Journal of GeoEngineering, Vol. 12, No. 2, pp. 59-64, June 2017
http://dx.doi.org/10.6310/jog.2017.12(2).2

EFFECTIVE STRESS BASED BEARING CAPACITY EQUATIONS FOR SHALLOW FOUNDATIONS ON UNSATURATED SOILS

Yi Tang [1], Hossein A Taiebat [2], and Kostas Senetakis [3*]

ABSTRACT

The bearing capacity of shallow foundations is conventionally estimated using the equations derived for saturated or dry soils. Recent research shows a growing interest in the bearing capacity of unsaturated soils due to the conservative results of the conventional bearing capacity equations. In this study, alternative equations are proposed based on the effective stress principle, to estimate the bearing capacity of shallow footings on unsaturated soil considering both uniform and varied suction profiles. The validity of the proposed equations is established by comparing their predictions with experimental data available in the literature. The main purpose of this work is to provide equations those can be easily used by the practising engineers for preliminary or final estimations of bearing capacity of shallow footings on unsaturated soils. These equations require a small number of soil properties which can be easily obtained.

Key words: Bearing capacity, plate load test, shallow foundation, unsaturated soil, effective stress principle.

1. INTRODUCTION

The conventional bearing capacity equations are widely used in practice to estimate the bearing capacity of footings on saturated or dry soils (Terzaghi 1943; Meyerhof 1953; Hansen 1970; Vesic 1973). However, a large portion of soils under shallow footings may be in unsaturated conditions, especially in arid areas (Loret and Khalili 2000; Vanapalli and Mohamed 2013). In these cases the bearing capacity can be significantly increased in comparison to the fully saturated conditions due to suction developed within the soil (Costa *et al*. 2003; Rojas *et al*. 2007; Oh and Vanapalli 2011; Vanapalli and Mohamed 2013). However, the conventional bearing capacity equations do not take into account the influence of suction.

Design of a footing based on the "partial factors of safety" method or the "load and resistance factor design" method considers the probability of occurrence of different conditions when defining the load factors or the strength reduction factors. If a foundation soil becomes completely saturated only for short periods of time during the service of a footing, surely larger strength reduction factors can be incorporated to achieve a more economical design. This is based on the assumption that the bearing capacity of footings on unsaturated soil can be evaluated correctly.

On the other hand, there has been a growing need to understand the response of plate load tests on unsaturated soils to be able to evaluate or back-calculate the strength parameters of the ground properly (Costa *et al*. 2003; Oh and Vanapalli 2013). The conventional interpretation of the results of plate load tests performed on unsaturated soil would lead to unrealistically large strength and stiffness parameters if the effect of matric suction is not considered (Costa *et al*. 2003). Yet, there has only been relatively limited research on the topic of bearing capacity of shallow foundations on unsaturated soil.

The bearing capacity (q_u) of shallow foundations with concentric loading on fully saturated or completely dry soil is generally calculated by Terzaghi's (1943) equation:

$$q_u = c'N_c + q'N_q + 0.5\gamma' BN_r \tag{1}$$

where c' is the cohesion, N_c, N_q and N_γ are the bearing capacity factors which depend on the angle of shear strength, ϕ', of the soil, q' is the effective overburden pressure, γ' is the effective unit weight of the soil, and B is the footing width.

Fredlund *et al*. (1978) proposed an equation to calculate the shear strength of unsaturated soil using the concept of two independent stress state variables, *i.e.*, the net stress, $(\sigma - P_a)$ and the suction, $s = (P_a - P_w)$:

$$\tau = c' + (\sigma - p_a)\tan\phi' + (p_a - p_w)\tan\phi^b \tag{2}$$

where σ is the total stress, P_a and P_w are the pore air and pore water pressures, respectively. ϕ^b is the friction angle of the unsaturated soil with respect to the change in suction when the net stress remains constant.

Based on this shear strength theory, Oloo *et al*. (1997) presented a bearing capacity equation for surface footings on unsaturated soil by extending Terzaghi's equation and considering the effect of suction (s) as apparent cohesion:

$$q_u = \left[c' + s_e\tan\phi' + (s - s_e)\tan\phi^b\right]N_c + 0.5\gamma\, BN_\gamma \tag{3}$$

where s_e is the air entry or air expulsion value marking the transition from saturated to unsaturated conditions. Equation (3) implies that the bearing capacity increases linearly with suction at a

Manuscript received February 6, 2017; revised March 27, 2017; accepted April 12, 2017.

[1] Researcher, Geotechnical Engineering Department, Nanjing Hydraulic Research Institute, Nanjing, China.
[2] Senior Lecturer, School of Civil and Environmental Engineering, The University of New South Wales, Australia.
[3] Assistant Professor (corresponding author), Department of Architecture & Civil Engineering, City University of Hong Kong, HKSAR, China (e-mail: ksenetak@cityu.edu.hk).

constant rate of $\tan\phi^b$ when the suction is larger than the air entry value. However, experimental data show that this relationship is not linear (Costa *et al*. 2003; Rojas *et al*. 2007; Oh and Vanapalli 2011; Vanapalli and Mohamed 2013). To overcome this shortcoming, Vanapalli and Mohamed (2013) proposed the following equation:

$$q_u = \left[c' + s_e \tan\phi' + (s_{\mathrm{AVE}} - s_e) S_r^{\psi} \tan\phi'\right] N_c\, \xi_c\, d_c + \gamma D N_q\, \xi_q\, d_q + 0.5\gamma B N_\gamma\, \xi_\gamma\, d_\gamma \tag{4}$$

where S_r is the degree of saturation and D is the depth of the footing. ξ_c, ξ_q and ξ_γ are shape factors. d_c, d_q and d_γ are depth factors. ψ is a fitting parameter which should be determined by experimental tests or calculated by an empirical relationship as $\psi = 1 + 0.34(I_p) - 0.0031(I_p^2)$, in which I_p is the plasticity index of the soil. Equation. (4) includes the effects of "average suction" (s_{AVE}) in the soil on the bearing capacity of footings. The average suction in the "stress bulb zone" (from the footing base to a depth of 1.5B below the footing base) should be calculated following a procedure described by Vanapalli and Mohamed (2013).

Recently, Vahedifard and Robinson (2016) proposed a bearing capacity equation for unsaturated soil using the effective degree of saturation:

$$q_u = \left[c' + s_e \tan\phi' + (s_{\mathrm{AVE}} - s_e) S_{eff,\mathrm{AVE}} \tan\phi'\right] N_c\, \xi_c + q N_q\, \xi_q + 0.5\gamma B N_\gamma\, \xi_\gamma \tag{5}$$

where $S_{eff} = (S_r - S_{res})/(1 - S_{res})$ is the effective degree of saturation, in which S_{res} is the residual degree of saturation. $S_{eff,\mathrm{AVE}}$ is the average effective degree of saturation corresponding to the average suction in the stress bulb zone. The effect of varied suction profile is considered in Eqs. (4) and (5) by using the average value of suction.

This paper presents alternative equations based on the effective stress principle for the evaluation of the bearing capacity of shallow footings on unsaturated soil with uniform and varied suction profiles. The proposed equations require smallest number of parameters in comparison to all previously proposed formulae. The predictions of the bearing capacity based on the proposed equations are compared with data from model footing tests and in-situ plate load tests found in the literature.

2. BEARING CAPACITY EQUATIONS FOR UNSATURATED SOIL BASED ON EFFECTIVE STRESS PRINCIPLE

By extending the effective stress principle to unsaturated soils, Bishop (1959) proposed the following equation to calculate the effective stress in unsaturated soil:

$$\sigma' = (\sigma - p_a) + \chi(p_a - p_w) = (\sigma - p_a) + \chi s \tag{6}$$

where σ' is the effective stress and χ is the effective stress parameter. The contribution of suction on the effective stress and thus the strength of unsaturated soil is taken into account by χs. The effective stress principle could be used in association with complete elasto-plastic frameworks to describe this behaviour of unsaturated soils. The general simplicity and accuracy of the effective stress principle in unsaturated soils has been highlighted by many studies, which may differ slightly through the choice of χ (Bishop 1959; Kohgo *et al*. 1993; Khalili and Khabbaz 1998; Lu and Griffiths 2004). χ is equal to 1 when the soil is fully saturated and equal to 0 when the soil is completely dry.

Based on the shear strength data of different types of soils, Khalili and Khabbaz (1998) proposed a unique expression for χ as follows:

$$\chi = \begin{cases} 1 & \text{for} \quad \dfrac{s}{s_e} \le 1 \\ \left(\dfrac{s}{s_e}\right)^{-0.55} & \text{for main drying curve} \quad \dfrac{s}{s_e} > 1 \end{cases} \tag{7}$$

This definition of χ has been validated for a broad range of different soils and been widely adopted to well capture the behaviour of unsaturated soils (Loret and Khalili 2000; Khalili *et al*. 2004; Masin 2010). Using this expression the shear strength of unsaturated soil can be simply predicted with the effective shear strength parameters for saturated soils except for the air entry value, which can be determined in any soil laboratory. For most practical problems in which $p_a = 0$, the shear strength of unsaturated soil can be calculated as:

$$\tau = c' + (\sigma + \chi s)\tan\phi' \tag{8}$$

in which the contribution of suction on the shear strength is taken into account by $\chi s \tan\phi'$.

Suction in the vadose zone above the water table may not be uniformly distributed (Lu and Griffiths 2004; Vo and Russell 2016). At a steady state suction may be greater at the top of the vadose zone and may decrease with depth and vanish at the ground water level. Theoretical studies have been performed to derive the suction profile in unsaturated soil at steady state (Lu and Griffiths 2004; Vo and Russell 2016). Vo and Russell (2016) found that linear approximations of χs profiles could be assumed for both evaporation and infiltration and the associated errors are small. A linear χs profile in unsaturated soil may be defined as:

$$\chi s = \chi s_0 - \rho z \tag{9}$$

where χs_0 is the value of χs at the base of the footing, z is the depth below the footing base and $\rho = \partial(\chi s)/\partial z$ is a constant defining the variation of χs with depth.

It has been proved by Vo and Russell (2016) that the cohesion (c') and $\chi s \tan\phi'$ have similar and independent effects on the shear strength and the bearing capacity of unsaturated soil. The effect of the linear variation of χs with depth can be incorporated into the bearing capacity equation similar to a linear cohesion profile in saturated soil. The effect of the constant component of suction, s_0, on the bearing capacity is included in the first term of the bearing capacity equation. The influence of gradient of suction plays the same role as soil density in the bearing capacity based on the theory of plasticity and the method of characteristics (Davis and Booker 1973; Martin 2004) and thus is considered in the third term of the bearing capacity equation. Therefore, a bearing capacity equation for unsaturated soil may be expressed as:

$$q_u = (c' + \chi s_0 \tan\phi')N_c d_c + qN_q d_q + 0.5B(\gamma - \rho)N_\gamma d_\gamma \quad (10)$$

In this equation, the bearing capacity of unsaturated soil is estimated conveniently by the effective stress parameter, χ, and suction together with the drained shear strength parameters, c' and ϕ'. Equation (10) is consistent with the traditional bearing capacity equation for dry soil when suction is equal to 0. When the soil is fully saturated, the submerged unit weight of soil, γ', should be used. When the suction in the soil is smaller than the air entry value, χ is equal to 1 and suction acts similar to a negative pore pressure. When the suction is greater than the air entry value, the contribution of suction on the bearing capacity may be simply considered by $\chi s \tan\phi'$. Equation (10) also provides a smooth transition at the point of air entry value due to the continuous definition of χ.

If suction is uniformly distributed under a footing or the representative average suction in the stress bulb zone is available, Eq. (10) can be simplified to be:

$$q_u = \left[c' + (\chi s)_{\mathrm{AVE}} \tan\phi'\right] N_c d_c + qN_q d_q + 0.5\gamma B N_\gamma d_\gamma \quad (11)$$

in which $(\chi s)_{\mathrm{AVE}}$ is the value of χs corresponding to the average suction in the stress bulb zone. Equation (11) is consistent with Eqs. (4) and (5) when the average suction is smaller than the air entry value. Assuming the water table under a footing is lower than the bottom of the stress bulb, Eqs. (10) and (11) can be used to estimate the bearing capacity of the footing on unsaturated soil with typical χs profiles which are shown in Fig. 1.

A model footing test or an in-situ plate load test is mostly likely to be performed under a constant moisture content condition due to the low permeability of unsaturated soil. It has been found by Tang *et al*. (2016) that χs could be assumed constant and the initial value can be used in the interpretation of the test results without loss of significant accuracy. In this study the initial values of χs are used in the calculations of bearing capacity.

3. VALIDATION

In this section, the bearing capacity of footings on unsaturated soils evaluated based on Eqs. (10) or (11) is compared with the measured bearing capacity of model footing tests and plate load tests presented in the literature. Surface and embedded footings on different types of soils are considered. Details of these tests and soil parameters are summarized in Table 1. Note that more plate load tests on unsaturated soils can be found in the literature, however, many did not provide data on the values of suction under the footing, or the tests were performed on loose materials where punching failure happened rather than the general shear failure.

Rigorous values of the bearing capacity factors N_c, N_q, and N_γ for rough circular and strip footings, presented by Martin (2004), are used in the calculation of the bearing capacity of model footing tests in this study using Eqs. (4), (10) and (11). The effect of footing shape is included in these bearing capacity factors and thus the shape factors are not required. The bearing capacity calculated using Eq. (4) and rigorous bearing capacity factors is denoted as "Vanapalli and Mohamed (2013) (1)" in Figs. 2 to 7. As suggested by Vanapalli and Mohamed (2013) and Vahedifard and Robinson (2016), the values of N_c and N_q from Terzaghi (1943), the values of N_γ from Kumbhokjar (1993) and the values of shape factors from Vesic (1973) are also used in the calculation using Eqs. (4) and (6). The bearing capacity calculated using Eq. (4) and this set of factors is denoted as "Vanapalli and Mohamed (2013) (2)" in Figs. 2 to 7. Depth factors are obtained by the relations proposed by Hansen (1970). The effective stress parameter, χ, is calculated using Eq. (7).

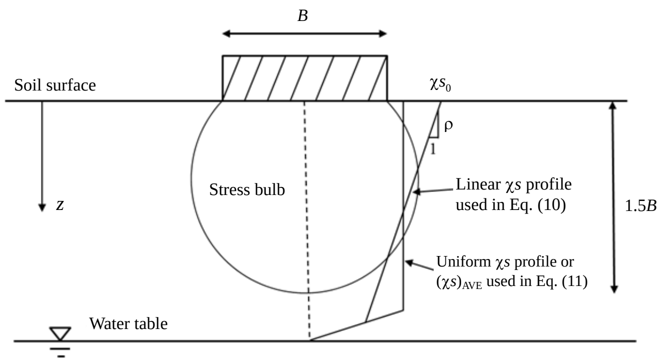

Fig. 1 Typical uniform and linear χs profiles in unsaturated soil under a footing

Table 1 Details of bearing capacity tests and soil parameters

Tests	Footing size (m)	Footing depth (m)	s_e (kPa)	c' (kPa)	ϕ' (°)	ψ	γ_d (kN/m^3)
Rojas *et al.* (2007)	$d^* = 0.31$	0	18	3	26	4.6	15.7
Oh and Vanapalli (2011)	0.10 × 0.10	0	3	0.6	39	1	16.05
Oh and Vanapalli (2013)	0.05 × 0.05	0	7.45	3.5	21	5.5	15
Wuttke *et al.* (2013)	0.477 × 0.079	0	1.9	0	46.9	1	13.5
Vanapalli and Mohamed (2013)	0.15 × 0.15	0 and 0.15	3	0.6	39 and 35.3	1	16.02

* d : diameter of circular footing

Rojas *et al*. (2007) carried out in-situ plate load tests on unsaturated lean clay using a circular steel plate of diameter 0.31 m. Loading of the plate was stopped when either a settlement equal to 10% of the plate diameter or a maximum pressure of 650 kPa was achieved. For the latter case, the load-settlement curves were extrapolated to obtain the load bearing capacity of the plate corresponding to settlement equal to 10% of the footing diameter. The variations of suction measured at depths of 0.1 m, 0.3 m, 0.6 m, and 0.9 m below the plate were also provided by Rojas *et al*. (2007). Figure 2 compares the experimentally obtained bearing capacity (Rojas *et al.* 2007) with the bearing capacity predicted by different equations. For the bearing capacity calculated using Eqs. (4), (6) and (11), the average value of the measured suctions at depth 0.1 m and 0.3 m below the plate were used. In Fig. 2, it is also shown the bearing capacity of the plate calculated using Eq. (10) for the linear χs profile. For this case, it is assumed that suction at the surface is equal to the one measured at a depth of 0.1 m, and varies linearly to that measured at a depth of 0.9 m. Figure 2 shows that both Eqs. (10) and (11) proposed in this study provide satisfactory estimations of the plate capacity.

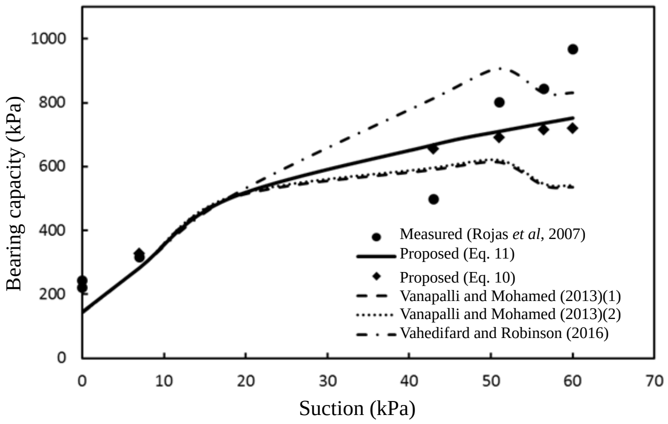

Fig. 2 Measured and predicted bearing capacity of plate on lean clay (Rojas *et al.* 2007)

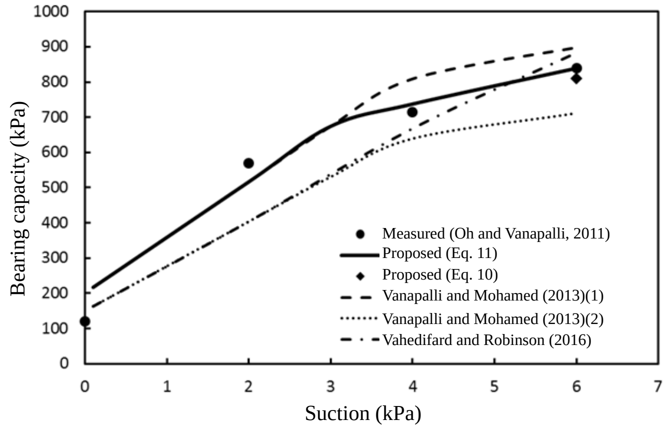

Fig. 3 Measured and predicted bearing capacity of plate on sand (Oh and Vanapalli 2011)

Oh and Vanapalli (2011) presented experimental data of plate load tests using a square plate with size of 0.1 m × 0.1 m on unsaturated coarse sand. This set of data was also reported by Mohamed and Vanapalli (2006). The air entry value of the soil reported by Oh and Vanapalli (2011) is 3 kPa which is used in the calculations here except for Eq. (5). However, Vahedifard and Robinson (2016) assumed the air entry value to be 5.7 kPa and calculated the bearing capacity using Eq. (5). Their predictions are shown in Fig. 3. The variation of suction with depth was presented for one of the tests, which is used here to evaluate the prediction of Eq. (10). Figure 3 compares the experimental data with the bearing capacity predicted by different methods. The predictions by the proposed equations match the experimental data very well.

Wuttke *et al*. (2013) conducted load tests on a small strip footing on a poorly graded unsaturated sand. The size of the footing was 0.477 m in length and 0.079 m in width. The maximum suction induced in the sand was limited to 4 kPa. Figure 4 compares the experimental data with the bearing capacities predicted by different methods. All the bearing capacity equations over-predict the experimental results to some extent. This can be attributed, perhaps, to the punching shear failure expected for footing on a relatively loose soil, as it may be the case here since the soil had a dry unit weight of 13.5 kN/m^3. The soil-water characteristic curve (SWCC) consists of a main drying path and a main wetting path. The predictions presented in Fig. 4 are based on the drying path of the SWCC although predictions based on the wetting path also show a similar trend.

Oh and Vanapalli (2013) performed load tests on unsaturated fine grained soil using a 0.05 m × 0.05 m square footing. Figure 5 presents the measured bearing capacity corresponding to 0.1B settlement for suction values from 55 kPa to 160 kPa together with the predicted values. The bearing capacity predicted by Eq. (11) agrees well with the experimental data. Equation (4) underestimates and Eq. (5) overestimates the bearing capacity for unsaturated conditions.

Vanapalli and Mohamed (2013) presented the results of loading tests on a square plate of 0.15 m × 0.15 m resting on the surface and 0.15 m below the surface of saturated and unsaturated coarse sand. The air entry value of the tested sand is 3 kPa. As suggested by Vanapalli and Mohamed (2013) and Vahedifard and Robinson (2016), $\phi' = 39°$ and $\phi' = 35.3°$ are used in the

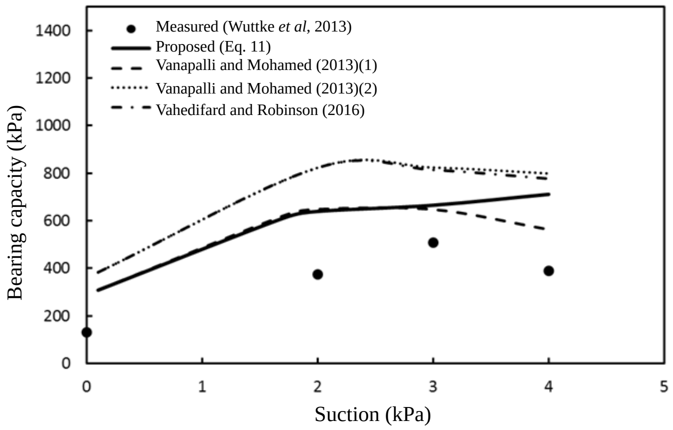

Fig. 4 Measured and predicted bearing capacity of strip footing on sand (Wuttke *et al*. 2013)

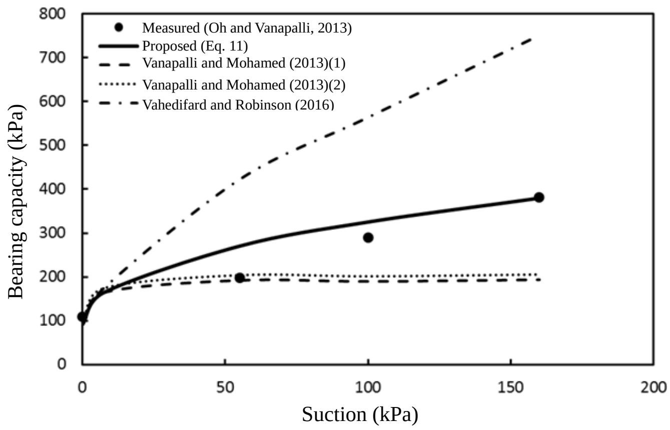

Fig. 5 Measured and predicted bearing capacity of plate on fine grained soil (Oh and Vanapalli 2013)

calculations for surface and embedded footings, respectively. The measured bearing capacity of the surface footing on saturated soil is larger than that predicted by the conventional bearing capacity equation. Figure 6 compares the experimental data for surface footing with the bearing capacity predicted by different methods. The bearing capacity predicted by the equation proposed by Vahedifard and Robinson (2016) is based on their assumption

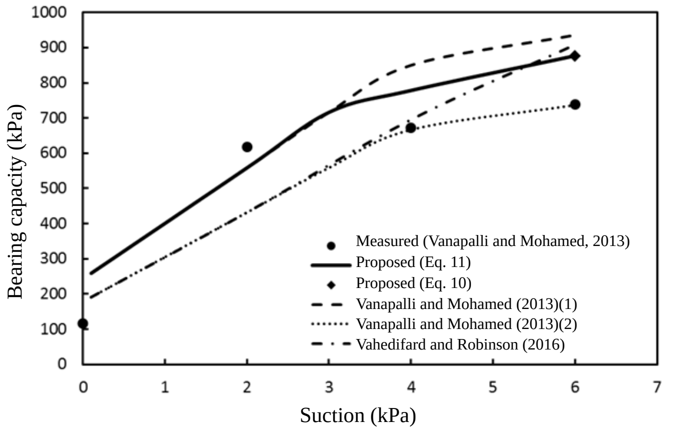

Fig. 6 Measured and predicted bearing capacity of surface footings on sand (Vanapalli and Mohamed 2013)

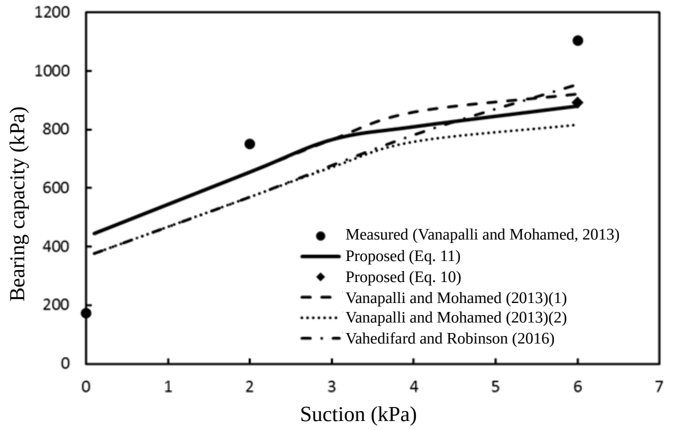

Fig. 7 Measured and predicted bearing capacity of embedded footings on sand (Vanapalli and Mohamed 2013)

that the air entry value is 5.7 kPa. The prediction of Eq. (10) is also presented in this figure for only one of the tests for which the suction profile was available. The measured capacity of the embedded footing on saturated soil is less than half of the one predicted by the conventional method (Fig. 7). The measured capacities of the footing on unsaturated soil are also larger than those predicted by different methods (Fig. 7). Similar results were obtained by Vahedifard and Robinson (2016). This may be due to the smaller friction angle used in the calculation for the embedded footing. However, this figure shows that the proposed equations can capture the increase in the bearing capacity due to suction.

Figure 8 compares the measured bearing capacity and those values predicted by the proposed equations for the tests presented in this paper. It can be seen that the errors associated are less than 25% for most cases.

Compared to the equations available in the literature it can be seen from Figs. 2 to 7 that the proposed equations provide better estimations of bearing capacity. It can be found that the equation proposed by Vahedifard and Robinson (2016) may overestimate the bearing capacity when the suction is greater than the air entry value, perhaps, because the contribution of suction from zero to the air entry value is always accounted by $s_e \tan\phi'$ in this equation. However, the effect of air entry value has been included in the definition of χ. When the suction is greater than the air entry value, the value of χ is less than 1 and the contribution of suction up to the air entry value should be accounted by $\chi s_e \tan\phi'$. While, the equation by Vahedifard and Robinson (2016) overestimates the bearing capacity for the tests performed by Rojas *et al*. (2007) and Oh and Vanapalli (2013) when the suction is greater than the air entry value, for the tests performed by Oh and Vanapalli (2011) and Vanapalli and Mohamed (2013), the largest suction value considered is slightly greater than the air entry value and therefore the differences are not obvious. The differences would be more significant for larger suction values.

4. RECOMMENDATION

It is a common practice in foundation analysis and design to consider the soil as fully saturated. In cases that the foundation is found on unsaturated soil and the ground water level is at such a depth that unsaturated conditions will dominate no matter the possible rise of the ground water table, it is important to consider the effect of suction in the soil for a more realistic design. The proposed equations require a relatively small amount of soil parameters and are recommended to the practising engineers for evaluation of the bearing capacity of footings or for back-calculating soil parameters from plate load testing on unsaturated soils. Apart from shear strength parameters and unit weight, Eqs. (10) and (11) require suction, s, and the effective stress parameter, χ, which is a function of s_e. Therefore, the only extra parameters required by the proposed equations, apart from those required by the conventional method, are s and s_e which can be found from the SWCC of the foundation soil.

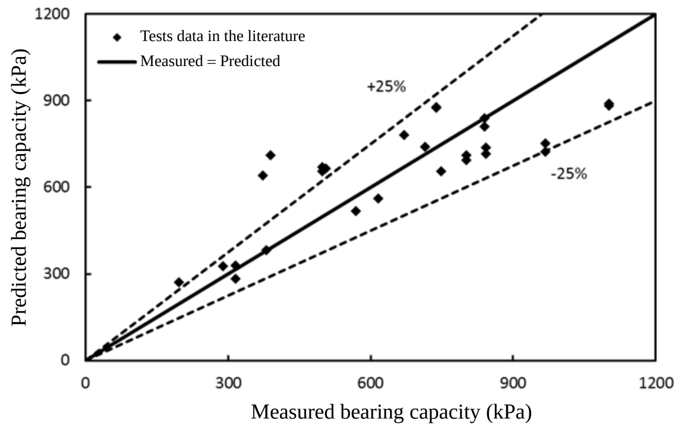

Fig. 8 Comparison between measured and predicted bearing capacity based on the proposed equations in this study

It has been found that the SWCC is void ratio dependent (*e.g.*, Masin 2010; Salager *et al*. 2010; Russell 2014). The air entry value changes with void ratio and this enables different values of χ for a specific s. Ignoring the void ratio dependency of SWCC may lead to inaccurate estimations of bearing capacity. Russell (2014) defined the air entry value as a function of the void ratio (e) of the soil:

$$s_e = Ce^{-D_s} \tag{12}$$

where D_s is the fractal dimension of the particle size distribution and C is a positive constant which was derived by Russell (2014) in terms of particle and pore scale properties. If the SWCC for a particular void ratio is obtained in the laboratory, the value of the constant *C* in Eq. (12) can be back calculated and used to update s_e and the SWCC for any other void ratios encountered in the field.

5. CONCLUSIONS

In this paper, equations are presented to predict the bearing capacity of shallow footings on unsaturated soil based on the effective stress principle. The validity of the equations is examined through comparison of their predictions with the published data of plate loading tests on unsaturated soils. The predictions of the proposed equations are shown to be satisfactory, although there are uncertainties in the magnitudes of the bearing capacity of plate load tests obtained experimentally. The proposed equations are simple for practical use since the only extra parameters required for unsaturated soil are the suction and the SWCC of the foundation soil. The relationship between the air entry value and void ratio through the particle size distribution facilitates the application of this method for interpretation of plate load tests where the SWCC may not be readily available for the void ratio of interest.

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