

PLASTIC SETTLEMENT EVALUATION OF EMBEDDED RAILROADS UNDER REPEATED TRAIN LOADING

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ABSTRACT

This study proposes a procedure to evaluate the total cumulative plastic settlement due to repeated train loading for designing embedded railroad track foundations. The approach is based on Li and Selig's method which was developed for ballasted railroads. An illustrative embedded track structure modified from Kaohsiung light rail is analyzed for demonstration. In addition, a parametric study is conducted to investigate the influence of the thickness, stiffness and plastic strain parameters of the subgrade soil and the stiffness of the prepared subgrade on the plastic settlement. The results show that a larger thickness of the subgrade does not necessarily produce larger settlement; the influence of soil strength on the plastic settlement is more significant than that of subgrade modulus; the CL soil can give the largest plastic strain for a low deviatoric stress ratio less than 0.8; the influence of modulus of the prepared subgrade is less significant to the plastic settlement than the thickness of the prepared subgrade.

Key words: Cohesive soil, embedded track, finite element method, plastic strain, plastic settlement, repeated load.

1. INTRODUCTION

Light rail is an urban transit system which has been widely used in Europe and American cities. In recent years, this system has been gradually introduced to metropolitan areas of Taiwan. The first light-rail transit system has begun to be built in Kaohsiung since 2013 and was scheduled to begin operation by the end of 2015. For reducing the traffic impact, some sections of Kaohsiung light rail share space with road traffic where tracks and trains run along the streets. Because the space is shared, the tracks are designed as an embedded type to have the same elevation with the surrounding ground. For embedded tracks on soft subsoil, the permanent deformation of the subsoil due to train loading is one of the important factors which controls the design life as well as the maintenance cost. Excessive settlement may cause pockets to form in the subgrade. Water can collect in the pockets and further deteriorate the subgrade. This may endanger the train riding safety and influence the road traffic when the tracks are excavated for repair.

Current design methods for railroad track foundations are mainly for ballasted railroads. Burrow *et al.* (2006) reviewed some design methods of railroad track foundations. The design methods reviewed were Li and Selig's method (1998a), International Union of Railways method (UIC719R 1994), British Railway method (Heath *et al.* 1972), UK Network Rail code (NR code 039, 2005) and West Japan Railway Company method

(WJRC 2002a, 2002b). In their review, major factors considered in the design procedures include axle load, sleeper type, length and spacing, rail section and speed, annual and cumulative tonnage, and subgrade condition. In those methods, Li and Selig's method completely considers the aforementioned factors and provides a quantitative procedure to design the thickness of the trackbed layers (ballast and sub-ballast layers) of railroad tracks for controlling the plastic deformation of the subgrade. Chai and Miura (2002) applied Li and Selig's method to analyze the permanent deformation of roads due to traffic loading. In their study, they proposed a modified empirical equation to compute the cumulative plastic strain of cohesive soils under repeated loading for the subsoil in normal-to slightly over-consolidated state and considering the effect of initial static deviatoric stress.

Without sleepers and ballast layers, an embedded railroad track structure generally combines the rail, track slab, foundation slab and subgrade, as shown in Fig. 1. The track slab replaces sleepers to support the rail and the foundation slab replaces ballast layers to sustain the track structure. The combined thickness of the track slab and foundation slab is generally 0.4 ~ 0.5 m. For tracks on poor subgrade, prepared subgrade below the foundation slab whose engineering properties have been improved can be included in design. According to UIC 719 code (2008), a fixed thickness of the prepared subgrade for Kaohsiung light rail was adopted. In UIC 719 code, the minimum thickness of the prepared subgrade is recommended based on the bearing capacity quality of the subgrade. Therefore, to have a cost-effective foundation design of embedded tracks, it is desirable to predict the train load induced time-dependent settlement for determining if the prepared subgrade is required and its thickness. In this study, we apply Li and Selig's method to develop a procedure for evaluating the permanent settlement of embedded railroads due to repeated train loading and further designing the prepared subgrade of embedded railroads when poor subgrade is encountered. Besides, a parametric study is performed to investigate the influence of the thickness, stiffness and plastic strain parameters of the subgrade soil and the stiffness of the prepared subgrade on the plastic settlement of railroads.

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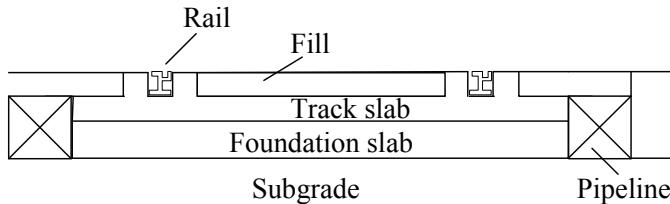


Fig. 1 Embedded track structure

2. LI AND SELIG'S METHOD

Fine grained subgrade soils subjected to repeated train loading will cause permanent settlement due to cumulative plastic shear strain, cumulative consolidation and cumulative compaction (Li and Selig 1996). Li and Selig (1996) proposed a method to compute the permanent settlement of fine grained subgrade. The method is briefly described as follows,

First, the cumulative plastic strain ε_p at each depth is computed:

$$\varepsilon_p (\%) = a \left(\frac{\sigma_d}{\sigma_s} \right)^m N^b \quad (1)$$

in which σ_d is the soil deviatoric stress; σ_s is the compressive strength of the soil; N is the number of repeated stress applications; a , b , and m are material parameters, which need to be determined for various soil types.

The above plastic strain model mainly considers the influences of the soil stress state, soil physical state, and soil type. For the soil stress state, the deviatoric stress is considered. The soil physical state as defined by the soil moisture content and dry unit weight is taken into account by soil static strength. The material parameters a , b , and m are dependent on soil type. Since no simple test technique exists to directly determine a , b , and m , the values in Table 1 based on back-calculated results are recommended when repeated loading tests are not available (Li and Selig 1998a). They generally increase with increasing clay content and soil plasticity, *i.e.*, with the trend of ML to MH to CL to CH (Li and Selig 1996). The parameter a is the reference plastic strain which represents the soil plastic strain induced when the soil is monotonically loaded to failure; the parameter b is an exponent which reflects the accumulation rate of plastic strain under repeated loading; the parameter m is an exponent which indicates the softening phenomenon of deviatoric stress on soils.

The next step is to integrate the plastic strains over the depth of the deformable part of the subgrade and then the total cumulative plastic deformation ρ can be determined as,

$$\rho = \int_0^T \varepsilon_p dt \quad (2)$$

in which T is the thickness of the subgrade.

With this method, Li and Selig (1998a, 1998b) further proposed a design method for ballasted railroad foundations to design the thickness of the granular ballast layers. In the proposed design method, the main work is to obtain the deviatoric stress in the subgrade due to loading. Through the computation of the plastic settlement with time, the track foundation can be designed to control the plastic settlement. For design, Li *et al.* (1998b) suggested that the allowable total cumulative plastic deformation ρ be set to 25 mm.

Table 1 Parameters a , b , m (Li and Selig 1998a)

Model parameters	Soil classification by USCS			
	ML	MH	CL	CH
a	0.64	0.84	1.1	1.2
b	0.10	0.13	0.16	0.18
m	1.7	2.0	2.0	2.4

3. PROCEDURE OF ANALYSIS

This study applies Li and Selig's method to analyze the plastic settlement for designing embedded railroad track foundation. The analysis procedure has the following three stages:

STAGE I. Collect the Information about the Design Traffic, Track Layer Condition, and Subgrade Characteristics

Design Traffic

Two parameters, the dynamic wheel load P_d and total equivalent number of design load applications, are required to represent the design traffic. According to the American Railway Engineering Association (AREA) (1996), the dynamic wheel load can be expressed as

$$P_d = \left(1 + \frac{0.0052V}{D} \right) P_s \quad (3)$$

where V is the train speed (km/h), D is the wheel diameter (m), P_s is the static wheel load.

Track Layer Condition

Obtain the thickness and elastic modulus of the track slab and foundation slab.

Subgrade Characteristics

Obtain the soil type, resilient modulus, and compression strength of the subgrade soil. When the prepared subgrade is included, the resilient modulus of the prepared subgrade is required.

STAGE II. Compute the Deviatoric Stress in the Subgrade Soil

A three-dimensional finite element model is built based on the track structure arrangement and loading. Considering the effect of three-dimensional stress, the deviatoric stress can be computed as

$$\sigma_d = \left(\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2} \right)^{0.5} \quad (4)$$

where σ_1 , σ_2 and σ_3 are the maximum, middle and minimum principal stresses, respectively. Since P_d is transformed from P_s based on Eq. (3), we also can compute σ_d based on P_s first and then modify it with Eq. (3).

STAGE III. Compute the Plastic Settlement and Adjust the Track Layer Arrangement and the Thickness of the Prepared Subgrade

Based on the σ_d profile obtained in Sect. 3.2, the number of repeated stress applications N , and material parameters a , b and m , the total cumulative plastic settlement can be computed using Eqs. (1) and (2). Check if the computed plastic settlement exceeds the allowable value. If yes, change the track layer arrangement or include the prepared subgrade until the plastic settlement meets the criterion.

4. ILLUSTRATIVE EXAMPLE

In this section, we use an embedded track structure modified from Kaohsiung light rail (Chen *et al.* 2014) as shown in Fig. 2 to demonstrate the application of the aforementioned procedure.

Kaohsiung light rail is situated in the alluvial plain area of the Jenai River and the Chienzhen River. The shallow soil layer of depth varying from 2 to 12 m along the route is generally silty clay of SPT-N less than 10. The groundwater levels were high, usually 1~3 m deep below the ground surface. The unconfined compressive strength of soil varied from 40 kPa to 60 kPa. The soil modulus for clay estimated from in-situ vertical plate loading testing was in a range between 10~50 MPa.

Analysis Parameters

The track structure has two tracks (Tracks L and R). The associated analysis parameters for the track foundation are as follows:

- Train and Traffic Conditions

Length of train: 35 m

Train speed: 50 km/h

The train loading conditions are shown in Fig. 3. Eight axles, each with 130 kN (single wheel load is 65 kN).

Wheel diameter: 59 cm

Equivalent number of load applications: 48820 times/year

Design life: 30 years

Allowable cumulative plastic settlement: 25 mm

- Parameters of the Track and Foundation Slabs

Track slab: Width of 6.2 m, thickness of 0.165 m, elastic modulus $E = 26291$ MPa, poisson ratio $\nu = 0.2$

Foundation slab: Width of 6.2 m, thickness of 0.335 m, elastic modulus $E = 21786$ MPa, poisson ratio $\nu = 0.2$

- Parameters of the Subgrade

Soil type: Soft clay (CL)

Thickness of subgrade: 10 m, as shown in Fig. 2.

Referring to the soil condition of the site of Kaohsiung light rail, assume the compression strength of the subgrade $\sigma_s = 40$ kPa, the resilient modulus $E = 10$ MPa (assuming $E = 250 \sigma_s$) and poisson ratio $\nu = 0.49$. According to Table 1, the parameters a , b , m are 1.1, 0.16 and 2.0, respectively.

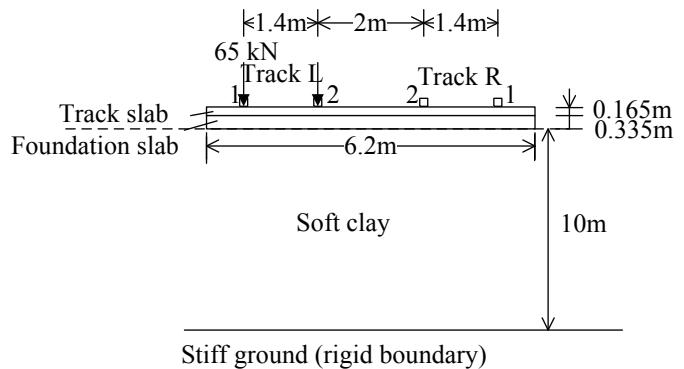


Fig. 2 Example track structure

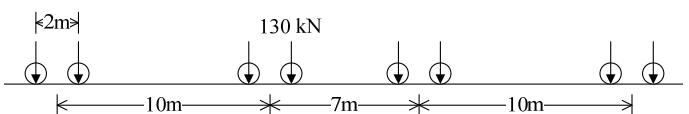


Fig. 3 Train loading conditions

Analysis Model and Results

Based on the above analysis parameters, a three-dimensional finite element numerical model is built using ANSYS (2004) as displayed in Fig. 4. In the global coordinate system of the model, Z and X axes are set to be parallel to the longitudinal and transverse directions of the railroad, respectively, and Y axis is along the direction of depth. Three-dimensional 20-node solid elements (SOLID 186) are adopted in the model to simulate the track slab, the foundation slab and the subgrade. The elements are isotropically elastic. The element size is finer near the track region. For the track slab, the range of the element size is $0.35 \sim 0.7$ m (X) \times 0.165 m (Y) \times 0.5~1 m (Z); for the foundation slab, the range of the element size is $0.35 \sim 0.7$ m (X) \times 0.1675 m (Y) \times 0.5~1 m (Z); for the subgrade, the range of the element size is $0.35 \sim 1$ m (X) \times 1 m (Y) \times 0.5~1 m (Z). The aspect ratio of each element is limited to below five. Interface elements are set between the foundation slab and the subgrade. The size of the whole model is 101 m (Z) \times 120 m (X) \times 10 m (Y) and roller supported boundaries are set.

With the application of the static single track train loading on the model (as displayed in Fig. 4), Fig. 5 shows the elastic deformation contour. Since loading is applied at single side (the left way track), it causes an uneven stress distribution in the soil and therefore leads to an asymmetric soil deformation pattern. Figure 6 shows the vertical displacement in the longitudinal direction from the leftmost axle to the rightmost axle (length of 29 m), L_1 and L_2 are for the left way, and R_2 and R_1 are for the right way. In the transverse direction, the maximum displacement is at L_1 ; the displacement decreases from left to right; the minimum displacement is at R_1 . The maximum displacement in the longitudinal direction is at the middle of the train.

From the FEM results, the principal stresses σ_1 , σ_2 and σ_3 at the nodes of interest can be recorded and the deviatoric stress can be computed accordingly based on Eq. (4). In ANSYS, the deviatoric stress can be automatically calculated. Figure 7 displays the static deviatoric stress below the foundation slab upon the axle loading for transverse sections at different longitudinal locations from the middle of the train. It can be seen that the distributions of these sections are close and the maximum deviatoric

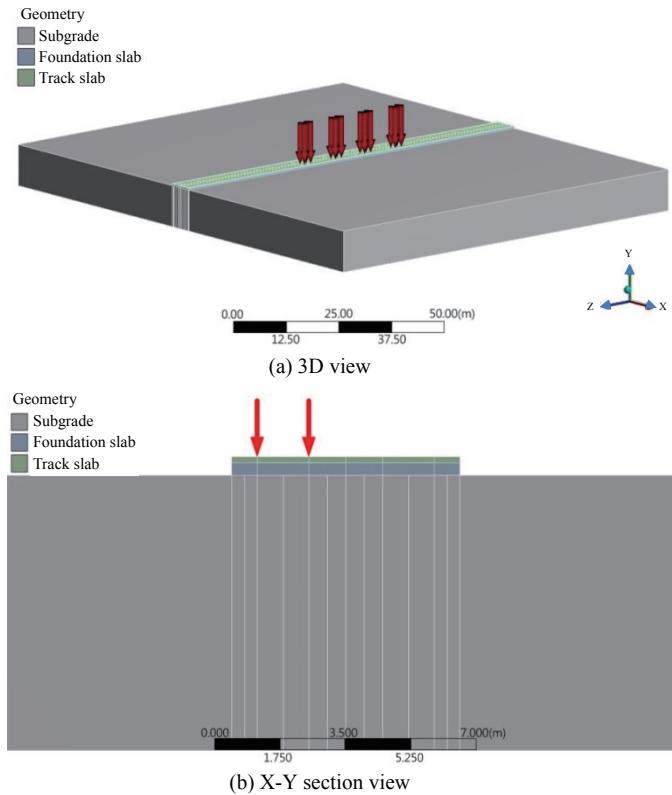


Fig. 4 ANSYS finite element model

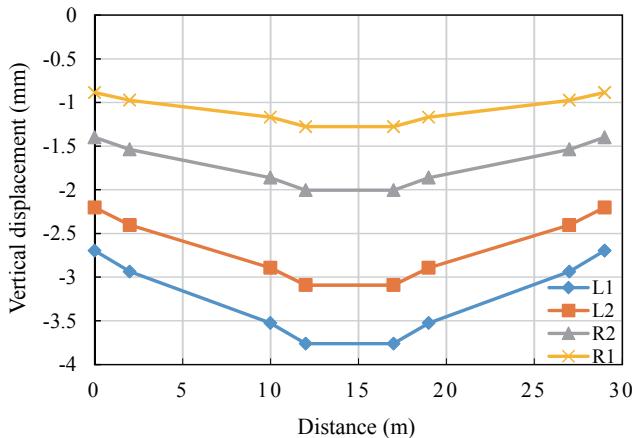


Fig. 6 Longitudinal elastic vertical displacement at L1, L2, R2, R1 racks

stress occurs at the left end of the slab for the transverse section at the location of the nearest wheel to the middle of the train (*i.e.*, 2.5 m from the middle of the train), which implies the maximum plastic strain would happen at this position. Focusing on the maximum plastic settlement, as shown in Fig. 8, on the cross-section at 2.5 m from the middle of the train we choose Position A (left end of the slab) to demonstrate the process and results of plastic settlement analysis. Besides, for comparison, at the same section Position B at the middle of the slab is selected.

Firstly, the static deviatoric stress distributions with depths at Positions A and B are retrieved. Further, they are multiplied by a modification factor to consider the dynamic effect which is $P_d / P_s = 1 + 0.0052 \times 50/0.59 = 1.44$, as shown in Fig. 9. At Position A (at the end of the slab), the largest deviatoric stress is

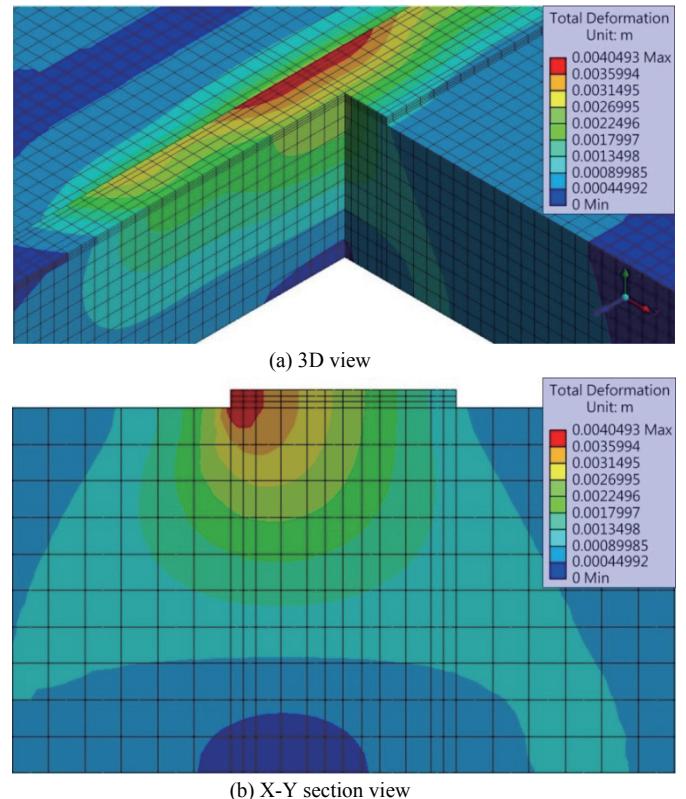


Fig. 5 Elastic deformation contour

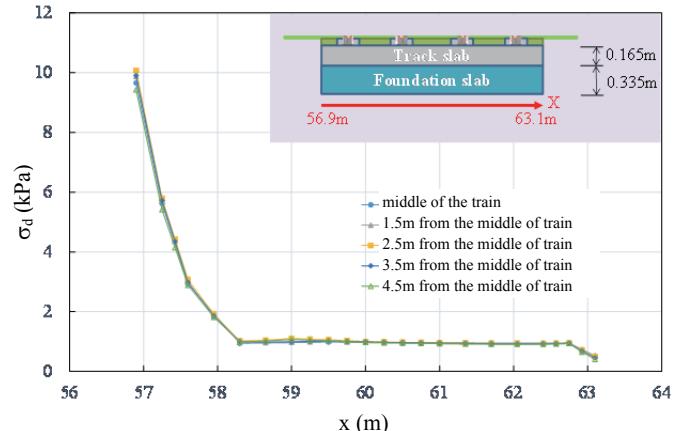


Fig. 7 Deviatoric stress distribution below the foundation slab at different longitudinal locations

14.5 kPa which is at the bottom of the slab and the stress decreases as the depth increases. The stress decreases to about 3.73 kPa at a depth of 10 m. At Position B, the deviatoric stress is the smallest at the bottom of slab of about 1.43 kPa, gradually increasing with depth and reaching the maximum of 5.3 kPa at a depth of about 3 m; after that depth the stress decreases gradually with depth and eventually the stress will be close to that of Position A below a depth of about 6 ~ 7 m.

Based on Fig. 9, we can compute the total cumulative plastic settlement with time using Eq. (2) as shown in Fig. 10. From this figure, it can be seen that at Position A between the third and fourth years, the settlement will exceed the allowable value of 25 mm; however, at Position B the settlement does not exceed the allowable value in the design life of 30 years. To make the

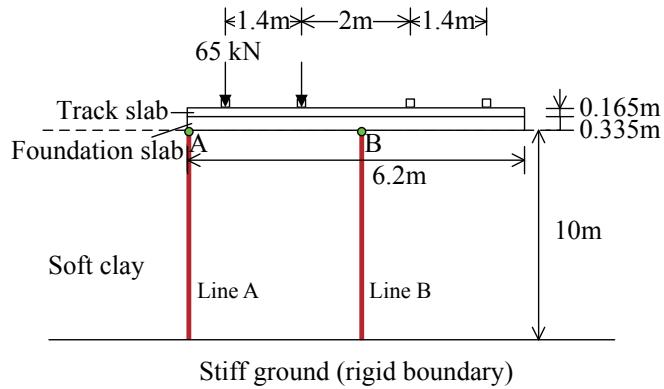


Fig. 8 Observation positions for soil stresses and plastic settlement

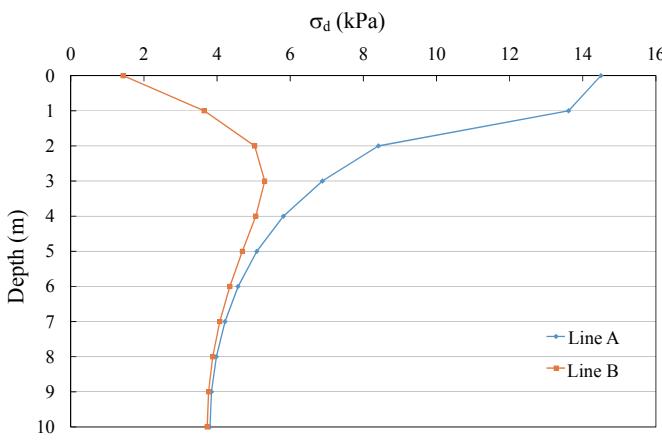


Fig. 9 Deviatoric stress distribution with depth at Positions A and B (application of P_d)

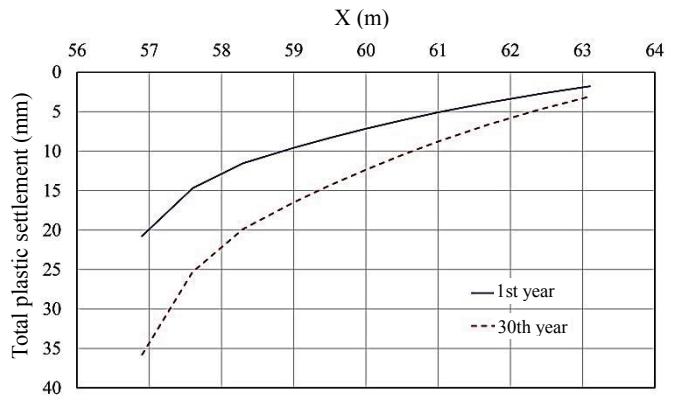


Fig. 11 Total cumulative plastic settlement profiles on the cross-section at the location of the nearest wheel to the middle of the train after first and 30th years

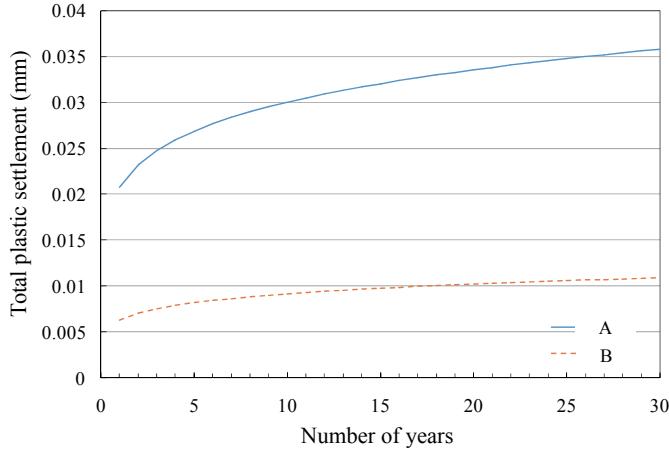


Fig. 10 Total cumulative plastic settlement at Positions A and B

settlement at Position A not exceed the allowable settlement, the prepared subgrade beneath the foundation slab is needed. Following the same procedure, the plastic settlement profile of this section after the first and 30th years can be built as shown in Fig. 11. It can be seen that the plastic settlement at the left end of the slab is the maximum, decreasing from the left side to the right side.

According to UIC 719 code, the minimum resilient modulus of 80 MPa is required for prepared subgrade. Therefore, assuming that the resilient modulus of the prepared subgrade is 100 MPa, we analyze the cases of the prepared subgrade thickness of 0.5 m, 1.0 and 1.5 m to evaluate the influence of the prepared subgrade. Figure 12 displays the deviatoric stress profiles with depth for the specified prepared subgrade thicknesses and compares them with that of the case without the prepared subgrade. From the figure, it can be seen that with increasing prepared subgrade thickness, the deviatoric stress at Position A is reduced mainly within a depth of about 2 m; the deviatoric stress at Position B is reduced mainly within a depth of 5 m.

Figure 13 shows the total cumulative plastic settlement with time for the prepared subgrade thicknesses of 0.0, 0.5 and 1.0 m. From the figure, it can be seen that when the prepared subgrade thickness of 1.0 m is adopted, the plastic settlement can be limited to below 25 mm within the design operation life. The thickness of the prepared subgrade appears to be larger than the minimum thickness specified by UIC 719 code.

From the above demonstration, the proposed procedure based on Li and Selig's method can be successfully applied to evaluate the plastic settlement of embedded railroad track foundations and further to quantify the influence of the prepared subgrade on the plastic settlement.

5. PARAMETRIC STUDY

On the basis of the above example, this section further applies the proposed procedure to investigate the influence of the thickness, stiffness and plastic deformation parameters of the subgrade soil and the stiffness of the prepared subgrade on the plastic settlement evaluation.

Influence of Thickness of Subgrade Soil

Three values of subgrade thickness are adopted for comparison: 5, 10 and 20 m. Figure 14 shows the deviatoric stress profiles with depth for the three cases. It can be seen that the deviatoric stress basically decreases with increasing thickness of the subgrade. Note that except for the case of 5m-thick subgrade, the

deviatoric stress profiles at Positions A and B coincide below depths of 8 ~ 10 m. Figure 15 displays the plastic settlement at Position A. It can be seen that the case of 10 m-thick subgrade has the largest settlement while the case of 5 m-thick subgrade has the smallest settlement. Although the case of 5 m-thick subgrade has larger deviatoric stress, the subgrade thickness is smaller and therefore the contribution of plastic strain is limited. On the other hand, for the case of thickness 20 m which has a larger thickness, since the deviatoric stress decays with depth, the plastic settlement is not the largest. Figure 16 further displays the settlement and settlement ratio profiles with depth for the 10 m-thick subgrade. It can be seen that the major contribution to plastic settlement (90% of total plastic settlement) is within a depth of 7 m (1.13 times of track slab width). Therefore, the plastic settlement contribution for the soil after a depth of 7 m is minor.

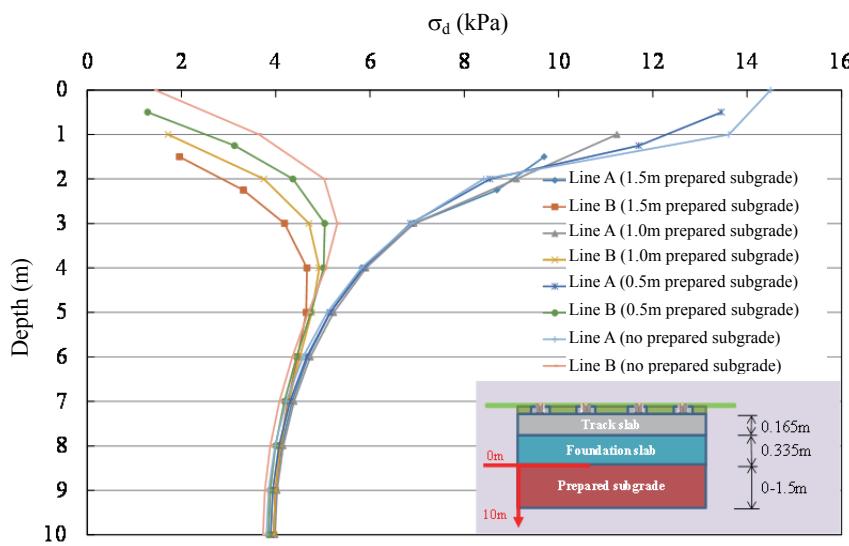


Fig. 12 Deviatoric stress profiles with depth for different thickness of prepared subgrade

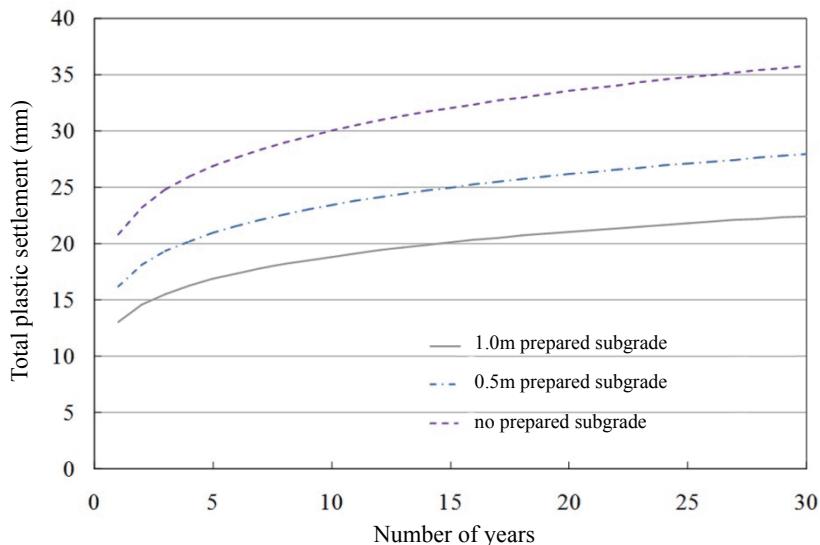


Fig. 13 Total cumulative plastic settlement at Position A for different prepared subgrade thickness of 0.0, 0.5, 1.0 m

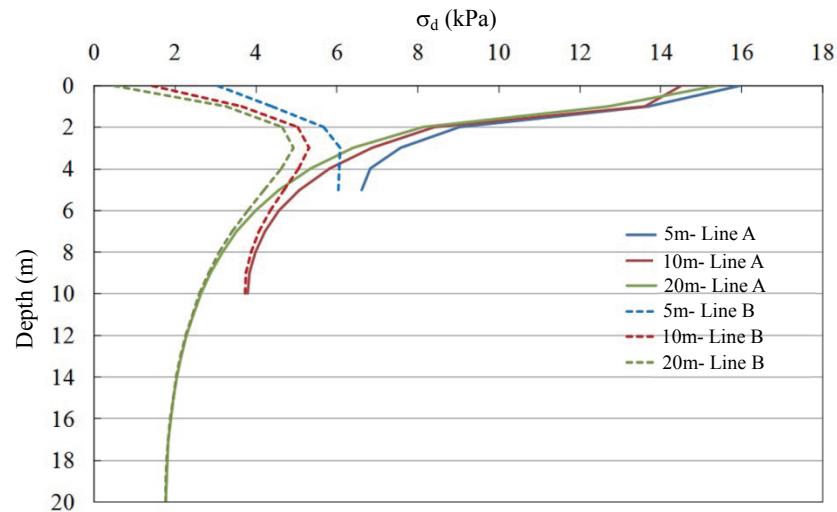


Fig. 14 Deviatoric stress profiles at Positions A and B for different subgrade thicknesses

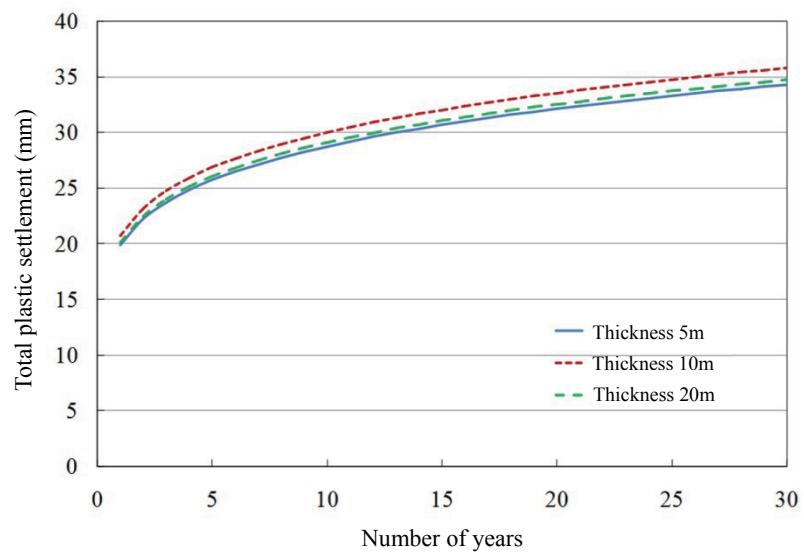


Fig. 15 Total cumulative plastic settlement at Position A for different thicknesses of subgrade

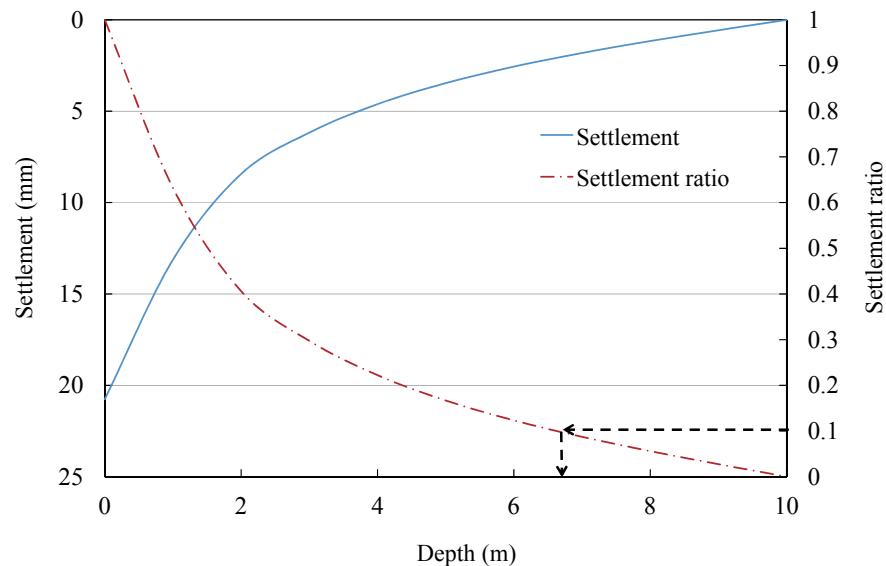


Fig. 16 Settlement and settlement ratio of 10 m thick subgrade after one loading cycle

Influence of Resilient Modulus of Subgrade

Three values of subgrade modulus are adopted for comparison: 10, 20 and 40 MPa, and the corresponding compression strengths are proportionally set to be 40, 80 and 160 kPa, respectively. Figure 17 shows the deviatoric stress profiles with depth for the three cases. At Position A, the deviatoric stress at most depths of soil slightly increases with increasing subgrade

modulus; however, at Position B, the deviatoric stress slightly decreases with increasing subgrade modulus. Figure 18 compares the plastic settlement for the three cases. It can be seen that irrespective of small difference in deviatoric stress, the plastic settlement for subgrade modulus 10 MPa is the largest because of the smallest soil strength. The influence of the soil strength is more significant than that of the subgrade modulus.

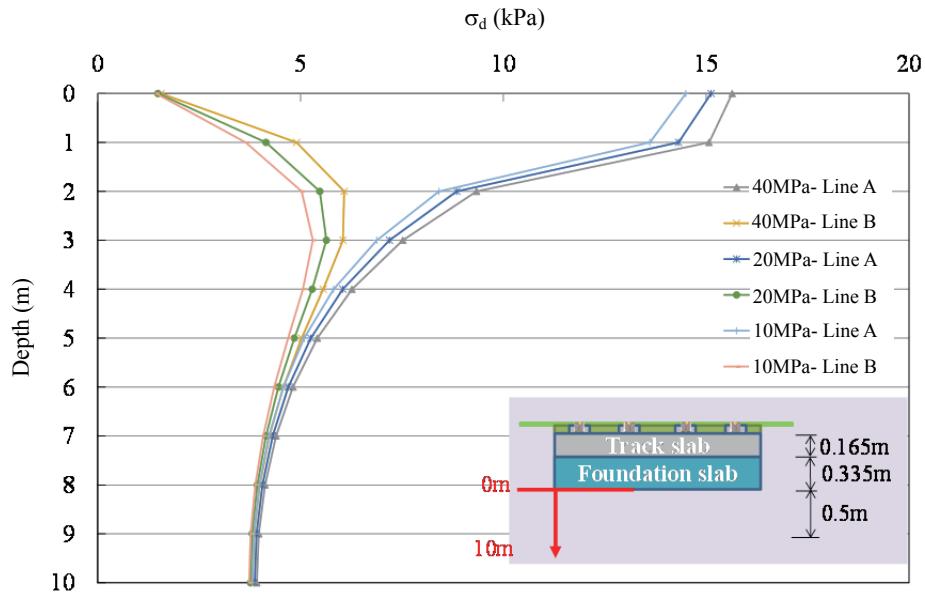


Fig. 17 Deviatoric stress profiles at Positions A and B for different subgrade moduli

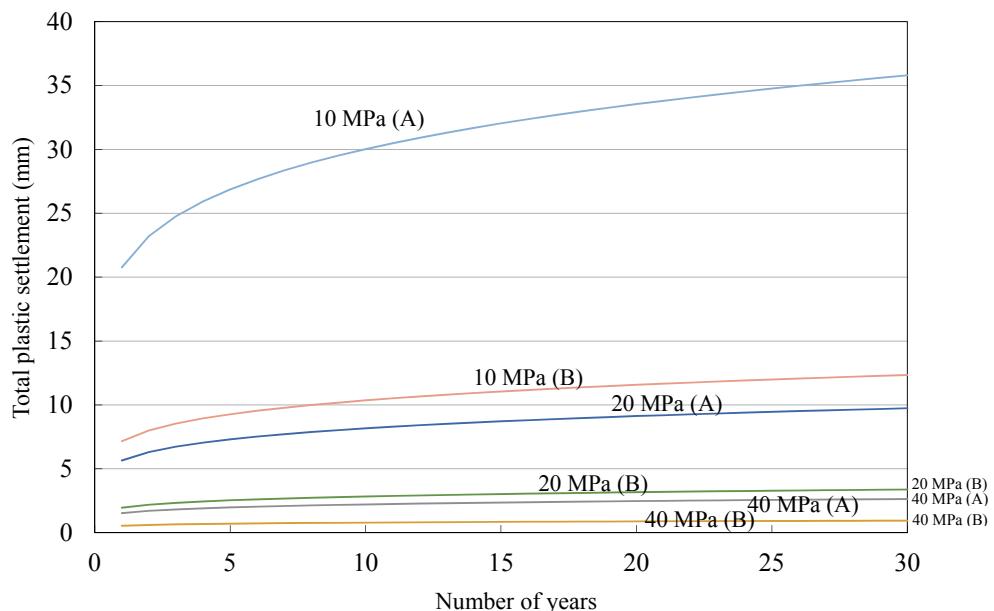


Fig. 18 Total cumulative plastic settlement for different moduli of subgrade

Influence of Plastic Strain Parameters

Li and Selig (1996) stated that material parameters a , b , m generally increase with increasing clay content and soil plasticity, a higher value is related to a higher plastic strain accumulation, and a more cohesive soil generally is more susceptible to plastic-strain accumulation. However, a larger m does not lead to a larger plastic strain for the deviatoric stress ratio (σ_d / σ_s) less than one. For example, Fig. 19 compares the total plastic settlement of the 10 m-thick subgrade using the plastic strain parameters of CH and CL soils. From the figure, it can be seen that the CH soil does not lead to larger plastic settlement than the CL soil. Figure

20 shows the relationships of plastic strain and deviatoric stress ratio after one loading cycle for ML, MH, CL, CH soils. The plastic strain increases with increasing deviatoric stress ratio. Generally, the ML soil gives the smallest plastic strain and the MH soil gives the second smallest plastic strain in the most range of stress ratio. When the deviatoric stress ratio is less than about 0.8, the CL soil gives the largest plastic strain; after this ratio, the CH soil gives the largest plastic strain. Therefore, in Fig. 19, the CL subgrade produces larger plastic settlement than the CH subgrade because of low deviatoric stress ratios.

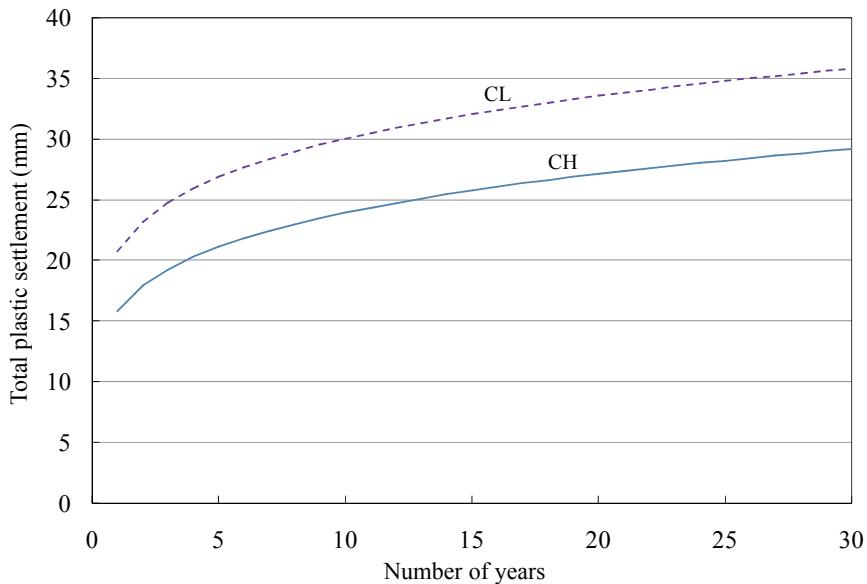


Fig. 19 Total cumulative plastic settlement at Position A for CH and CL soils

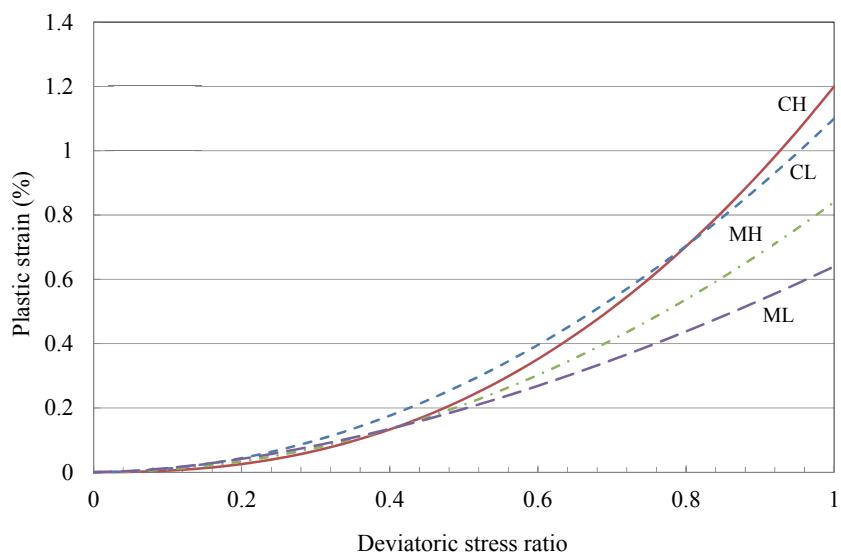


Fig. 20 Relationships of plastic strain and deviatoric stress ratio

Influence of Resilient Modulus of Prepared Subgrade

The illustrative example in the previous section has shown that the thickness of prepared subgrade is effective to reduce plastic settlement of the soft subgrade. Considering a 0.5 m-thick prepared subgrade, four values of resilient modulus of the prepared subgrade are adopted to investigate the influence of the modulus of the prepared subgrade. Figure 21 shows the deviatoric stress profiles. It can be seen that the variation of the resilient modulus of the prepared subgrade affects the profiles mainly within a depth of 2 m. For a smaller modulus, the maximum de-

viatoric stress occurs at the bottom of the prepared subgrade; with increasing modulus, the maximum deviatoric stress occurs at depths of about 0.5 ~ 1 m below the prepared subgrade. Figure 22 shows the total cumulative plastic settlement for the cases. It can be seen that an increase in the modulus of the prepared subgrade results in a slight decrease in the plastic settlement. As a whole, the difference on the deviatoric stress profile at the shallow depth would not cause a significant difference in the total cumulative plastic settlement.

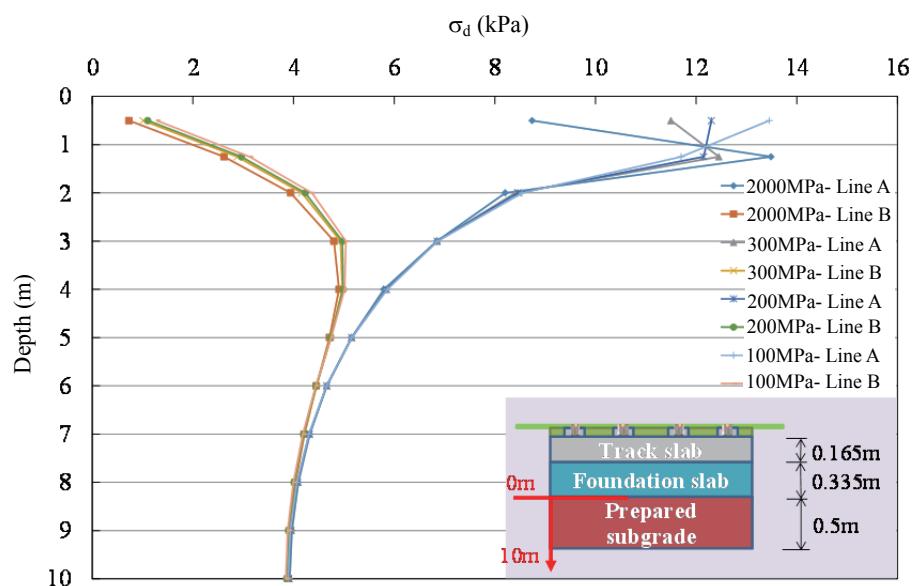


Fig. 21 Deviatoric stress profiles at Positions A for different modulus of prepared subgrade

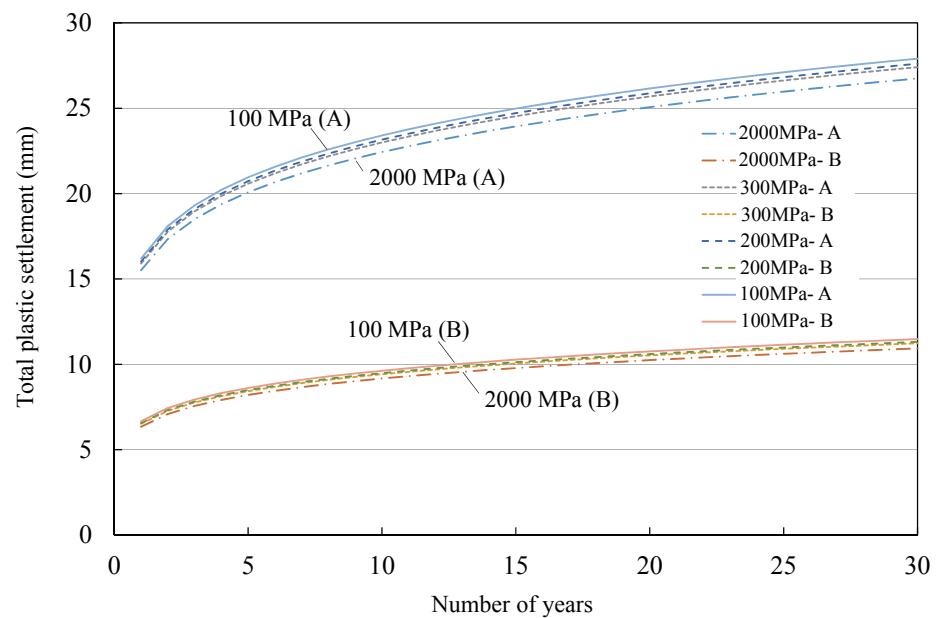


Fig. 22 Total cumulative plastic settlement for different modulus of prepared subgrade

6. CONCLUSIONS

Based on this study, the following conclusions can be drawn:

1. For the foundation design of embedded railroad track, Li and Selig's method can be applied to evaluate the plastic settlement of the railroad due to repeated train loading and to design the thickness of the prepared subgrade for reducing the plastic settlement.
2. A larger thickness of fine grained subgrade does not always produce a large plastic settlement. An increase in the subgrade thickness represents an increase in the range of soil that possibly produces plastic strain, but on the other hand the deviatoric stress may decrease due to a larger range of stress distribution.
3. With increasing the subgrade modulus and strength proportionally, the influence of soil strength on the plastic settlement is more significant than that of subgrade modulus.
4. Comparing the plastic strain parameters for ML, MH, CL and CH soils, the CH soil which represents more cohesive soil does not always produce the largest plastic strains. For a smaller deviatoric stress ratio, the CL soil produces the largest plastic strain; for the deviatoric stress ratio larger than 0.8, the plastic strain of the CH soil becomes the largest.
5. The thickness of prepared subgrade is effective to reduce plastic settlement. The modulus of the prepared subgrade can affect the distribution pattern of deviatoric stress at shallow depths of soil; however, its influence is less significant to the total plastic settlement.

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