

IMPROVING THE STRENGTH OF NON-HOMOGENOUS SOILS USING THE STONE COLUMN TECHNIQUE

Maryam Gaber^{1*}, Anuar Kasa², and Norinah Abdul Rahman³

ABSTRACT

A numerical solution was provided in order to improve the understanding of the behaviour of stone columns in an embankment system. A two-dimensional finite element method was adopted in this study to estimate the bearing capacity and safety factor against the deep-seated failure of an embankment over stone column-improved non-homogenous soft soil. The factors influencing the behaviour of stone columns were investigated, including the spacing ratio, diameter, friction angle, unit weight, elastic modulus and Poisson's ratio of the stone columns and the height of the embankment fill. 15-node triangular elements in the Plaxis 2D (v8.2) software based on the plane strain concept were used to examine these features. According to the results of this study, it was observed that the load carrying capacity was developed as the diameter and the stiffness of the stone columns were enhanced, where the internal friction angle of the stone columns exerted the greatest influence compared to all the other parameters. The safety factor for most of the cases ranged between 2.2 to 2.6, whereby the lower value was recommended.

Key words: Finite element method, stone columns, bearing capacity, safety factor.

1. INTRODUCTION

The seating of embankment foundations on weak soil may cause the embankment to undergo large vertical and horizontal displacements. Some of the ground improvement techniques that have been adopted to relieve settlement are stone columns (Greenwood 1970; Hughes *et al.* 1975), pre-consolidation with prefabricated vertical drains (Yeung 1997; Shen *et al.* 2005), vacuum pre-consolidation (Chu *et al.* 2000; Indraratna *et al.* 2004), and deep mixed columns (Krenn and Karstunen 2008; Huang and Han 2009).

Stone column is a common ground improvement technique used to alter the condition of the subsoil. The advantages of using stone columns to support embankments are: (i) increased bearing capacity, (ii) reduced settlement, and (iii) accelerated consolidation settlement. This method requires the replacement, typically between 10 ~ 40%, of a weak subsoil with stiffer granular materials. Due to the higher stiffness of the stone column in comparison to the native soil, the stress will be concentrated in the stone column, thereby reducing the stress in the surrounding soil (Aboshi *et al.* 1979). The distribution of stress is generally defined in terms of the stress concentration ratio (SCR), which is expressed as:

$$SCR = \frac{\sigma_s}{\sigma_c} \quad (1)$$

where

σ_s = stress in the column

σ_c = stress in the surrounding soil

The stress concentration ratio is one of the most important factors to be considered when designing stone columns. Since there is no accurate method for obtaining a rational estimate of this ratio, it has to be determined either through an empirical estimation of the field measurements or based on the engineer's previous experience. This ratio is crucial in predicting the beneficial effects of stone column-reinforced grounds, especially in settlement and stability analyses.

Theories of the predicted settlement of stone columns, the load transfer, and evaluation of the ultimate bearing capacity were developed over a period of four decades by numerous researchers (Greenwood 1970; Hughes and Withers 1974; Aboshi *et al.* 1979; Priebe 1976; van Impe and de Beer 1983; Balaam and Poulos 1983).

Several publications have shown that the stress concentration ratio for stone column-reinforced foundations range between 2 to 6, with typical values being between 3 to 4 (Goughnour and Bayuk 1979; Ambily and Gandhi 2007). Greenwood (1991) reported a much higher ratio of 25, which was measured in very soft clay at a low stress level. Most previous studies focused on designs of isolated stone columns seated only in homogenous soils (Baumann and Bauer 1974; Ng and Tan 2014). Since very few researchers have studied the behaviour of stone columns that penetrate non-homogenous soils under long-term loading conditions using a plane strain model, this paper focused on this novel approach.

2. PARAMETRIC STUDY

In the parametric study of the construction of embankments on stone columns, one parameter was varied at a time, while all the other parameters were kept constant. The following section

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presents a detailed discussion of the geometry of the embankment along with the stone column-reinforced ground. In the analysis, the diameter of the stone column (d), spacing ratio (S/d), deformation modulus and other properties of the stone column material (friction angle ϕ , Poisson's ratio ν and unit weight γ), and the height of the embankment (H) were varied, as summarized in Table 1.

3. DESCRIPTION OF STONE COLUMN-REINFORCED EMBANKMENT FOUNDATION

The properties of the stone columns, soil and embankment fill were chosen from the Ipoh-Rawang Double Track (IRDT-MALAYSIA) project. The site location of this project is shown in Fig. 1. Vibro replacements (vibro stone columns) were installed at 23 separate locations covering a total track length of about 7 km.

The embankment top had a minimum width of 14.9 m with a side slope of 1V:2H constructed on 9.5 metres of non-homogeneous soil. In ordinary cases of embankment constructions, the stone column has a diameter of 0.8 m with a center to center spacing of 1.8 m and a height of 2 m.

Table 1 Values of influencing factors

Parameter	Range
Stone column diameter (m):	$d = 0.8, 0.9, 1.0, 1.2$
Spacing ratio:	$S/d = 1.875, 2.25, 2.5, 2.75, 3.125$
Stone column material:	
Deformation modulus (MPa)	$E = 50, 65, 80, 100, 130, 150$
Angle of internal friction ($^\circ$)	$\phi = 35, 38, 40, 43, 45$
Poisson's ratio (-)	$\nu = 0.25, 0.35, 0.4, 0.45, 0.49$
Unit weight (kN/m^3)	$\gamma = 15, 17, 20, 23, 25$
Height of embankment (m):	$H = 1.8, 2, 2.2, 2.5, 3, 3.5$

4. SUBSOIL PROFILE

Ground improvement works were necessary for about 32 km of the IRDT project. The new track passes through highly variable poor soils, with a mixture of very soft silts, soft clays and loose sand up to a depth of 24 m. The cone tip resistance (q_c) values of the very soft silts and clays often range between 150 to 250 kPa. Vibro stone columns were installed to depths of 8 m until 18 m to support the embankments, with heights varying from 2 to 11 m. Figure 2 shows the constructed embankment with time as well as the settlement over 1,231 days measured in field and FEM estimation. The settlement increased at a rapid rate with time until approximately 380 and 150 days based on field and FEM respectively, after which, a very small rate of increase in the settlement with time was recorded, and the settlement did not exhibit any increase after 650 days in field measurement. This indicated that the consolidation had ended. The total final settlement was 48 mm measured in the field and 44 mm according to the FEM calculation.

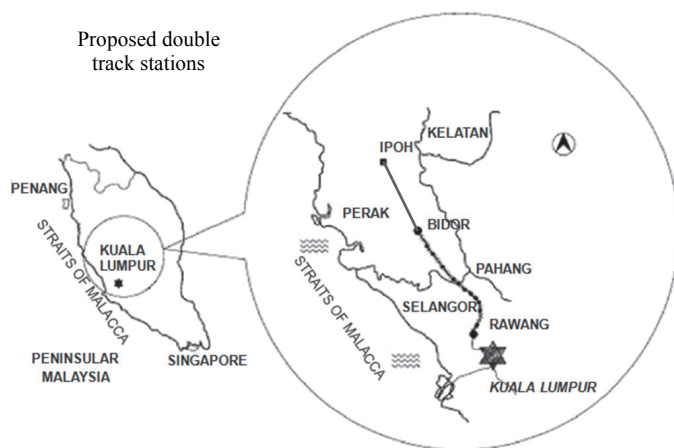


Fig. 1 Site location

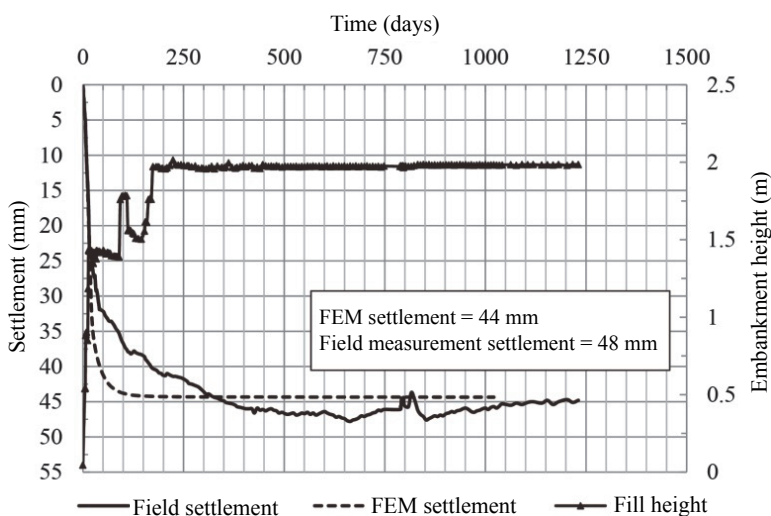


Fig. 2 Typical settlement data with applied 2 meters height embankment

5. NUMERICAL MODEL AND ANALYSIS

In this study, a finite element analysis approach was selected to numerically simulate the behaviour of the stone columns and the foundation soils. A commercial analysis was performed using the finite element program, PLAXIS 2D (v8.2). For the purpose of symmetry, only half of the embankment was modelled to reduce the computation time. A plane strain model with 15-node triangular elements was built. A relatively refined mesh arrangement was used to achieve greater accuracy during the initial consolidation process (the average size of the elements was about 630×10^{-3} m).

The columns were installed in a square grid with spacings, S , supporting an embankment with a height, H . The 9.5 m long column penetrated the entire non-homogenous soil and rested on a rigid stratum. Standard fixities were used for the horizontal and vertical boundaries. The boundary effect was investigated to extend the right boundary successively up to 40 m from the toe of

the embankment. The stone columns were modelled using a linear elastic perfectly plastic model based on the Mohr-Coulomb failure criterion. The Mohr-Coulomb model is defined by five parameters: friction angle (ϕ), effective cohesion (c), dilatancy angle ($\psi = \phi - 30^\circ$, as given in Brinkgreve (2008)), effective Young’s modulus (E), and Poisson’s ratio (ν). The parameters of the Mohr-Coulomb model used in the numerical analysis are summarized in Table 2.

Figure 3 shows the 2D model with a refining mesh that was used in the analysis. Interface elements were used to simulate the interaction between the stone columns and the soil; without the interface, there would be no slipping and gapping between the stone columns and the surrounding soil. In this study, the interfaces are set rigid ($R_{inter} = 1.0$) which mean the interface elements have same strength properties of surrounding soil (Brinkgreve 2002).

Table 2 Soil parameters

Mohr-Coulomb	Fill material	Soft clay (layer1)	Silty sand (layer2)	Firm clay (layer3)	Sand (layer4)	Stone column	Firm clay
Type	Drained	Undrained	Undrained	Undrained	Undrained	Drained	Undrained
γ_{unsat}	kN/m ³	17	14	17	16	17	16
γ_{sat}	kN/m ³	18	16	20	18	18	17
k_x	m/day	1.00	7.36E-5	0.10	7.36E-5	1.00	7.36E-5
k_y	m/day	1.00	3.68E-5	0.10	3.68E-5	1.00	3.68E-5
E_{ref}	kN/m ²	20,000	1,000	3,000	2,400	3,600	15,000
ν	-	0.3	0.4	0.333	0.333	0.333	0.4
c_{ref}	kN/m ²	5.0	10.0	1.0	30.0	0.1	23.0
ϕ	°	30	0	28	0	28	28
ψ	°	0	0	0	0	0	0

Note: To avoid numerical instability, a cohesion value of 0.1 kN/m² was used

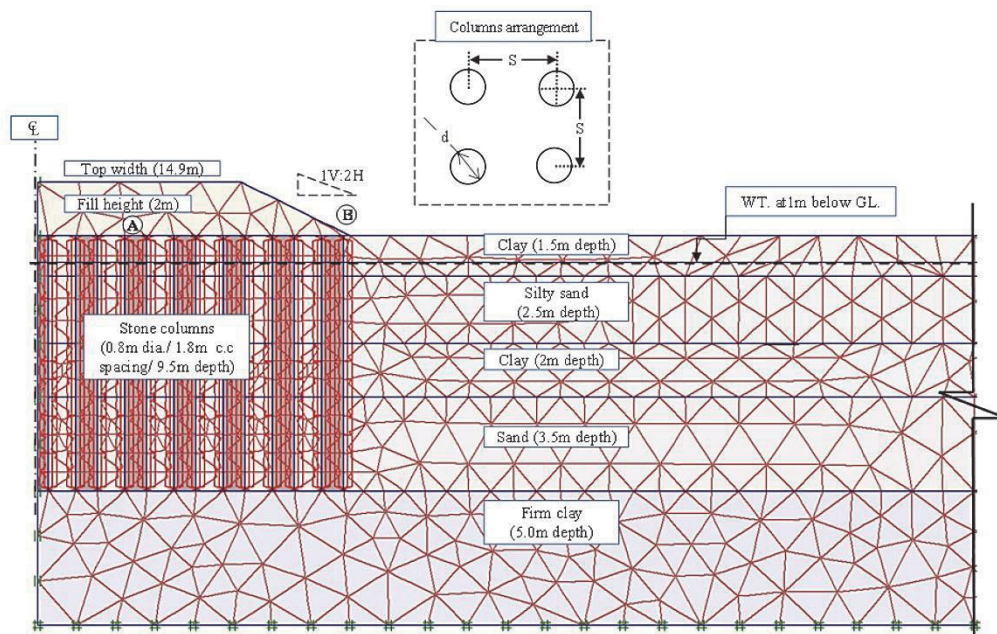


Fig. 3 Numerical model used in the 2D analysis

The soil was assumed to be normally consolidated with an overconsolidation ratio (OCR) of 1. On the other hand, the initial stress was generated by K_o , which was calculated using the formula by Jaky (1944), namely $K_o = 1 - \sin \phi$, since the installation effects of the stone columns were not taken into account in the current work. The water pressure was fully hydrostatic and, based on the general phreatic level, was set at 1 metre below the ground surface to consider the impact of ground water on embankment stability during the analysis. However, the boundary conditions for the consolidation analysis had to be closed at the left vertical boundary to take into consideration the line of symmetry or the fact that no flow entered, thereby preventing a horizontal flow. The right vertical boundary also had to be closed since it was far enough from the embankment to have any significant impact on the results. The bottom was open to allow a free flow of excess pore pressure. A consolidation analysis was performed in which the embankment fill was assumed to be constructed in one stage, and it took 21 days to be completed, while the second phase of the model analysis took a period equal to 1,000 days to take into consideration long-term conditions. In addition, the (Phi-c) reduction calculation was selected to find the safety factor (SF) against the stability of the embankment after each phase.

6. RESULTS AND DISCUSSION

6.1 Deformation Modes

The deformation mode of the stone columns is shown in Fig. 4. The formation of internal bulging was observed close to the upper part of the inner columns. Bulging occurred in the

stone columns when the applied load was higher than the confining stress. The surrounding soil provided some lateral support to prevent further expansion of the columns. As the confining stress increased with depth, bulging occurred in the upper part of the stone column (Madhav and Miura 1994). Shivashankar *et al.* (2011) studied the behaviour of a stone column installed in layered soils. Maximum bulging was noted at a depth of one column diameter from the top, and the total length of the stone column exhibiting bulging was observed to be 2 ~ 3 times the column diameter. According to Mohanty, and Samanta (2015) the vertical extent of the bulging increases with increase in the thickness of the top soft clay up to two times the diameter of the stone column.

Ambily and Gandhi (2007) investigated the performance of single and group columns. They reported that the bending of a column is dependent on its position in the group. In addition, bending increases further away from the center of the column. However, Fig. 4 clearly shows that bulging does not occur along the depth of the column, and that the failure at the edge of the column is not caused only by the bulging, but can also be due to the sliding that occurs far from the group. On the other hand, Priebe (1995) assumed that constant bulging occurs along the column, and used the cavity expansion theory to estimate the settlement improvement factor. As mentioned above, the bulging along the column length that was observed in this study was not constant. The horizontal displacement profile along the edge of the column, as shown in Fig. 5, revealed that the maximum displacement in the model occurred at about 1.0 m below the surface of the ground. This magnitude was observed in most of the cases that were studied.

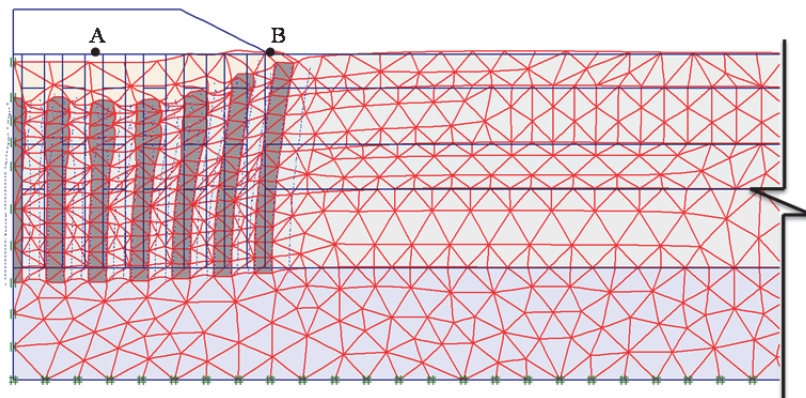


Fig. 4 Deformation mode

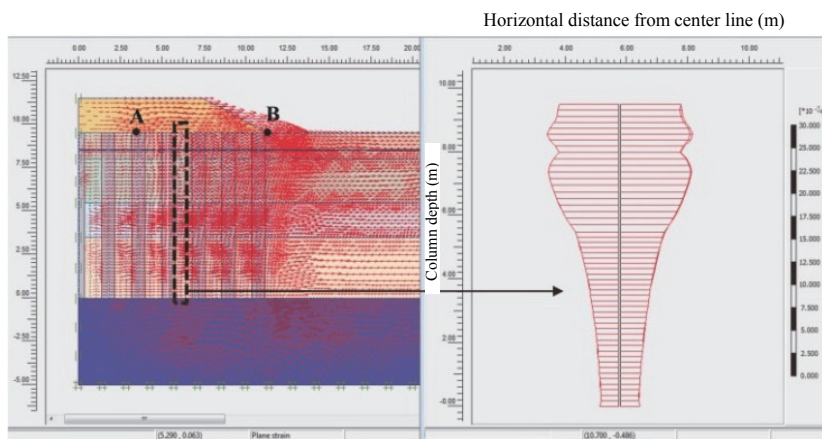


Fig. 5 Horizontal displacement profile at 5.8 m from the center line

6.2 Bearing Capacity

The ultimate bearing capacity of a stone column depends on the geometry of the stone column and the properties of the material as well as those of the native soil. On the other hand, the effect of the column length on the ultimate bearing capacity of a long column is negligible. Since the applied load is transferred from the column to the surrounding soil, only a small amount of the load will be transferred to the base of the column (Pitt et al. 2003).

Several researchers adopted different approaches to estimate the bearing capacity of a single column and a group of columns, (Hughes and Withers 1974). Etezad et al. (2006) published a report on the analytical treatment of the bearing capacity and failure mechanisms. They depended on the results of the output from a combination of finite element analyses and field trials for the adoption of failure mechanisms.

In this paper, the finite element method based on PLAXIS was carried out to compute the bearing capacity of the stone columns. Several criteria have been proposed to define the ultimate bearing capacity of foundations based on the load-settlement curves. Some of these criteria are described by Lutenege and Adams 1998; including the 0.1B, the tangent intersection, the Log-Log and the Hyperbolic methods. During this study, Tangent method has been selected to evaluate the bearing capacity of stone column.

Figure 6 presents the results of a typical single column plate-load test carried out in (IRDT) project. In the first cycle loading, the allowable design load was applied and maintained for a 24 h duration while, in the second cycle, a maximum load of 1.5 times of the design load has been applied. The acceptance requirement of the load test was that the settlement should not exceed 50 mm under the allowable load design and 80 mm under 150% of the allowable design load (Arulrajah et al. 2009).

Figure 7 shows the load-settlement curve of the untreated embankment. The final settlement was recorded as 56.54 mm under a stress of 31 kPa and the ultimate bearing capacity (q_{ult}) was adopted about 17.2 kPa. On the other hand, Figs. 8 and 9 show the results of the load-settlement curve for all the treated cases in the study that were calculated at point A.

Increasing the diameter of the stone columns clearly resulted in an improvement in the bearing capacity compared to the untreated state. The load-settlement relationships for four different diameters that were examined showed the same development, with a slight increase in the bearing capacity when the diameter was increased (Fig. 8(a)). The stone column with a diameter of 1.2 m had the highest bearing capacity of about 37 kPa. The stress-settlement behaviour of the stone columns for all the spacing ratios (S/d) was the same, as shown in Fig. 8(b). The load-settlement curve indicated that the spacing ratio (S/d) of 1.875 was better than the greater spacing ratios in increasing the bearing capacity of the stone columns. More than 20% improvement achieved when the spacing ratio was reduced from 3.125 to 1.875. The results indicated that the bearing capacity of the ordinary stone columns increased with a decrease in the spacing distance between the columns as well as with an increase in the stone column diameter, which was similar to the behaviour reported by Afshar and Ghazavi (2014). Figure 8(c) shows the impact of the embankment height on the bearing capacity of the column. The load-settlement graph shows that reinforcing the weak soil with stone columns at a lower embankment height provided the best bearing capacity amongst all the investigated cases.

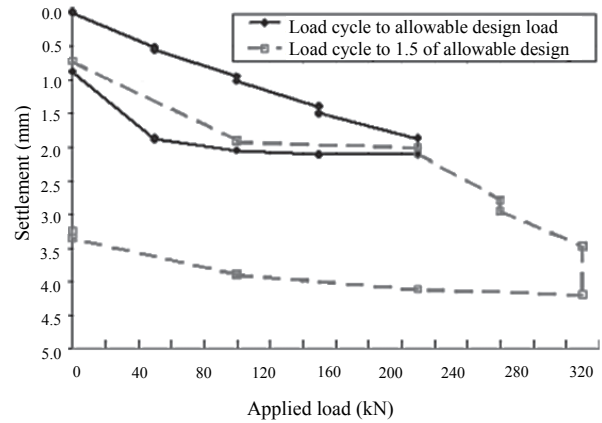


Fig. 6 Sample of load-settlement curves of a single column plate-load test (chain 352066-IRDT project)

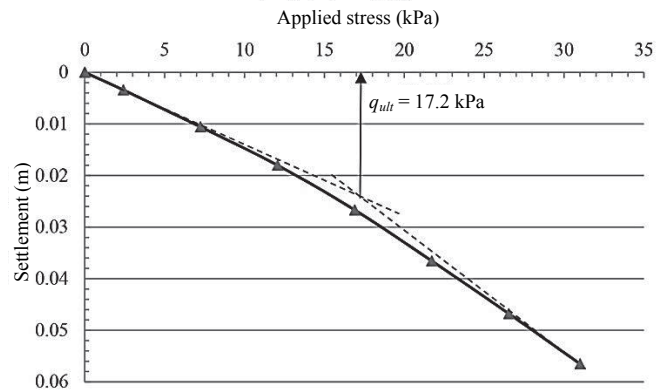


Fig. 7 Load-settlement curve for the foundation of unimproved embankment at point A

Improving the characteristics of the stone columns, such as in terms of the internal friction angle and unit weight under the embankments, significantly improved the bearing capacity of the foundation soil, as can be seen in Figs. 8(d) and 8(e). Increasing the friction angle of crushed stone column from 35° to 45° led to increase the bearing capacity of stone column up to 30%. The graph in Fig. 9(a) shows that the elastic modulus of the stone columns had less of an effect on the bearing capacity compared to the factors that were previously studied, although increasing the elastic modulus of the stone columns contributed to an increase in the bearing capacity. This result was in agreement with the report by Reihani and Dehghani (2014). On the other hand, Poisson’s ratio did not seem to have a noticeable effect on the bearing capacity of the soil or on any of the other factors that were studied, as can be seen in Fig. 9(b). Falsafi, and Motahari (2015) reported that increasing the stone material’s Poisson coefficient has a slight influence on the stone column behavior which is agree with this study’s results.

6.3 Safety Factor (SF)

A safety (Phi-c) analysis was conducted in this study to assess the global safety factor (SF) against a deep-seated failure

of embankments over stone column-improved non-homogenous soil based on the plane strain model. This option is available in the Plaxis 2D, where the safety factor is computed by reducing the strength parameters of the soil until failure occurs. The total multiplier ΣM_{sf} defines the magnitude of soil strength parameters at given stage in the analysis. The strength parameters are consecutively reduced automatically until failure of the structure

occurs which at this point the safety factor is given by the follow expression:

$$SF = \frac{\text{available strength}}{\text{strength at failure}} = \Sigma M_{sf} \text{ at failure} \quad (3)$$

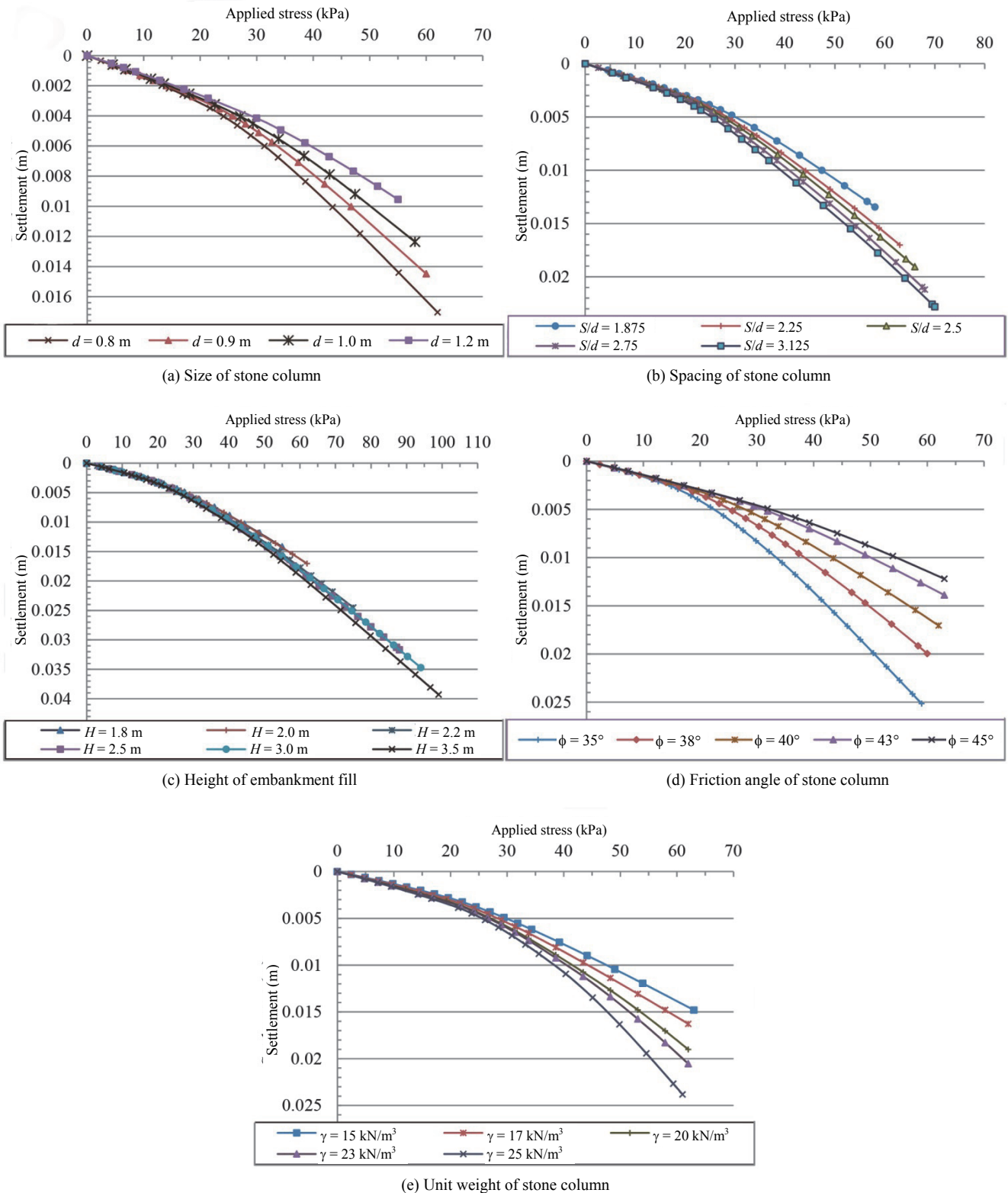


Fig. 8 Factors influencing the bearing capacity of reinforced soil foundation at point A

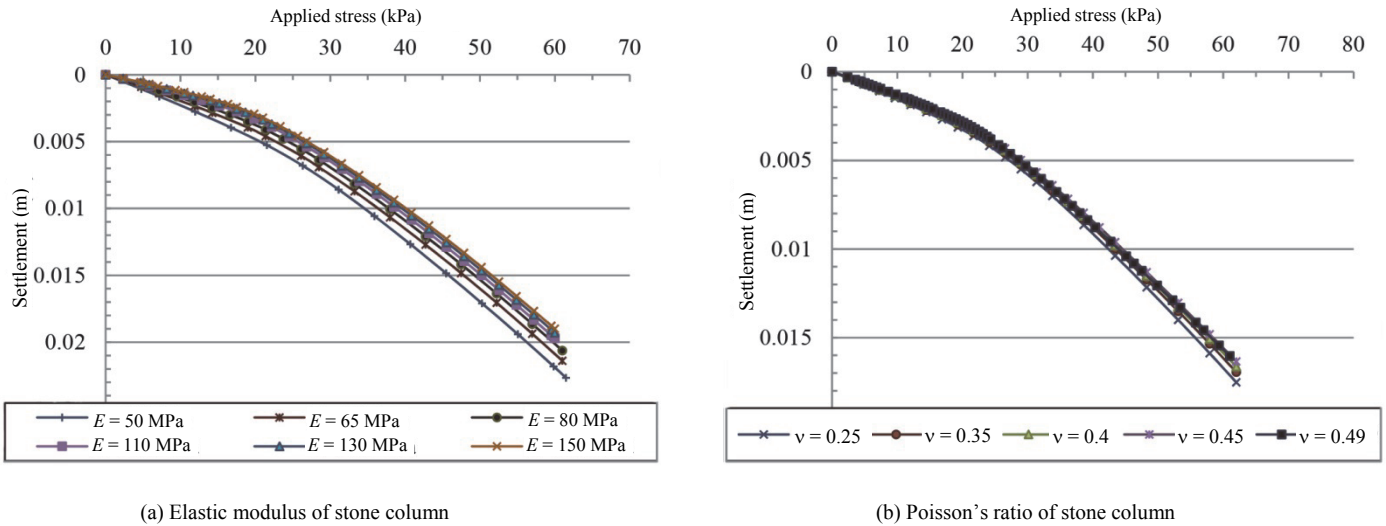


Fig. 9 Factors influencing the bearing capacity of reinforced soil foundation at point A

A safety analysis was conducted after each stage of construction. The safety factor was evaluated by plotting the displacement against the parameter, ΣM_{sf} , at a selection point on the slope head (point B), as shown in Fig. 10. Since the displacement was set at zero at the beginning of each Phi-c reduction analysis so the total displacement seen in this graph is not relevant and it indicates whether or not a failure mechanism has developed (Brinkgreve 2002).

This study investigated several factors influencing the safety factor against deep-seated embankment failure over the group of stone columns during construction and service conditions, as stated in Table 1. The impact of each factor on the safety factor under both conditions is presented in Fig. 11.

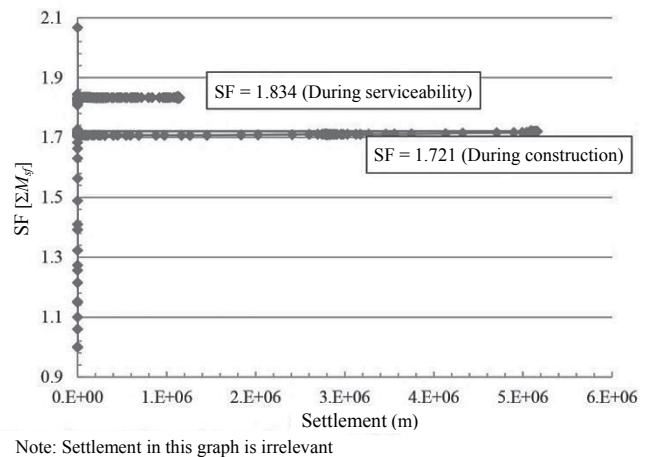
The effect of the diameter of the stone column on the safety factor is shown in Fig. 11(a). The value of the safety factor increased gradually as the value of d was increased in both the short and long-terms conditions. Figure 11(b) illustrates the effect of the spacing ratio of the stone columns on the value of the safety factor. The value of the spacing ratio in the investigation was between 1.875 and 3.125. The result showed that the behaviour of the safety factor decreased gradually as the spacing ratio (S/d) was increased. The maximum safety factor of 2.465 was observed during the serviceability stage, when the spacing ratio was 1.875, while the minimum safety factor was 2.218 during the construction stage when the spacing ratio was 3.125.

The effect of the internal friction angle of the stone column material on the safety factor of the embankment over the stone column-improved soil is shown in Fig. 11(c). It indicated that a superior stone column material resulted in a higher safety factor for the embankment system. Figure 11(d) shows the plot of the effect of the embankment height on the safety factor. The stability of the embankment is most critical when the embankment is at its highest during the construction and serviceability stages (short and long-term), as the subsoil will become stronger over time when the excess pore pressure dissipates. Therefore, lower safety factors of 1.77 and 1.88 were adopted in the stability analyses for the construction and serviceability stages, respectively.

Figures 11(e) to 11(g) show the effect of the elastic modulus (E), unit weight (γ) and Poisson's ratio (ν) of the stone columns on the safety factor. The benefit of increasing these parameters was less significant compared with the other parameters investigated in this study. However, a slight change in the safety factor was observed when the values of these parameters were changed.

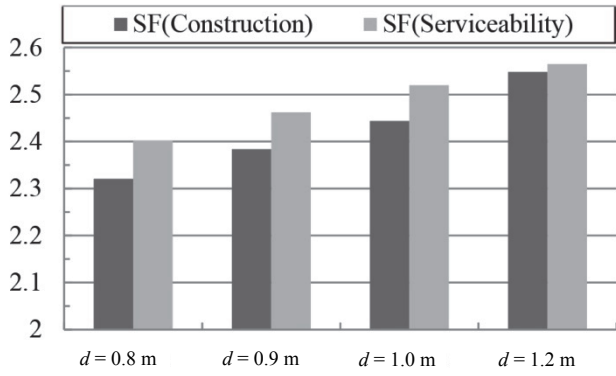
Figure 12 illustrates an example of the maximum shear-strain rates with a critical slip surface. It shows the critical slip surface and safety factor in the untreated case (before the use of a stone column) and the treated case in the study, where a stone column with a friction angle of 45° was used. The figure shows that the safety factor increased from 1.834 to 2.401 when the ground was improved through the use of the stone columns.

As a result, if a higher safety factor is used, then unnecessary and costly ground improvement works will have to be undertaken. It is more significant to carry out an appropriate planning analysis and design rather than to employ a higher safety factor to cover weaknesses in the design methodology.

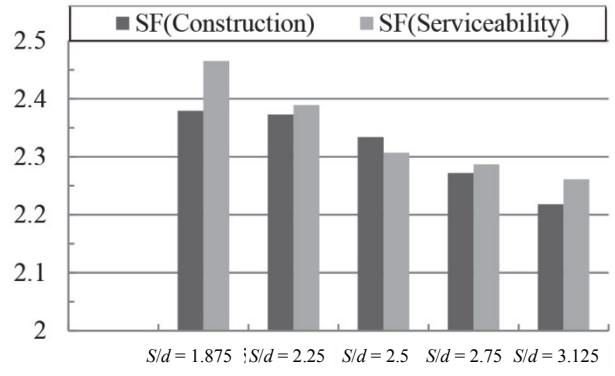


Note: Settlement in this graph is irrelevant

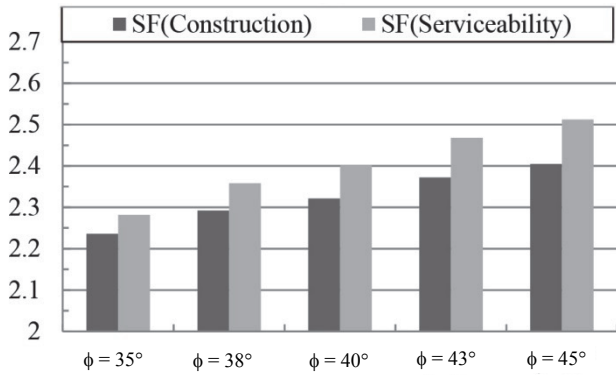
Fig. 10 SF graph of untreated soil foundation of two phases



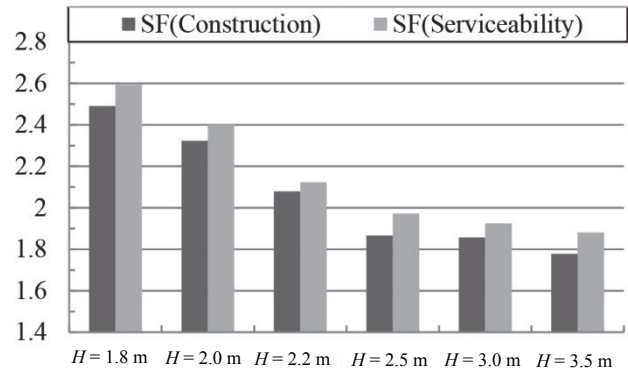
(a) Size of stone column



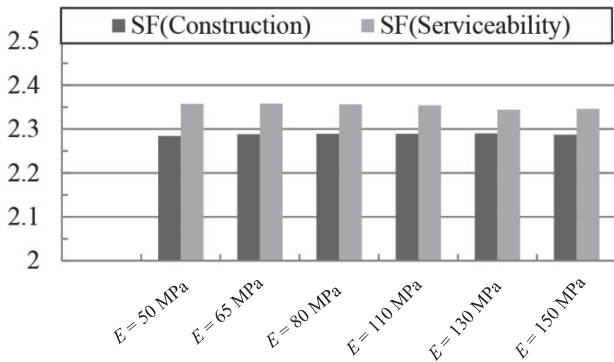
(b) Spacing of stone column



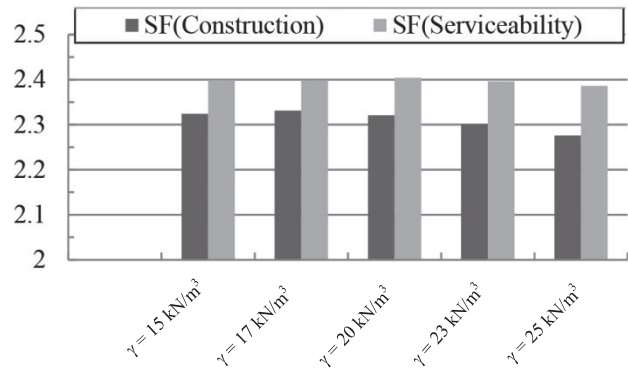
(c) Friction angle of stone column



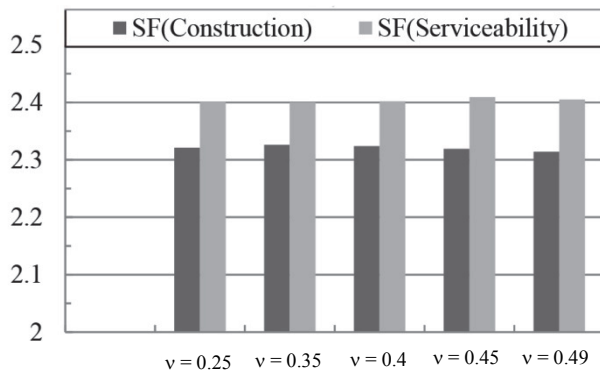
(d) Height of embankment fill



(e) Elastic modulus of stone column



(f) Unit weight of stone column



(g) Poisson's ratio of stone column

Fig. 11 Factors influencing FS behavior

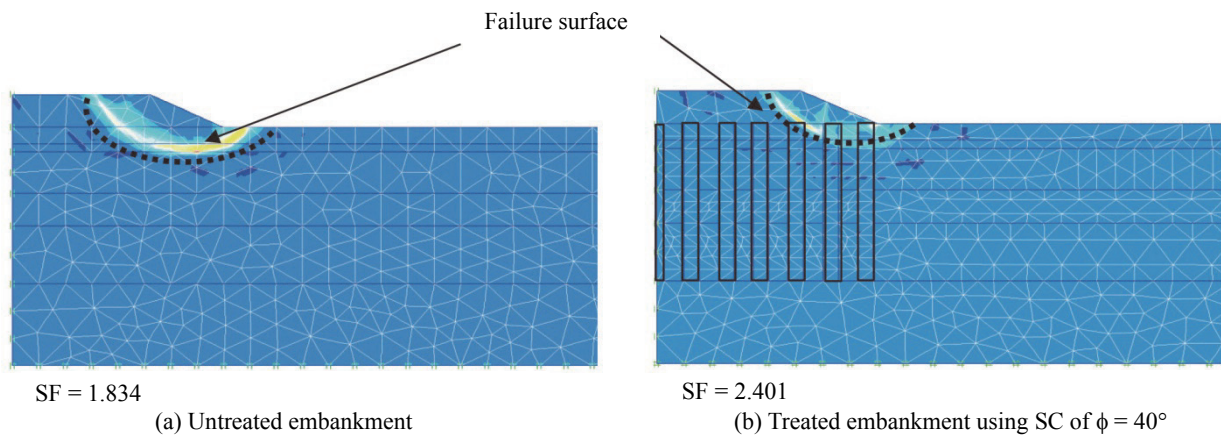


Fig. 12 Embankment stability over untreated and treated models of stone columns

7. CONCLUSIONS

A series of 2D numerical analyses was carried out to evaluate the bearing capacity and safety factor with regard to a soil foundation reinforced with a group of stone columns. The soil foundation was non-homogeneous, and the analyses employed an elastic, perfectly plastic, constitutive model that was based on the Mohr-Coulomb failure criterion. The following conclusions were made based on the results obtained in this study.

1. This study highlighted some of the variables that had been taken into account in a few previous studies, such as Poisson's ratio and the unit weight of the stone columns, to verify the impact of most of the characteristics of the columns.
2. Improving the characteristics and geometries of the stone columns resulted in a significant increase in the bearing capacity of the stone columns.
3. The size, spacing and friction angle of the stone columns, the stone column material, and height of the embankment fill affected the safety factor values against the deep-seated failure of the embankment. However, a safety factor of 2.2 was adopted for the construction and serviceability stages.
4. One important point that emerged from this numerical study was that the friction angle, diameter of the stone columns, spacing of the columns, height of the embankment and spacing ratio are the most important design parameters in determining the performance of an embankment with stone columns.

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REFERENCES

- Aboshi, H., Ichimoto, E., Enoki, M., and Harada, K. (1979). "The 'Compozer' — A method to improve characteristics of soft clays by inclusion of large diameter sand columns." *Proceedings of International Conference on Soil Reinforcement*, 211–216.
- Afshar, J.N. and Ghazavi, M. (2014). "Geotechnical Engineering A simple analytical method for calculation of bearing capacity of stone-column." *International Journal of Civil Engineering*, **12**(1), 15–25.
- Ambily, A.P. and Gandhi, S.R. (2007). "Behavior of stone columns based on experimental and FEM analysis." *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, **133**(4), 405–415.
- Arulrajah, A., Abdullah, A., Bo, M.W., and Bouazza, A. (2009). "Ground improvement techniques for railway embankments." *Proceedings of the ICE — Ground Improvement*, **162**(1), 3–14.
- Balaam, N.P. and Poulos, H.G. (1983). "The behaviour of foundations supported by clay stabilised stone columns." *Proceedings of the Eight European Conference on Soil Mechanics and Foundation Engineering*, Helsinki, 199–204.
- Baumann, V. and Bauer, G.E.A. (1974). "The performance of foundations on various soils stabilized by the vibro-compaction method." *Canadian Geotechnical Journal*, **11**(4), 509–530.
- Brinkgreve, R. (2002). *PLAXIS 2D Version 8.2*. Delft University of Technology and PLAXIS BV, the Netherlands.
- Chu, J., Yan, S.W., and Yang, H. (2000). "Soil improvement by the vacuum preloading method for an oil storage station." *Geotechnique*, **50**(6), 625–632.
- Etezad, M., Hanna, A.M., and Ayadat, T. (2006). "Bearing capacity of groups of stone columns." *Proceedings of the 6th European Conference on Numerical Methods in Geotechnical Engineering*, Graz, 781–786.
- Falsafi, A. and Motahari, M.R. (2015). "Improving the bearing capacity of footing on soft soils using stone columns." *International Journal of Current World Environment*, **10**(1), 1037–1042.
- Indraratna, B., Bamunawita, C., and Khabbaz, H. (2004). "Numerical modeling of vacuum preloading and field applications." *Canadian Geotechnical Journal*, **41**(6), 1098–1110.
- Greenwood, D.A. (1970). "Mechanical improvement of soils below ground surface." *Proceedings of Ground Engineering*, The Institution of Civil Engineering, London, UK, 9–20.
- Greenwood, D.A. (1991). "Load tests on stone columns." *Deep Foundation Improvements: Design, Construction and Testing*, 148–171.
- Goughnour, R.R. and Bayuk, A.A. (1979). "A field study of long term settlements of loads supported by stone columns in soft ground." *Proceedings of International Conference on Soil Reinforcement*, 279–286.
- Huang, J. and Han, J. (2009). "3D coupled mechanical and hydraulic modeling of a geosynthetic-reinforced deep mixed column-supported embankment." *Geotextiles and Geomembranes*, **27**(4), 272–280.
- Hughes, J.M.O. and Withers, N.J. (1974). "Reinforcing of soft cohesive soils with stone columns." *Ground Engineering*, **3**(7), 42–49.

- Hughes, J.M.O., Withers, N.J., and Greenwood, D.A. (1975). "Field trial of the reinforcing effect of a stone column in soil." *Geotechnique*, **25**(1), 31–44.
- Jaky, J. (1944). "The coefficient of earth pressure at rest." *Journal for Society of Hungarian Architects and Engineers*, 355–358 (in Hungarian).
- Krenn, H. and Karstunen, M. (2008). "Numerical modelling of deep mixed columns below embankments constructed on soft soils." *Proceedings of the 2nd International Workshop on Geotechnics of Soft Soils*, Glasgow, Scotland, September, 159–164.
- Lutenegger, A.J. and Michael T.A. (1998). "Bearing capacity of footings on compacted sand." *Proceeding of the 4th International Conference on Case Histories in Geotechnical Engineering*, St. Louis, Missouri, 1216–1224.
- Madhav, M.R. and Miura, N. (1994). "Soil improvement. Panel report on stone columns." *Proceedings of the 13th International Conference on Soil Mechanics and Foundation Engineering*, New Delhi, India, 163–164.
- Mohanty, P. and Samanta, M. (2015). "Experimental and numerical studies on response of the stone column in layered soil." *International Journal of Geosynthetics and Ground Engineering*, **1**(3), 27. doi:10.1007/s40891-015-0029-z.
- Ng, K.S. and Tan, S.A. (2014). "Design and analyses of floating stone columns." *Soils and Foundations*, JGS, **54**(3), 478–487.
- Pitt, J.M., White, D.J., Gaul, A., and Hoevelkamp, K. (2003). *Highway Application for Rammed Aggregate Piles in Iowa Soils*. Iowa Department of Transportation, Final Report.
- Priebe, H.J. (1976). "Evaluation of the settlement reduction of a foundation improved by vibro replacement." *Die Bautechnik*, **53** (H.5), 160–162 (in German).
- Priebe, H.J. (1995). "The design of vibro replacement." *Ground Engineering*, **72**(3), 189–191.
- Reihani, B. and Dehghani, M. (2014). "The modeling of ground improvement with stone columns." *International Journal of Scientific Engineering and Technology*, **3**(9), 1210–1212.
- Shen, S.L., Chai, J.C., Hong, Z.S., and Cai, F.X. (2005). "Analysis of field performance of embankments on soft clay deposit with and without PVD-improvement." *Geotextiles and Geomembranes*, **23**(6), 463–485.
- Shivashankar, R., Babu, M.R.D., Nayak, S., and Kumar, V.R. (2011). "Experimental studies on behavior of stone columns in layered soils." *Geotechnical and Geological Engineering*, **29**(5), 749–757.
- van Impe, W.F. and de Beer, E. (1983). "Improvement of settlement behaviour of soft layers by means of stone columns." *Proceedings of the 8th International Conference on Soil Mechanics and Foundation Engineering*, Balkema, Rotterdam, the Netherlands, 309–312.
- Yeung, A.T. (1997). "Design curves for prefabricated vertical drains." *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, **123**(8), 755–759.