_{j=1}^{m}\left\{(a+bx_j^L-y_j^L)x_j^L\right\}} \tag{A-2}$$