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A NEW ANALYTICAL MODEL FOR NATURAL PERIOD ANALYSIS OF ELEVATED TANKS CONSIDERING FLUID-STRUCTURE-SOIL INTERACTION

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ABSTRACT

The main purpose of this research is suggesting new relations which can be used to calculate impulsive and convective period for elevated tanks considering fluid-structure-soil interaction (FSSI) with an emphasis on foundation embedment effect. No previous study on the discovery of impulsive and convective period that emphasizes FSSI has been directly developed. In this respect, six simplified models of elevated tanks are assessed by analytical methods. Single degree of freedom, coupled and uncoupled multi degree of freedom for fluid-structure interaction, as well as the mass-spring substructure method for soil-structure interaction is presented. The applicability of these six models which are emphasized and illustrated for impulsive and convective periods is to discover elevated tanks behaviors with five different subsoil classes. Extracted impulsive and convective periods from suggested relations are compared by FEM, analytical analysis and regular methods from seismic codes. Maximum difference between suggested formulation for impulsive period and FEM analysis occurs in soft soil by maximum of 18 percent deviation. Also, results reveal that coupled and decoupled analysis of FSSI have negligible differences in similar conditions. Moreover, it is shown that embedment ratio has more remarkable effects on impulsive period and its effect would be significant in soft soil conditions.

Key words: Elevated tanks, fluid-structure-soil interaction, impulsive period, convective period.

1. INTRODUCTION

With reference to previous reports, there are several elevated tanks that were damaged or showed a large loss during previous earthquakes that have occurred globally. These uncomfortable and tragic events have demonstrated that in addition to vessel design of elevated tanks, staging structure system of the elevated tanks is more important than the other structural types of liquid vessels (Dutta *et al.* 2004). A large proportion of such losses have been observed during current earthquakes, which have occurred due to reasons like unsuitable design of support systems, construction of tanks on loose soil, and neglecting the effects of the soil-structure interaction (SSI) (Dutta 1995). Designs of shaft type supporting structures of elevated tanks are greatly vulnerable under seismic forces due to earthquakes. Reinforced concrete circular shaft type staging is extensively employed in the construction of elevated tanks within the developing and developed countries (Dutta *et al.* 2004).

An investigation into the fluid-structure interaction (FSI) method was suggested by most researchers due to a total recognition of hydrodynamic liquid effects on vessels of elevated tanks (Ibrahim 2005; Livaoghlou and dogangun 2007). Methods which can be used to describe the interaction between fluids and solids have been one of the major focal points for the research within the field of computational engineering for the recent years (Ozdemir *et al.* 2010; Moslemi *et al.* 2011).

Many different analytical and semi analytical methods which can be used to verify and develop more advance numerical methods have been developed for liquid tank connective problems. These methods are often limited to analysis of tanks with simple geometries, such as rectangular or cylindrical tanks. Several useful and applicable analytical FSI methods were developed by Westergaard (1933). Several regular approximate methods were also developed by Housner (1963) and Haroun (1985).

A satisfactory spring mass analogue used to characterize basic dynamics for two mass model of elevated tank was proposed by Housner (1963) after the Chilean earthquake of 1960, and is found to be more appropriate and being commonly used in most international codes. The pressure generated within the fluid due to the dynamic motion of the tank can be divided into impulsive and convective parts. When a tank containing liquid with a free surface is subjected to horizontal earthquake ground motion, the tank wall and liquid are subjected to horizontal acceleration. The liquid in the lower region of the tank termed as impulsive liquid mass acts like a mass that is rigidly connected to the tank wall. Liquid mass in the upper region of the tank termed as convective liquid mass undergoes sloshing motion. ACI 350.3 utilizes the Housner method. This method essentially assumes that hydrodynamic effects due to seismic loading can be evaluated approximately as the sum of the following two parts. The impulsive part, which represents the portion of the stored liquid that moves in unison with the structure and, the convective part, which represents the effect of the sloshing action of the liquid.

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The natural period which consider sloshing mass of water called convective period and also the period of impulsive mass is called impulsive period.

Regarding vessels design approach and vessels international codes recommendation (ACI-350), also pervious liquid tanks designer experts such as Housner, it should be evaluate separately to find impulsive period for body of structure, vessel and no fluctuation part of liquid also convective period to find the natural period of inlet liquid turbulence. Both of the natural periods are important to find the real dynamic behavior of the vessels. The aim of this separately evaluation is finding the effect of inlet liquid and also body of structure on dynamic behavior of elevated tanks. In fact the effects of inlet liquid is evaluated by convective period which normally set in part of velocity sensitive in response spectrum. Also the vessel and its shaft dynamic behavior are evaluated as impulsive period which normally set on acceleration sensitive part of response spectrum.

In order to represent these two masses and to include the effect of their hydrodynamic pressure in analysis, two-mass uncoupled model is adopted for elevated tanks. In spring-mass model, convective mass (M_c) is attached to the tank wall by the spring having stiffness (K_c), whereas impulsive mass (M_i) is rigidly attached to the tank wall. Basis of this method is explained in many different literatures and codes such as API 650, Euro code 8 and ACI 350.3 (Livaoglu and Dogangun 2006). The two mass idealizations can be treated as two uncoupled single degree of freedom system as shown in Fig. 1.

The most accurate method for liquid storage modeling such as elevated tanks modeling is fluid-structure-soil interaction (FSSI). In this method, effects of convective pressure, vessel shape and shaft, soil stiffness and masses and also foundation mass and foundation embedment would be assessed. Suggested analytical FSI models are solely used for fluid effects on structure (Chen and Barber 1976; Mori *et al.* 2015; Moslemi *et al.* 2011). In addition, no universally accepted parameters for soil effects on elevated tanks response has been found in engineering analysis.

Various research methods have been used in the study of elevated tanks in recent decades. Haroun and Ellaithy (1985) developed a model for analyzing elevated rigid tanks that is exposed to shifting and rotation. Resheidat and Sunna (1990) investigated the behavior of a rectangular elevated tank considering the soil-foundation structure interaction during earthquakes. They ignored the sloshing effects on the seismic behavior of elevated tanks and the radiation damping effect of soil. Haroun and Temraz (1992) analyzed two-dimensional x-braced elevated tanks supported on the isolated footings in order to investigate the impact of dynamic interaction between the tower and the supporting soil-foundation system. In this study, they ignored the sloshing effects. Livaoglu and Dogangun (2007) investigated the seismic behavior of fluid-elevated tank-foundation soil systems in domain frequency. Livaoglu and Dogangun (2007) analytical and numerical analyzed the procedure for seismic analysis of fluid-elevated tank-soil systems by using known method of Housner and EC-8 code. Dutta (2007) proposed alternate tank staging configurations for reduced torsional vulnerability. Marashi and Shakib (2008) performed an ambient vibration test for the evaluation of dynamic characteristics of elevated tanks. Livaoglu *et al.* 2011 conducted a comparative study on the seismic behavior of elevated tanks considering both fluid structure and soil-structure interaction effects. Ghanbari and Abbasi Maedeh (2015) had a new method study on dynamic response of ground supported tanks considering FSSI effects.

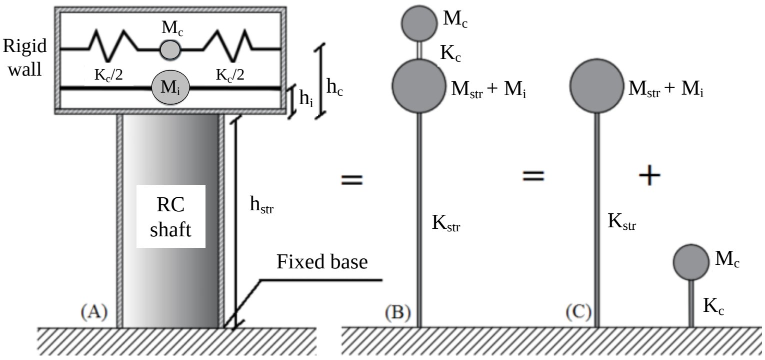

Fig. 1 A: Schematic of RC elevated tank with fixed base condition, B: Coupled analytical FSI model with base condition, C: Uncoupled analytical model with fixed base condition

The majority of the studies were dedicated to appraise the behavior of the fluid and the supporting structure by using the fixed base assumption and the soil effect on elevated tanks behavior was ignored. Moreover, there were no previous studies to develop a direct engineering relationship which can be used to find FSSI effect on impulsive and convective period of elevated tanks.

The purpose of this research is to develop the new functional relationship which can be used to find impulsive and convective period of elevated tanks considering FSSI. Those developed relations will be able to modify and improve the suggested analytical FSI models to FSSI models. For validation of suggested relationships, obtained results are compared by calculated values of FEM, analytical methods and regular international code suggestions. In addition, by defining an elevated tank case study, impulsive and convective period variation would be assessed by analytical methods and suggested relations. Base shear and overturning moment for different forms of suggested analysis modeling, with an emphasis on National Earthquake hazards reduction program (NEHRP) recommendation, in a high-risk seismic zone are calculated.

2. PRINCIPAL EQUATIONS AND ASSUMPTIONS

To determine the natural period of elevated tanks regarding FSSI, the following assumptions were considered; elevated tank vessels are identical to a superstructure intended as a single degree of freedom system composed of a single mass M_{str} that are installed on a shaft with stiffness K_{str} at an elevation of $h_{str} + h_{c.g}$ (Fig. 2), where $h_{c.g}$ is vessel center of gravity height with an emphasis on its shape (Kianoush and Chen 2006). The considered foundation which is in full contact with the surrounding soil would be a rigid circular pad with radius r and depth of embedment e.

The mass and the mass moment of inertia of the foundation are represented by M_f and I_f, respectively. The soil stiffness matrix of the surrounding soil is represented by a 2 × 2 matrix, where K_x, K_θ and $K_{x\theta}$ are the horizontal, rocking and horizontal-rocking coupling terms of the corresponding static stiffness matrix, respectively (Wolf 1985).

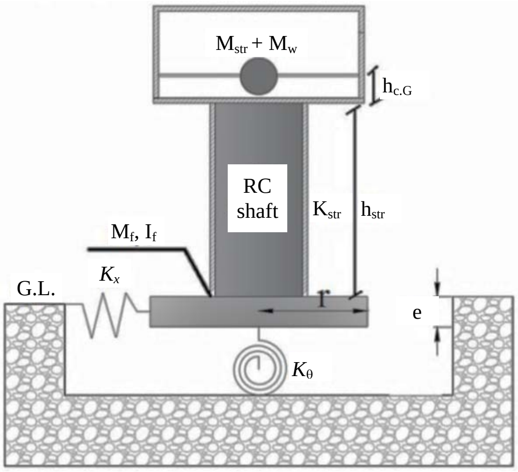

Fig. 2 Lumped mass SDOF system of elevated tank considering soil-structure interaction

$$\begin{bmatrix} K_x & K_{x\theta} \\ K_{x\theta} & K_\theta \end{bmatrix} \tag{1}$$

Soil stiffness is attached to the central point of the rigid circular foundation (Kramer 1996). The explained soil stiffness for circular rigid foundations supported at the surface of a homogeneous half space equations are presented in Table 1 (Paris and Kausel 1985, 1988; FEMA 450 2003; FEMA 368 2001; FEMA 273 1996).

Where G, R, and ν are shear modulus, Poisson's ratio of soil and radius of equivalent circular foundation. The foundation radius for translational and rotational degree of freedom was also calculated (Gazetas and Stoke 1991; Gazetas 1991). These stiffnesses are also estimated using the expressions given in federal emergency management agency (FEMA) for embedment and foundations that rest on a surface stratum of soil underlain by a stiffer deposit that has a shear wave velocity that is more than twice of the surface layer (Paris and Kausel 1985, 1988).

It has been generally recognized that the interaction between soil and structure can indeed affect the response of structures, particularly for structures on relatively flexible soils. (Veletsos and Nair 1975; Wolf 1985; Jahankhah *et al.* 2013).

Table 1 Soil stiffness formulas considering foundation embedment

Soil stiffness mode	Equation
Horizontal stiffness (kN/m)	$K_x = \frac{8GR}{2-\upsilon}\left(1+\frac{e}{r}\right)$
Rocking stiffness (kN·m)	$K_\theta = \frac{8GR^3}{3(1-\upsilon)}\left(1+2.3\left(\frac{e}{r}\right)+0.58\left(\frac{e}{r}\right)\right)^3$
Horizontal-rocking stiffness (kN/m)	$K_{x\theta} = \frac{e}{3}(K_x)$

To determine natural period of a SDOF system, the following equation would be defined and solved. Consider a spring, fixed at one end and having a mass attached to the other; this would be SDOF oscillator. For a single degree of freedom oscillator, a system in which the motion can be described by a single coordinate, the natural frequency depends on two system properties: Mass and stiffness; (providing the system is un-damped). In general, for a system of n degree of freedom, the equation of motion is a set of second linear differential equation as given in Eq. (2) (Abbasi Maedeh *et al.* 2016; Chopra 2000).

$$[K]\{U\}+[C]\{\dot{U}\}+[M]\{\ddot{U}\}=\{F\} \tag{2}$$

The vector $\{U\}$ function of time denotes the displacement response at all degrees of freedom. The matrices $[K]$, $[G]$ and $[M]$ represent stiffness matrix, damping matrix and mass matrix respectively, which are constant for a linear system. The vector $\{F\}$ denotes the prescribed loads at the corresponding degree of freedom as a function of time. In order to estimate the fundamental frequency of the system under harmonically varying load, Eq. (3) can be written as:

$$\{U\}=\{Z\}\cos(\omega t-\alpha) \tag{3}$$

where $\{Z\}$ and $\{U\}$ represent the amplitude of the load and displacement, respectively. Substituting Eq. (3) into Eq. (4) the following equation is driven:

$$\left[[K]-\omega^2[M]\right]\{Z\}=\{0\} \tag{4}$$

Equation (4) has specific answer only when determinant of the above mentioned matrix is equal to zero:

$$\left[[K]-\omega^2[M]\right]=\{0\} \tag{5}$$

In solving Eq. (5), three vibration frequencies corresponding to three degree of freedom are driven. The radian frequency ω_n, can be found. The natural frequency equation for the lumped mass assumption of water tank considering SSI system presented in Fig. 2 is written as follows by substituting mass and stiffness of a lumped mass system to Eq. (5). Assembled SSI system matrix are presented in Eq. (6).

$$\begin{bmatrix} K_{str} & -K_{str} & 0 \\ -K_{str} & K_{str}+K_h & K_{x\theta} \\ 0 & K_{x\theta} & K_\theta \end{bmatrix} - \{\omega^2\}\begin{bmatrix} m_{str} & 0 & m_{str}(h_{str}+e) \\ 0 & m_f & m_f\left(\frac{e}{2}\right) \\ m_{str}(h_{str}+e) & m_f\left(\frac{e}{2}\right) & I_f+m_f\left(\frac{e}{2}\right)^2+m_{str}(e+h_{str})^2 \end{bmatrix}=0 \tag{6}$$

The expressions of stiffness's for this alternate configuration at a fixed support condition can be obtained from the literature given $K_{str}=E_cI_c/L_c^3$ (Chopra 2000).

Where E_c is the modulus of elasticity of the material of shaft, I_c the moment of inertia of column cross-section, and L_c the length of the shaft. Generally, concrete tanks are regarded as tanks with rigid walls, while steel tanks are regarded as tanks with flexible walls. Spring mass models for tanks with flexible walls are more cumbersome to use (Ghahramani *et al.* 2010a; 2010b). Moreover, difference in the parameters obtained from rigid and flexible tank models is not substantial (Dogangun and Livaoghlou 2004). The developed mass and stiffness matrix in Eq. (6), which is explained for SDOF considering SSI effect, is assumed not to have any convective part in vessel. In consideration of FSSI, the explained matrix should be modified to divide convective and impulsive masses. Furthermore, stiffness parameters and degree of freedom should be redefined. Indeed, the single degree of freedom system should be changed to multi degree of freedom in superstructure section.

3. PROPOSED ELEVATED TANKS MODELING METHODS

In the current study, analytical methods are used for elevated tanks modeling and analysis with and without emphasis on FSSI considering different forms of fixed base SDOF for superstructure (Model 1), fixed base FSI model (Models 2 and 3), SDOF superstructure with emphasis on SSI effect (Model 4) and FSSI models by two different categories (Models 5 and 6). In the present study, the suggested modeling methods that are used for the assessment of the natural period of elevated tanks and their behavior is illustrated in Fig. 3.

In Fig. 4, six schematic forms of suggested analytical models for elevated tanks superstructures assessment are shown. The first three forms of schematics display fixed base conditions of elevated tanks, while the other three represent SSI assumption.

Analytical Model 1 will be considered as systems with a SDOF in which their mass is concentrated at center of gravity ($h_{C.G}$). Structural mass M_{str} includes mass of container, one-third mass of shaft staging and mass of liquid. Mass of the container consists of mass of roof slab, container wall, gallery, floor slab, and floor beams. Analytical Model 2 is a satisfactory spring mass analogue used to characterize basic dynamics for two mass model of elevated tank and was proposed by Housner. An equivalent coupled and uncoupled system by fixed base condition is shown in Fig. 1 in which M_{str} represents the mass of container, one-third mass of shaft staging and impulsive mass of liquid (Housner 1963) and lateral stiffness of the structure K_{str} respectively, M_c, and K_c are convective mass and liquid stiffness (Models 2 and 3) (Housner 1963).

In analytical model of SSI with SDOF system, both the structure and soil have been idealized in a simplified procedure, as it has been developed for usual staging configuration. For analytical analysis, the tank container is assumed to act like a rigid cylindrical shell having maximum allowable water depth of h. The lumped mass system of an elevated tank is placed on an infinite soil area. SSI combination of mass and stiffness assembled matrix of this model is shown in Eq. (6). Schematics of SSI with SDOF of elevated tanks are revealed in Fig. 2 and the simplest model is shown in Fig. 4 (Model 4). The reason why there is no part of convective mass is that it is neglected from convective force and it is assumed that all mass of water would have a coordinated displacement with a specific frequency. The multi degree system of elevated tanks considering soil-structure interaction is shown in Fig. 5.

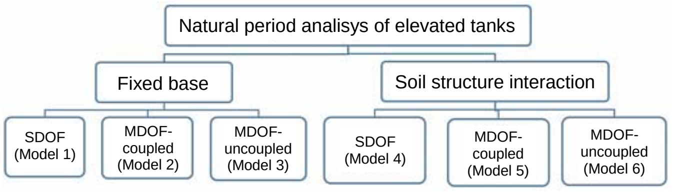

Fig. 3 Type of analytical suggested modeling for elevated tanks

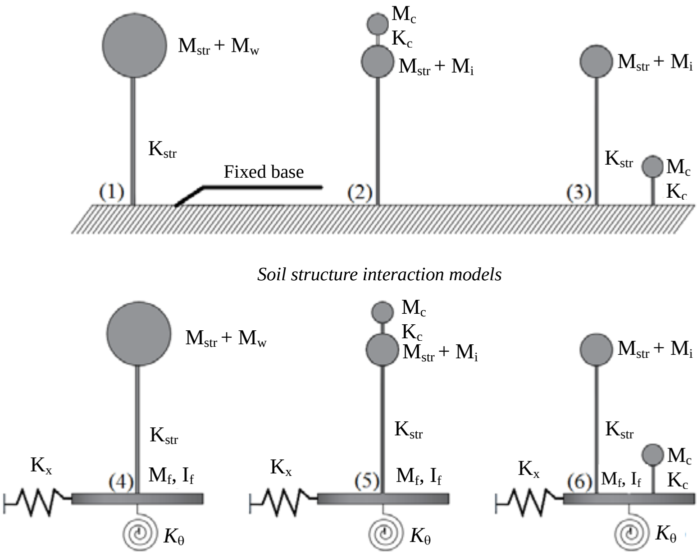

Fig. 4 Analytical schematics analysis of fixed base (Models 1, 2, 3) and SSI conditions (Models 4, 5, 6)

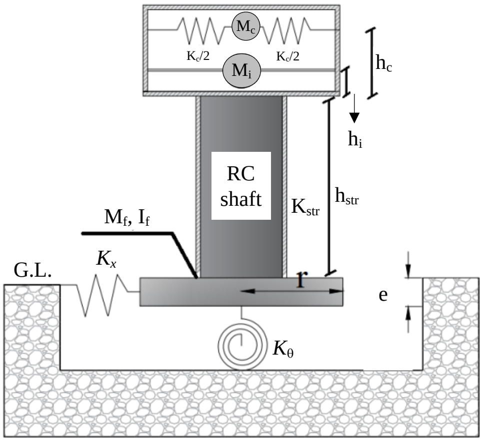

Fig. 5 Schematic model of elevated tank with an emphasis on fluid-structure-soil interaction system

Analytical model of FSSI system would be the most accurate method for liquid storage modeling such as elevated tanks. In this method, all stiffness and masses such as convective, vessel, shaft and soil stiffness would be evaluated in response to elevated tanks. Schematic model of coupled FSSI is shown in Fig. 6. The proposed analytical models were used to evaluate the effects of foundation embedment and foundation geometry on system natural period.

The assembled coupled matrixes that substituted into Eq. (5) with an emphasis on FSSI and foundation embedment effects are presented in the following equation:

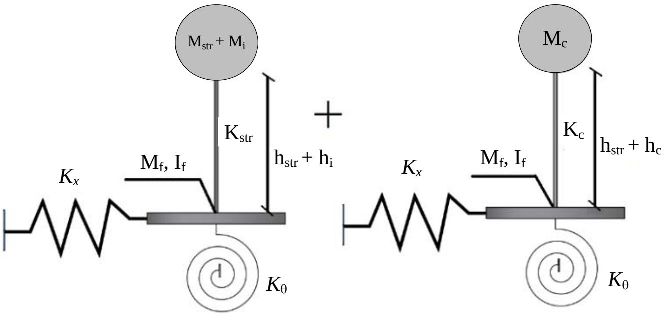

Fig. 6 Simplest uncoupled model of fluid-structure-soil interaction

$$\begin{bmatrix} K_c & -K_c & 0 & 0 \\ -K_c & K_c+K_{str} & -K_{str} & 0 \\ 0 & -K_{str} & K_{str}+K_x & K_{x\theta} \\ 0 & 0 & K_{x\theta} & K_\theta \end{bmatrix}$$
$$-\{\omega^2\}\begin{bmatrix} m_c & 0 & 0 & m_c(e+h_c+h_{str}) \\ 0 & m_{str} & 0 & m_{str}(e+h_i+h_{str}) \\ 0 & 0 & m_f & m_f\left(\dfrac{e}{2}\right) \\ m_c(e+h_c+h_{str}) & m_{str}(e+h_i+h_{str}) & m_f\left(\dfrac{e}{2}\right) & I_f+m_f\left(\dfrac{e}{2}\right)+m_{str}(e+h_i+h_{str})^2+m_c(e+h_c+h_{str})^2 \end{bmatrix}=0 \tag{7}$$

Regarding literature suggestions and many codes, FSI problems should be solved as an uncoupled method; therefore it is assumed that FSSI condition would be assessed as an uncoupled method. The suggested method of FSSI is illustrated in Fig. 7 (Model 6).

The assembled uncoupled for superstructure matrix that substitute into Eq. (5) is presented as following equation:

$$\begin{bmatrix} K_{str} & -K_{str} & 0 \\ -K_{str} & K_{str}+K_x & K_{x\theta} \\ 0 & K_{x\theta} & K_\theta \end{bmatrix}-\{\omega^2\}\begin{bmatrix} m_{str} & 0 & m_{str}(e+h_i+h_{str}) \\ 0 & m_f & m_f\left(\dfrac{e}{2}\right) \\ m_{str}(e+h_i+h_{str}) & m_f\left(\dfrac{e}{2}\right) & I_f+m_f\left(\dfrac{e}{2}\right)+m_{str}(e+h_i+h_{str})^2 \end{bmatrix}=0 \tag{8}$$

In addition, the assembled uncoupled matrix for convective fluid part that substitute into Eq. (5) is presented as Eq. (9).

$$\begin{bmatrix} K_c & -K_c & 0 \\ -K_c & K_c+K_x & K_{x\theta} \\ 0 & K_{x\theta} & K_\theta \end{bmatrix}-\{\omega^2\}\begin{bmatrix} m_c & 0 & m_c(e+h_c+h_{str}) \\ 0 & m_f & m_f\left(\dfrac{e}{2}\right) \\ m_c(e+h_c+h_{str}) & m_f\left(\dfrac{e}{2}\right) & I_f+m_f(e/2)+m_c(e+h_c+h_{str})^2 \end{bmatrix}=0 \tag{9}$$

4. SUGGESTED RELATIONSHIPS

To develop a direct relationship for finding natural period in each part (convective and impulsive) of elevated tanks with an emphasis on FSSI, according to eigenvalue equations and suggested assembled matrix of mass and stiffness, mathematical operations are carried out. Four values of natural period with an emphasis on four degrees of freedom are extracted from coupled initial assumptions (Wolf 1985). Regarding degrees of freedom arrangement on suggested matrix, first and second entries represent convective and impulsive eigenvalues that should be changed to natural periods by the regular relations. Two last entries of extracted eigenvalues represent the horizontal and rocking eigenvalues of foundation that were not assessed in the present study. Suggested formulations to calculate convective and impulsive periods of circular elevated tanks on RC shaft supporting with an emphasis on FSSI are developed as following equations:

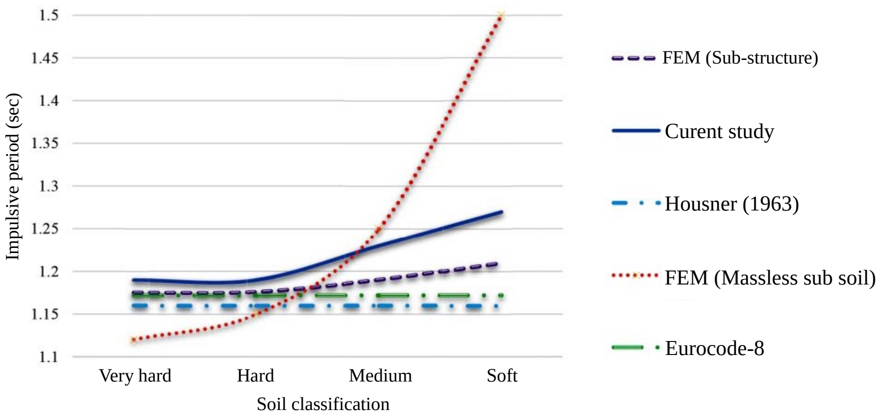

Fig. 7 Impulsive period comparison of suggested FSSI formulation and other numerical and analytical methods

$$T_c = 2\pi \times \sqrt{\frac{-[K_{x\theta} \times M_c \times \overline{h_c}]}{K_{hr} - K_{x\theta}^2} - \frac{M_c \times \left[-\overline{K} - (K_c - K_{x\theta}^2) - K_{s\theta} + (K_s \times K_{hr}) + (K_c \times K_{hr})\right]}{K_c \times \left[K_{s\theta} - (K_c \times K_{hr})\right]}} \tag{10}$$

$$T_i = 2\pi \times \sqrt{\frac{-M_{str} \times [K_{x\theta}^2 + K_{st} + K_{hr}]}{K_{s\theta} - (K_{str} \times K_{hr})} - \frac{K_{x\theta} \times M_{str} \times \overline{h_i}}{K_{hr} - K_{x\theta}^2}} \tag{11}$$

All of formulation parameters are explained in the body of the current study and new composite notations are explained in Table 2.

Table 2 The notations of developed formulation

Notation	Formulation
K_{hr}	$K_x \times K_\theta$
$\overline{K}$	$K_{str} \times K_c \times K_\rho$
$K_{s\rho}$	$K_{str} \times K_{x\rho}^2$
K_{st}	$K_{str} \times K_\rho$
$\overline{h_c}$	$e + h + h_c$
$\overline{h_i}$	$e + h + h_i$

In addition, regarding literature, convective natural period depends on the shape of the vessel of elevated tank and also h/D ratio. This dependency to vessel geometry is hidden to liquid stiffness and convective mass of each shape of vessel. The efficacy of soil stiffness on natural period is considered in the current study.

5. COMPARISON

Results of suggested relationships are compared with extracted results of FEM massless soil domain, FEM sub-structure model and regular analytical methods that was suggested in international codes. In Table 10, results of fixed base condition considering FSI are achieved. Current study recommendation relation extracted value is 1.17 seconds and the Livaoghlu (2006) value is 1.16 seconds. It is observed in case of fix base there are only 1% difference between them. To find fixed base condition from suggested formulation, rocking and horizontal soil stiffness would tend to infinite values to simulate the fixed base condition for elevated tanks. Maximum difference between FEM and suggested FSSI formulation in fixed base condition is shown as +1% for suggested formulation.

Another comparison is carried out by FEM analysis of a concrete shaft elevated tank and suggested relationships formulation. Results of this comparison are shown in Table 3.

FSSI effect on elevated tanks with an emphasis on different soil conditions which recommended by Livaoghlu (2006) is compared with proposed relationships, FEM and analytical methods. Results of Fig. 7 show the difference between several analysis methods on impulsive period conditions. Maximum difference occurred in massless soil domain. Critical difference occurred in soft soil, approximately 15% higher than FSSI suggested relation. Regarding sub-structure FEM modeling, it is observed that the maximum difference in soft soil would be approximately 5%.

Figure 8 shows that in convective mode, there are negligible differences in comparison with impulsive period. Results reveal that there are no obvious soil effects on convective period in the current study (Fig. 8).

Maximal differences between results occurred in massless FEM model, which are about +2 percent higher than suggested relationship. Similar to impulsive conditions, massless FEM has more overestimated predictions of periods in comparison with other methods. Finally, displayed graphs show that the impulsive period is more affected for soil conditions rather than convective period.

Table 3 Suggestion relationships results comparing with FEM analysis

	Concrete vessel (Moslemi *et al.* 2011)	Current study	Deviation (%)
Convective period (sec)	6.21	6.15	1
Impulsive period (sec)	0.51	0.55	8

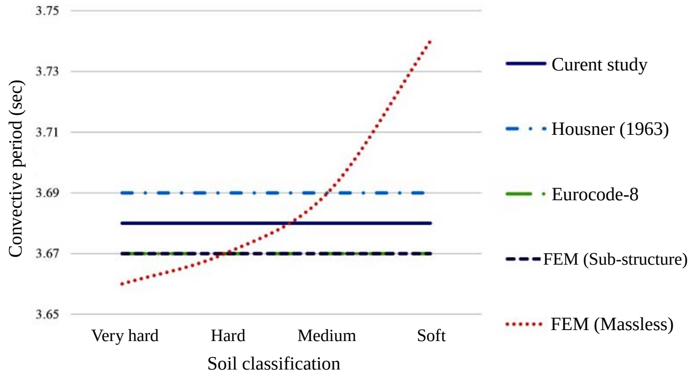

Fig. 8 Convective period comprising of developed formulation

5.1 Geometry and Material Characteristics of the Case Study

A reinforced concrete (RC) elevated tank with a vessel capacity of 256 m^3 is considered in natural period and seismicity analysis. The elevated tank has a shaft supporting structure with a total height of 16.40 meters from ground surface. This form of container and supporting structure has been used as a typical project for water supply in developed and developing countries for decades. The container is assumed to be filled with water to a density of 1,000 kg/m^3. Geometry of vessel, and shaft is depicted in Fig. 9.

Extra information of shaft supporting, vessel and foundation geometry is shown in Table 4.

In this current study, it is assumed that the elevated tank is built on dry and non-saturated clay soil with different characteristics. The stiffness of the springs for various kinds of clayey soil has been obtained from values of shear modulus G of soil according to the empirical relationship. N represents the number of blows to be applied in standard penetration test (SPT) of the soil. Following an established guideline, N is taken as 3, 6, 9, 15 and 30 for very soft, soft, medium, stiff and very stiff clay, respectively. The mechanical property of current study soil domain are reported in Table 5. Elevated water tanks with circular pad type of foundation are considered in the scope of the present study.

The basis of current study base shear and bending moment calculation theory is referred to national earthquake hazards reduction program (NEHRP) user guide which is one of the most prestigious codes all around the world. To calculate the base shear and overturning moment, a high-risk seismicity region and recommended ground acceleration of 0.4 g are from NEHRP. Furthermore, EPA (effective peak acceleration) and EPV (effective peak velocity) are explained factors in NEHRP (1996; 2001; 2003) guideline and are assumed to be 0.4. This value is chosen by an engineering judgment. Larger values are assigned to systems with excellent energy dissipation capacity and stability, as ensured by specific design and detailing procedures (Rai 2002).

For selecting a critical factor, 1.5 is considered for the elevated tank and high-risk seismicity location. In addition, the soil class of this study is taken on groups E and D of NEHRP soil classification. Factors of F_a and F_v are assumed with reference to Table 6. Supplementary information about high seismicity zone is explained in NEHRP (1996; 2001).

Regarding soil stiffness equations (rocking, horizontal and rocking-horizontal), values are calculated with an emphasis on foundation depth. All values of this stiffness are shown in Table 7.

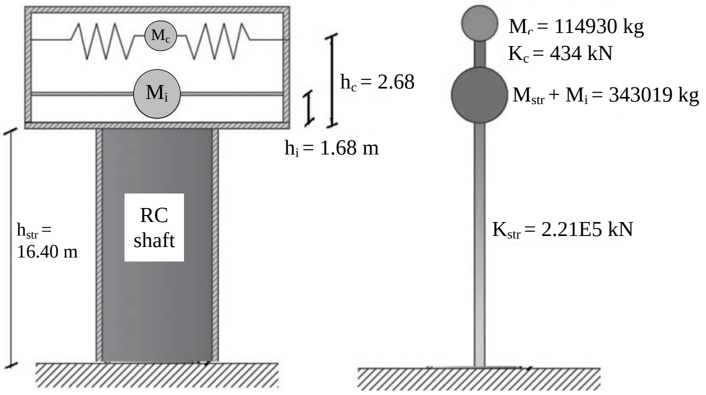

Fig. 9 Geometry of elevated tank and the simplest model

Table 4 Shaft, vessel, and foundation properties of the elevated tank

<table>
<tr><td rowspan="2">Mass</td><td colspan="3">Mass of water</td><td colspan="3">Mass of structure</td></tr>
<tr><td colspan="3">255,658 kg</td><td colspan="3">201,869 kg</td></tr>
<tr><td rowspan="2">Shaft and vessel material properties</td><td colspan="3">Elastic modulus</td><td colspan="3">density</td></tr>
<tr><td colspan="3">2.24E+10 Pa</td><td colspan="3">25 kN/m^3</td></tr>
<tr><td rowspan="2">Tank geometry</td><td colspan="2">Thickness of shaft</td><td colspan="2">Vessel diameter</td><td colspan="2">Water height</td></tr>
<tr><td colspan="2">150 mm</td><td colspan="2">8,600 mm</td><td colspan="2">4,400 mm</td></tr>
<tr><td rowspan="2">Foundation geometry</td><td colspan="2">Slab height</td><td colspan="2">Slab diameter</td><td colspan="2">Density</td></tr>
<tr><td colspan="2">1,500 mm</td><td colspan="2">5,000 mm</td><td colspan="2">25 kN/m^3</td></tr>
</table>

Table 5 Soil classification and mechanical properties

Soil type	SPT-N value	G (kN/m^2)	Unit weight of soil (kN/m^3)	Shear velocity wave V_s (m/s)
Very soft clay	3	30,994	13.5	150
Soft clay	6	53,964	17.0	176
Medium clay	9	74,641	18.5	198
Stiff clay	15	112,320	19.4	238
Very stiff clay	30	195,561	19.8	311

Table 6 NEHRP recommendation coefficient for high seismic zones

<table>
<tr><td colspan="3">EPA* and EPV* are assumed 0.4 for high seismicity risk area</td></tr>
<tr><td>Soil type</td><td>D</td><td>E</td></tr>
<tr><td>F_a^*</td><td>1.1</td><td>0.9</td></tr>
<tr><td>F_v^*</td><td>1.6</td><td>2.4</td></tr>
</table>

* EPA: Effective peak acceleration

* EPV: Effective peak velocity

* F_a and F_v: Soil dependent coefficients

Table 7 Soil stiffness considering soil classification and embedment ratio

Soil type	Stiffness type	Embedment ratio		
		0	0.5	1
Very soft clay	K_x (kN/m)	5.73E+09	8.60E+09	1.15E+10
	K_ρ (kN·m)	2.03E+11	4.50E+11	7.87E+11
	$K_{x\rho}$ (kN/m)	0.00E+00	7.17E+09	1.91E+10
Soft clay	K_x (kN/m)	1.00E+10	1.50E+10	2.00E+10
	K_ρ (kN/m)	3.53E+11	7.85E+11	1.37E+12
	$K_{x\rho}$ (kN/m)	0.00E+00	1.25E+10	3.33E+10
Medium clay	K_x (kN/m)	1.38E+10	2.07E+10	2.76E+10
	K_ρ (kN/m)	4.89E+11	1.09E+12	1.89E+12
	$K_{x\rho}$ (kN/m)	0.00E+00	1.73E+10	4.60E+10
Stiff clay	K_x (kN/m)	2.08E+10	3.12E+10	4.16E+10
	K_ρ (kN/m)	7.35E+11	1.63E+12	2.85E+12
	$K_{x\rho}$ (kN/m)	0.00E+00	2.60E+10	6.93E+10
Very stiff clay	K_x (kN/m)	3.62E+10	5.42E+10	7.24E+10
	K_ρ (kN/m)	1.28E+12	2.84E+12	4.96E+12
	$K_{x\rho}$ (kN/m)	0.00E+00	4.52E+10	1.21E+11

5.2 Results of Case Study

The Model 1 analysis in two different conditions (full and empty) shows that in the case of full capacity, the natural period would be 0.285 and in the empty case, it is detected to be 0.189. Evidently, results reveal maximum natural period, under 1 s and it is placed in acceleration sensitive area in seismic response spectrum. Natural period's analysis evaluation were also carried out for Model 2 and 3. The natural period is divided into impulsive and convective mode. Model 2 results show that the value of impulsive period remains under 1 sec and convective period would be 3.30 sec. It is approximately 12 times greater than natural period in Model 1 analysis condition.

Analysis for Model 3 was carried out and results of impulsive period reveal that the value of natural period is 0.249, which is 13% less than that of Model 1. All values of impulsive period for empty and full condition are shown in Fig. 10.

Considering the obtained results of sloshing analysis, all convective period values are placed on velocity or displacement sensitive parts of the response spectrum. The values of convective period in relation to fixed base analysis are shown in Table 8.

NEHRP provisions recommendation to base shear and overturning moment, excluding all fixed base conditions are shown in Table 9. It is observed that soil condition is more affected by base shear and overturning moment. In general, fixed base with lumped mass for soil type *D* (very hard clay), condition base shear is 7283.37 kN and for soil type *E* (very soft to hard clay) the base shear is estimated to be 5960 kN. It is observed that the base shear of soil type *D* is 22 percent higher than that of soil type *E*. By using FSI, the impulsive base shear would decrease around 30percent in each of soil conditions.

Results related to overturning moments are shown in Table 10. Similar to base shear, impulsive mode overturning moment in a particular soil condition and two mass analyses would be constant. There is a slight difference for convective mode overturning moment on Model 3 (uncoupled) when compared with other convective analysis.

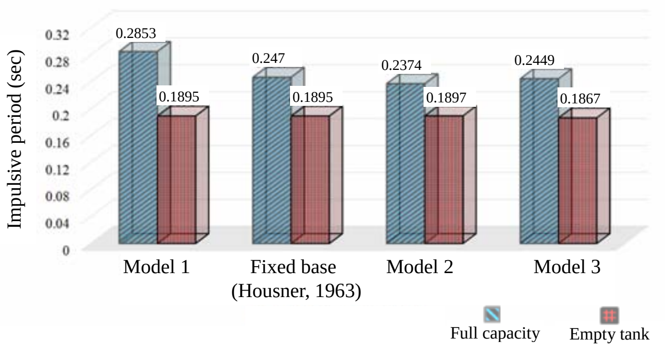

Fig. 10 Bar chart of impulsive period considering full and empty vessel condition

Table 8 Convective period values considering fluid-structure interaction method

	Fixed base (Housner 1963)	Model 2	Model 3
Convective period (sec)	3.2304	3.30	3.232

Table 9 Values of base shear considering fixed base condition with an emphasis on different soil categories for NEHRP

Model	Full tank				Empty tank			
	D		E		D		E	
	Impulsive	Convective	Impulsive	Convective	Impulsive	Convective	Impulsive	Convective
1	7.28E+03	0	5.96E+03	0	3.83E+03	0	3.14E+03	0
2	4.63E+03	4.62E+02	3.79E+03	2.60E+02	2.73E+03	0	2.23E+03	0
3	4.63E+03	4.95E+02	3.79E+03	2.79E+02	2.73E+03	0	2.23E+03	0

Unit of base shear is kN

Table 10 Values of overturning moments considering fixed base condition with an emphasis on soil categories for NEHRP

Model	Full tank				Empty tank			
	D		E		D		E	
	Impulsive	Convective	Impulsive	Convective	Impulsive	Convective	Impulsive	Convective
1	1.40E+05	0	1.15E+05	0	7.93E+04	0	6.05E+04	0
2	8.92E+04	9.19E+03	7.30E+04	5.11E+03	5.25E+04	0	4.30E+04	0
3	8.92E+04	9.74E+03	7.30E+04	5.48E+03	5.25E+04	0	4.30E+04	0

Unit of overturning moment is kN·m

The second part of the analysis indicates soil effects on FSI. The results of natural period analysis for Model 4 are shown in Fig. 11. The foundation embedment effects on natural period; base shear and overturning moment are evaluated. Considering soil classification by increasing the value of embedment ratio in similar soils, the natural period would decrease. Maximum effect of soil classification on impulsive period occurred when the embedment ratio is zero. In fact, the surface foundation would be extremely sensitive to soil stiffness variations. Period in this condition would decrease around 40% in soft to hard soil classifications.

Results of Model 5 (current study suggestion) and 6 are presented in Fig. 12. Similar to other analysis conditions, impulsive periods are less than 1 second. Maximum changes in impulsive periods with emphasis on FSSI would be observed when the embedment ratio is zero. Embedment ratio 0.5 and 1 reduce natural periods.

Results of convective periods (Fig. 13) show that there are no significant changes in different conditions of soil and embedment. Usually in all conditions, value of 3.23 s is a good prediction for convective natural period.

Results show that the impulsive period was more effective from soil type in compare with convective period. Figures 12 and 13 are carefully show this claim. It is observed that there are no significance variation by changing the soil in convective period part, despite of this there are significant variation considering soil stiffness changing.

Results of six suggested models reveal that SSI is more effective on non-embedment foundations. Base shear evaluation on suggested models show that the maximum effect of SSI would occur on SDOF elevated tanks. Furthermore, results show that maximum differences between coupled and decoupled methods would not be extremely significant in civil engineering problems.

Total base shear and overturning moment with emphasis on high seismicity risk for all suggested models are presented in Table 11. It is noted that there is no effect of embedment on base shear with emphasis on FSSI but there are a lot of effects on the overturning moment. In base shear assessment, it is observed that only Model 5 is affected by foundation embedment. This effect change of base shear is significant in soil type *E* but would be negligible in soil type *D*.

Maximum displacement for impulsive and convective parts for a surface foundation elevated tank is illustrated in Fig. 14. It is observed that the trend of liquid displacement is more remarkable in comparison with vessel mass. High period excitation would result in the generation of resonance in the liquid part and low period excitation would have an effect on the impulsive part of elevated tanks. Also, results of Fig. 14 indicate that soil stiffness variations would result in the phase difference in vessel displacement.

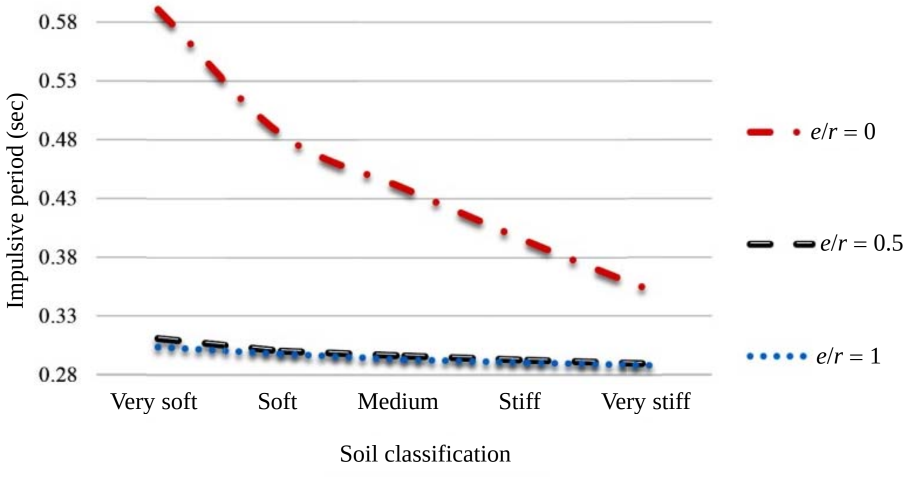

Fig. 11 Model 4 analysis results considering both of SSI embedment ratio effects

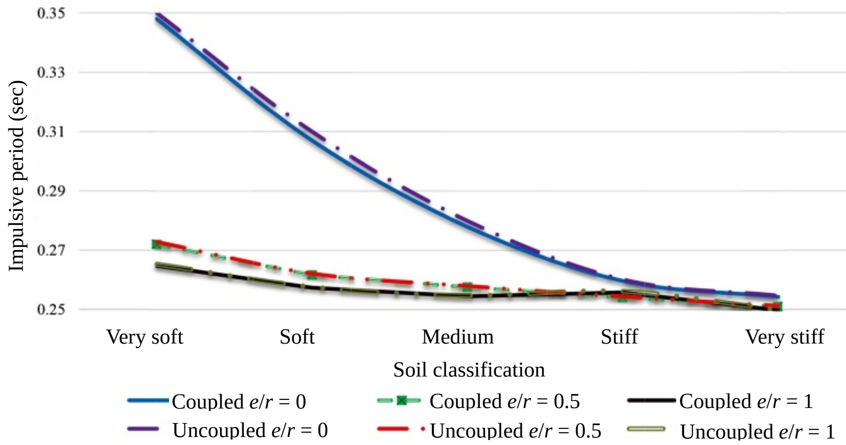

Fig. 12 Impulsive period of model 5 and 6 with emphasis on embedment ratios

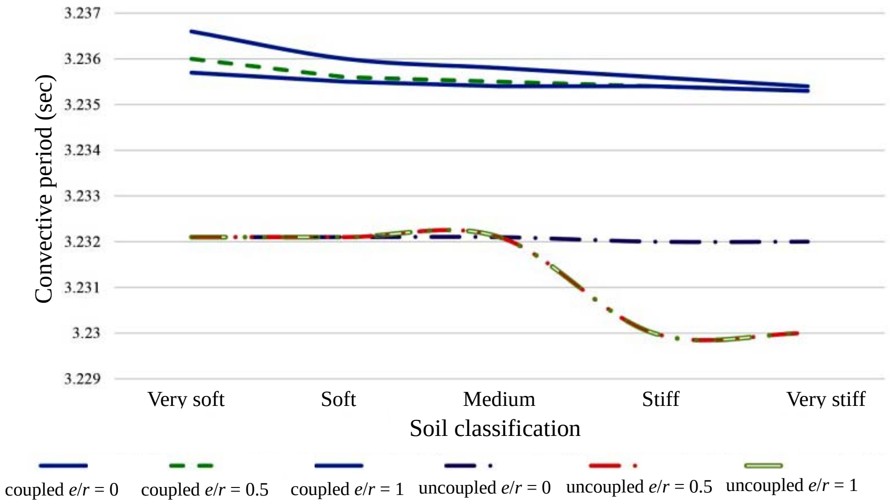

Fig. 13 Convective period of model 5 and 6 with emphasis on embedment ratios

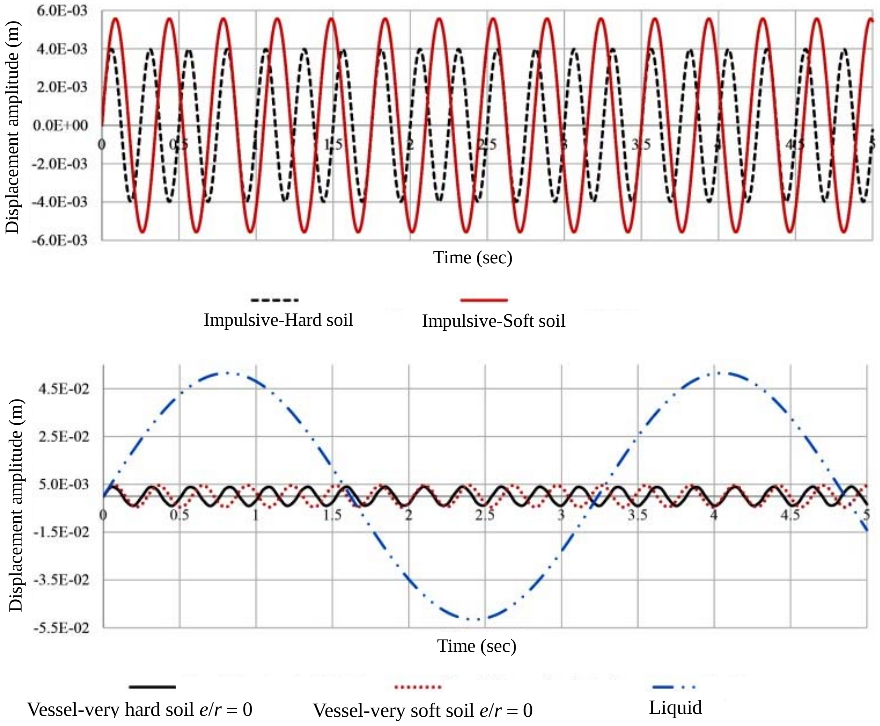

Fig. 14 Impulsive and convective parts displacement considering soil and embedment condition

Table 11 Total impulsive and convective base shear and overturning moment considering soil, fluid and embedment conditions

Model		Base shear (kN)			Overturning moment (kN·m)		
1	Soil type	Fixed Base			Fixed Base		
	E	5.96E+03			1.15E+05		
	D	7.28E+03			1.40E+05		
2	Soil type	Fixed Base			Fixed Base		
	E	3.79E+03			7.31E+04		
	D	4.65E+02			8.96E+04		
3	Soil type	Fixed Base			Fixed Base		
	E	3.80E+03			7.32E+04		
	D	4.66E+03			8.97E+04		
4	Soil type	$e/r = 0$	$e/r = 0.5$	$e/r = 1$	$e/r = 0$	$e/r = 0.5$	$e/r = 1$
	E	5.26E03 to 6.52E03	6.77E+03	6.77E+03	8.68E04 to 1.23E05	1.32E+05	1.37E+05
	D	8.20E03 to 8.28E03	8.28E+03	8.28E+03	1.54E05 to 1.56E05	1.62E+05	1.68E+05
5	Soil type	$e/r = 0$	$e/r = 0.5$	$e/r = 1$	$e/r = 0$	$e/r = 0.5$	$e/r = 1$
	E	5.52E+03	5.52E+03	5.52E+03	9.97E+04	1.03E+05	1.07E+05
	D	7.69E+03	7.69E+03	7.69E+03	1.22E+05	1.26E+05	1.31E+05
6	Soil type	$e/r = 0$	$e/r = 0.5$	$e/r = 1$	$e/r = 0$	$e/r = 0.5$	$e/r = 1$
	E	5.52E+03	5.52E+03	5.52E+03	9.97E+04	1.03E+05	1.07E+05
	D	7.69E+03	7.69E+03	7.69E+03	1.22E+05	1.26E+05	1.31E+05

5.3 CONCLUSIONS

In the present study, two relationships which can be used to assess the natural period of impulsive and convective part of RC elevated tanks considering FSSI are developed. Also, FSSI effects on an elevated tank in a specified soil domain and different embedment conditions has been studied. Results of this study are concluded as follows:

- Comparison between results of suggested relationships and numerical, regular, analytical and international code suggestions indicate that suggested relationships have an acceptable value for natural period. Obtained results have a good assessment in comparison with sub-structure FEM method in different soil classifications. Also, suggested relationships have maximally 15% deviation when compared with massless FEM method in soft soil conditions. For hard and medium soils, results of massless FEM have remarkable coincidence by suggested relationships.
- To develop suggested relationships for surface foundations, a part of each relation that indicate embedment and height effect is eliminated and suggested relationships are modified.
- Suggested relationships are proficient in determining the foundation depth effects on natural period of elevated tanks with an emphasis on FSSI. Accordingly, by increasing the embedment ratio, the natural period would decrease, so that for soft soil in embedment condition of 0.5, approximately 25% and for hard soil approximately 2% in comparison to surface foundation is observed.
- The influence of embedment ratios greater than 0.5 on natural period would decrease significantly. Moreover, soil classification effect in higher values of embedment ratio is also negligible.
- FSI study of elevated tanks demonstrates that in fixed base conditions, results of impulsive and convective period of coupled and uncoupled system (Models 2 and 3) have no significant variation when compared with regular analytical results such as Housner (1963). FSSI results gotten from the case study indicates that in similar soil conditions and embedment ratios there are no significant difference between coupled and decoupled analysis. In fact, it is observed that considering similarity of coupled and uncoupled results, the FSSI can be divided to FSI and SSI problem.
- The results of the current study indicate that there is no considerable effect of soil on convective part. Furthermore, the shape of vessel and height of liquid are more significant than soil condition effects on convective period.
- Assessment on base shear and overturning moment revealed that the maximum values of base shear occurred in case of SDOF and SDOF-SSI. In addition, it is observed that the maximum values of overturning would occur on SDOF-SSI by embedment ratio 1 and soft soil condition.
- Results of the present study showed that elevated tanks displacement is largely affected by soil condition and embedment ratio of foundation. Furthermore, it is observed that maximum displacement of liquid would be 10 times greater than impulsive part in a soft soil condition. Finally, the trend of liquid displacement has more remarkable effect when compared with vessel mass. High period excitation would result in generation of resonance in the liquid part and low period excitation would affect impulsive part of elevated tanks.

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