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EFFECTS OF PLASTIC POTENTIAL ON THE HORIZONTAL STRESS IN ONE-DIMENSIONAL CONSOLIDATION

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ABSTRACT

The coefficient of earth pressure at rest K_0 is obviously important in geotechnical problems. The influences of a plastic potential on the minor principal effective stress during one-dimensional consolidation are examined by finite element consolidation analysis. Results of experimental and numerical investigations indicate that the coefficient of earth pressure at rest during one-dimensional consolidation appears to be governed by the plastic potential and time effects of secondary compression.

Key words: One-dimensional consolidation, secondary compression, coefficient of earth pressure at rest, plastic potential.

1. INTRODUCTION

The relationship of the vertical compressibility and the applied load is used to predict one-dimensional consolidation settlement in a conventional analysis. In usual laboratory consolidation tests, the clay specimen is confined laterally by a rigid metal ring and loaded in the vertical direction. The change of the horizontal stress acting on clays is not measured during consolidation. However, the mechanism of one-dimensional consolidation exhibiting secondary compression should be investigated in the multi-dimensional stress field. Akai and Sano (1985) have found that the K_0 value in one-dimensional consolidation increases during one-dimensional consolidation and the rate of secondary compression decreases with the elapsed time.

And also it is well recognized that the use of an incorrect K_0 value for the initial stress condition in the field has serious effects on the prediction for the deformation of soft grounds. In order to understand the exact deformation behavior of clays, it is necessary to examine accurately the stress strain time relation. If a typical elasto-plastic clay model for example, Modified Cam clay is used in the finite element one-dimensional consolidation analysis, the calculated K_0 value may differ significantly from the initial value (Britto and Gunn 1987). It is also well known that the K_0 values given by Modified Cam clay model are larger than the initial observed K_0 values. The cause for the miscalculation depends on the plastic potential of Modified Cam clay.

It is the purpose of this paper to present finite element consolidation analysis with an elasto-visco-plastic clay model based on a new plastic potential and to discuss the effects of the plastic potential on the K_0 values and the rate of secondary compression.

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2. ELASTO-VISCO-PLASIC FINITE ELEMENT CONSOLIDATION ANALYSIS

2.1 Secondary Compression Model

The relationship of normally-consolidated clays between the effective stress and the total volumetric strain in one-dimensional consolidation is given by the components of primary and secondary compression as follows (Takeda *et al*. 2012)

$$v = v_p + v_s = \frac{\lambda}{f_0}\ln\left(\frac{p}{p_0}\right) = \frac{\lambda^*}{f_0}\ln\left(\frac{p}{p_0}\right) + \alpha\ln\left(\frac{\dot{v}_i}{\dot{v}_s}\right) \quad (1)$$

where λ and λ^* are the compression index defined by the total and primary compression respectively, f_0 is the initial specific volume, p is the effective mean stress, α is the coefficient of secondary compression defined by the volumetric strain, v is the total volumetric strain, v_p is the primary compression, $\dot{v}_s$ is the rate of secondary compression and $\dot{v}_i$ is the initial rate of $\dot{v}_s$. The superposed "·" implies time rate and the prime denoting effective stress is omitted in this paper.

Assuming that the rate of secondary compression and the amount of secondary compression at any time in secondary consolidation stage are $\dot{v}_{sf}$ and v_{sf}, $\dot{v}_s$ and $\dot{v}_i$ can be calculated from Eqs. (2) and (3), respectively.

$$\dot{v}_s = \dot{v}_i \cdot \exp\left(\frac{-v_s}{\alpha}\right) \quad (2)$$

$$\dot{v}_i = \dot{v}_{sf} \cdot \exp\left(\frac{v_{sf}}{\alpha}\right) \quad (3)$$

2.2 Yield Function, Plastic Potential and Visco-Plastic Flow Rule

The rate of effective stresses $\underline{\dot{\sigma}}$ at any loading stage is related to the rate of elastic strains $\underline{\dot{\varepsilon}_e}$ through a drained elastic stress strain matrix $\underline{D_e}$ as

$$\dot{\underline{\varepsilon}}_e = \dot{\underline{\varepsilon}} - \dot{\underline{\varepsilon}}_p - \dot{\underline{\varepsilon}}_s = \underline{D}_e^{-1} * \dot{\underline{\sigma}} \tag{4}$$

where subscripts e, p and s indicate elastic, plastic and creep (due to secondary compression) strain components.

The rate of plastic strain and creep strain, $\dot{\underline{\varepsilon}}_p$ and $\dot{\underline{\varepsilon}}_s$ is given by the following visco-plastic flow rule. The details of visco-plastic theory have been discussed elsewhere (Owen and Hinton 1980).

$$\dot{\underline{\varepsilon}}_p = \langle F \rangle \frac{\partial Q_p}{\partial \underline{\sigma}} \tag{5}$$

$$\dot{\underline{\varepsilon}}_s = \langle \dot{v}_s \rangle \frac{\partial Q_s}{\partial \underline{\sigma}} \tag{6}$$

where F represents the yield function and Q_p and Q_s are the plastic potential. The notation < > means that F and $\dot{v}_s$ are equal to zero, if F and $\dot{v}_s$ are less than zero.

Further, in order to express the actual visco-plastic behaviors of clays, it is very important to choose the appropriate yield function and plastic potential (Roscoe and Burland 1968). Equation (7) represents the yield function F and the potential function Q_p for the plastic strain used in this paper (Dafalias *et al.* 2002).

$$F(\equiv Q_P) = q^2 + M^2 p(p - p_0) + 2\gamma_p pq + \gamma_p^2 pp_0 = 0$$

for plastic strains (7)

$$Q_s = q^2 + M^2 p(p - p_0) + 2\gamma_s pq + \gamma_s^2 pp_0$$

for creep strains (8)

where q is the deviatoric stress, M is the slope of the critical state line, p_0 is the effective mean stress of K_0 normally consolidated clay and γ is constant. Subscripts "p" and "s" of γ correspond to plastic and creep strain components, respectively. The value of γ should be determined by triaxial tests and trial-and-error procedures until the calculated K_0 value coincides with the observed ones.

As can be seen from Eqs. (5) and (6), the elasto-visco-plastic analysis assumes that stresses in excess of yield are indeed legal (Perzyna 1963). So, this has the advantage that explicit relationships between the increment of stress and elsto-visco-platic strain are not required. Experience shows that computations tend to converge faster than those by a standard initial stress method using the elasto-plastic stress strain matrix $\underline{D}_{ep}$ (Smith 1982).

The solid line shown in Fig. 1 is the yield locus used in this paper. If the constant γ_p in Eq. (7) is assumed to be equal to zero, a broken line calculated by Eq. (7) corresponds to the yield locus of Modified Cam clay.

K_0 value depends on the value of γ_p and γ_s, however, it cannot be expressed explicitly in terms of γ_p and γ_s. In order to determine the value of γ_p for the elasto-plastic soil model, it is necessary to integrate Eq. (5) under the K_0 compression condition.

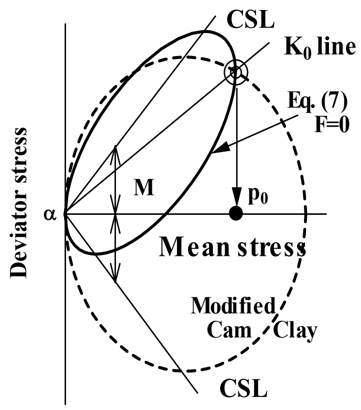

Fig. 1 Yield locus $F = 0$

2.3 Coupled Biot's Consolidation Equation

The application of the finite element discretization at any time to equilibrium and flow continuity equation leads to the following matrix formulation. Almost all of the subroutines used in author's program have been written in Smith (1982). For further details about the subroutines, please refer to specialized finite text books such as the one authored by Smith (1982).

$$\begin{bmatrix} \underline{K} & \underline{C} \\ \underline{C}^T & \Delta t * \underline{P} \end{bmatrix} \cdot \begin{bmatrix} \underline{d}_{t+\Delta t} \\ \underline{u}_{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{C}^T & \underline{0} \end{bmatrix} * \begin{bmatrix} \underline{d}_t \\ \underline{u}_t \end{bmatrix} + \begin{bmatrix} \underline{F} + \underline{F}_p + \underline{F}_s \\ \underline{0} \end{bmatrix} \tag{9}$$

where $\underline{K}$ is the stiffness matrix, $\underline{P}$ is the permeability matrix, $\underline{C}$ is the coupling matrix, $\underline{0}$ is the zero matrix, $\underline{d}_t$ is nodal displacements and $\underline{u}_t$ is nodal excess pore pressures. Subscript "t" means time. $\underline{F}_t$ indicates external nodal loads. $\underline{F}_p$ and $\underline{F}_s$ are equivalent nodal loads converted from the plastic strain and the creep strain, respectively.

The total strain components $\underline{\varepsilon}$ are calculated by using $d_{t+\Delta t}$ obtained from Eq. (9). Then, effective stresses $\underline{\sigma}$ and equivalent nodal loads $\underline{F}_s$ and $\underline{F}_p$ are obtained from Eqs. (10) and (11), respectively.

$$\underline{\sigma} = \underline{D}_e(\underline{\varepsilon} - \underline{\varepsilon}_p - \underline{\varepsilon}_s) \tag{10}$$

$$\underline{F}_s = \int \underline{B}^T \underline{D}_e \, \underline{\varepsilon}_s \, dxdy \quad \text{and} \quad \underline{F}_p = \int \underline{B}^T \underline{D}_e \, \underline{\varepsilon}_p \, dxdy \tag{11}$$

where $\underline{B}$ is strain displacement matrix and $\underline{\varepsilon}_s$ and $\underline{\varepsilon}_p$ is the creep strain components and plastic strain components , respectively.

3. EXPERIMENTAL ASSESSMENT

3.1 Elasto-Plastic Analysis

Soil parameters obtained from one-dimensional consolidation and triaxial tests are shown in Table 1. K_{0i} is the coefficient of earth pressure at rest obtained from K_0 consolidation test and used for the calculation of the initial stress in each elements. K_0 consolidation tests are carried by using a triaxial test. For one-dimensional consolidation the horizontal stress is adjusted to maintain the condition of zero lateral strain so that the difference of both volumetric and vertical strain should be zero.

Table 1 Soil parameters 1

λ	κ	ϕ′ (°)	K_{0i}	ν	α	c_v(cm²/min)	K (cm/min)
0.233	0.03	38.4	0.4	0.286	0.0018	0.08	4.4×10^{-6}

v is Poisson's ratio calculated from K_{0i} value. Permeability, k (= $m_v \cdot c_v \cdot \gamma_w$), is calculated from c_v (coefficient of consolidation), m_v (coefficient of volume compressibility), and γ_w (= unit weight of water).

In order to conduct the elasto-plastic analysis, it is necessary to postulate that $\lambda = \lambda^*$ and $\alpha = 0$. For finite element calculations the maximum drainage distance H is divided in 10 elements with the equal length as shown in Fig. 2. Elements are linear isoparametric quadrilateral and contain 8 nodal displacements and 4 nodal excess pore pressures.

Figure 3 shows the calculated volumetric strain time curves with observed ones. The calculated curves according to different γ_p values are shown by a solid line (γ_p = 0.52) and a dotted line (γ_p = 0) and also compared with results calculated by using program CRISP developed by University of Cambridge (Britto and Gunn 1987). The value of γ_p is obtained from the calculation based on an in-house program. CRISP adopts the associated flow rule ($F \equiv Q$) and γ_p = 0 in Eq. (7). The agreement among those calculated curves which correspond to the predictions based on Terzaghi's consolidation theory, is seen to be very close. However, some differences between the calculated and observed curve arise from the time dependent behavior of clays.

Figure 4 shows the effective stress path (p and q relation obtained from the calculated results in Fig. 2) during consolidation. Solid straight line with (γ_p = 0.52) shows that K_0 value remains constant during one-dimensional consolidation. The dotted curve with γ_p = 0 corresponds to that of Modified Cam clay model calculated by CRISP. Those K_0 values increase with consolidation. Modified Cam clay model can not reproduce the observed K_0 value. It must be emphasized that plastic potential has a predominant influence on the effective stress path and the K_0 value.

3.2 Elasto-Visco-Plastic Analysis

Assuming the ratio of the compression index λ^*/λ = 0.9 or 0.8 and using α = 0.0018 and $\gamma_p = \gamma_s = 0.52$, volumetric strain time curves shown in Fig. 5 are calculated. There is no significant difference in two curves according to the assumption about the ratio of compression index λ^*/λ. The agreement between the calculated and observed curves is seen to be more reasonable by considering secondary compression. However, the validity for the choice of ratio λ^*/λ is a big problem to be solved. New experimental technique is needed to measure separately each primary and secondary compressions during consolidation.

Figure 6 shows the effective stress path which becomes to a straight line. The K_0 value remains constant during consolidation.

A lot of consolidation tests carried out by the authors have indicated that the coefficient of K_0 was constant during consolidation. However, in the discussion of the behavior during secondary compression, Mesri and Castro (1986) concluded that secondary compression is not an effect caused by K_0 condition. Still mores, the majority of published experimental results showed an increase in K_0 with time (Schmertman 1983).

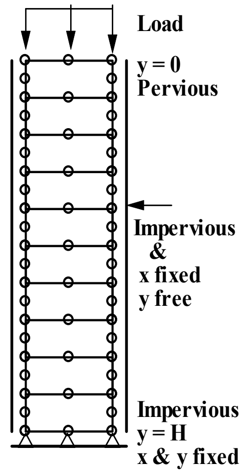

Fig. 2 Finite element mesh

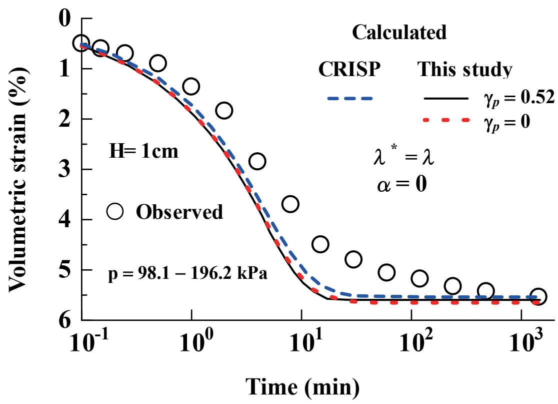

Fig. 3 Strain time curves 1 (elasto-plastic analysis)

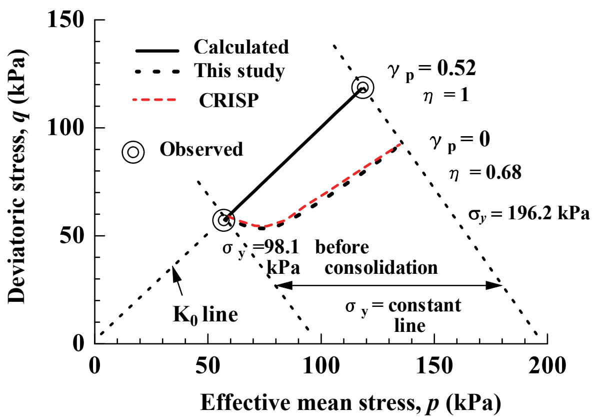

Fig. 4 Effective stress path 1 (elasto-plastic analysis) Calculated by FEM

3.3 K_0 Value During Secondary Compression

Akai and Sano (1985) have been conducted K_0 triaxial consolidation tests and demonstrated that the K_0 value increases with secondary compression and the rate of secondary compression gradually decreases with time. Figure 7 shows the strain and K_0 value time curves due to their long term K_0-triaxial consolidationtests. Soil parameters obtained from their test results are shown in Table 2. Constant γ_p included in the plastic potential

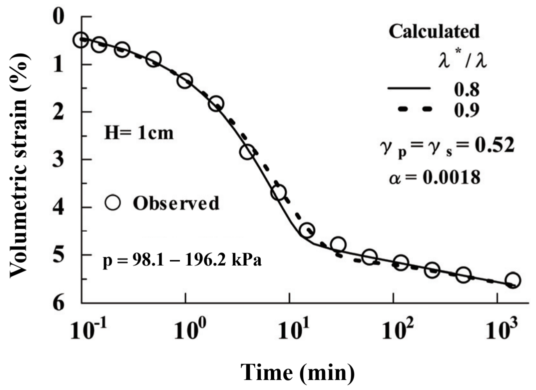

Fig. 5 Strain time curves 2 (elasto-visco-plastic analysis)

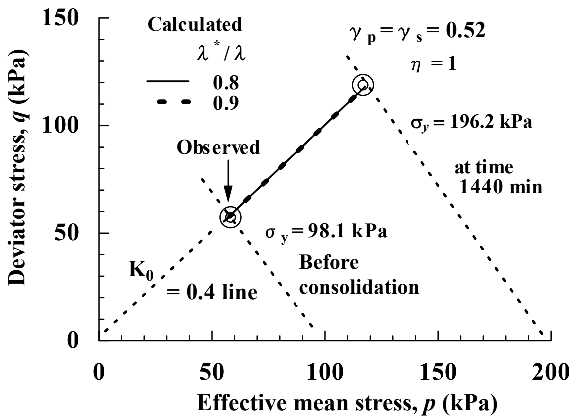

Fig. 6 Effective stress path 2 (elasto-visco-plastic analysis)

Table 2 Soil parameters 2 (after Akai and Sano 1985)

λ	κ	φ′	K_{0i}	ν	α	c_v (cm^2/min)	K (cm/min)
0.215	0.04	35.0	0.54	0.286	0.002	0.048	1.9×10^{-6}

Q_p, is again determined by the trial and error calculation. However, it is impossible to find out the appropriate value of γ_s at present. Using the following case, the volumetric strain time curves and the effective stress path are calculated.

Case 1: $\gamma_p = \gamma_s = 0.14$ calculated by in-house program

Case 2: $\gamma_p = 0.14$ and $\gamma_s = 0$ (assumed)

The calculated results are shown in Figs. 7 and 8. The K_0 value due to Case 1 remains constant during consolidation and volumetric strain time curves have some differences in the stage of secondary compression. According to Case 2, the K_0 value of the red broken line increases with secondary compression. However, there are large differences between the calculated and observed K_0 value.

The calculated volumetric strain time curves are linear with the logarithm of time although the observed rate of secondary compression decreases with time. This discrepancy depends on the proposed secondary compression model expressed by Eq. (2). Because the rate of secondary compression is a function of only α and v_s. Furthermore in order to calculate the large change of

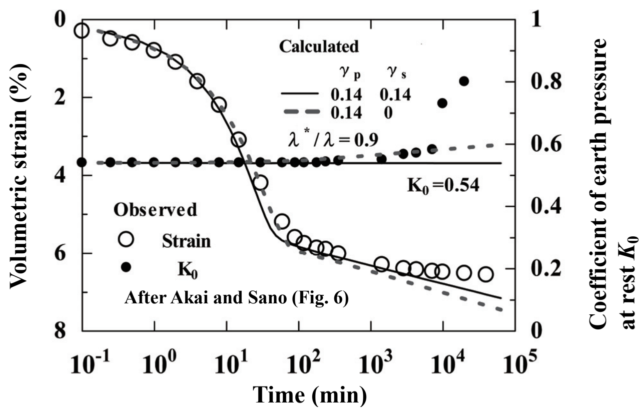

Fig. 7 Strain time curves 3 (elasto-visco-plastic analysis) Observed curves obtained from Akai and Sano

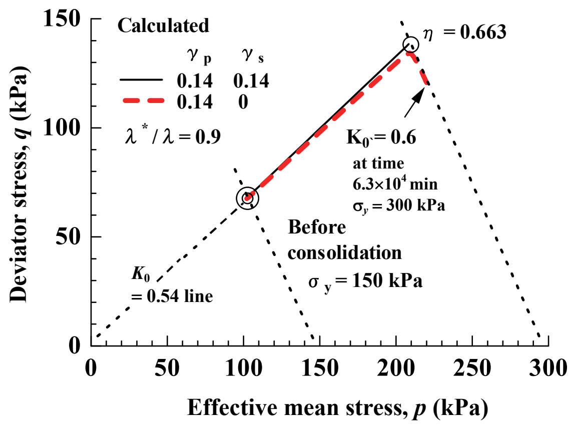

Fig. 8 Effective stress path 3 (elasto-visco-plastic analysis)

K_0 value, it is necessary to also modify the constant γ_p for plastic potential. It is considered that further experimental investigation is needed about the relation between secondary compression and K_0 value during one-dimensional consolidation.

4. CONCLUDING REMARKS

The finite element method of analyzing one-dimensional consolidation exhibiting secondary compression developed from the elasto-visco-plastic model based on a new plastic potential, appears to give some reliable predictions of both the K_0 value and volumetric strain time curves for laboratory consolidation tests. Finally, it is emphasized that the stress strain time relation of clays should be examined by not only the deformation but also the actual working stresses on a clay element.

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