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# ABSTRACT

Construction and industrial dynamic sources can produce environmental vibration problems for the supporting soils and adjacent structures. High vibrations and unacceptable dynamic settlements could disturb sensitive devices and even be the cause of structural damage and foundation failure. This paper aimed at investigating the technique to mitigate and control the dynamic effect of vibrating sources like generator using confined cell below the foundation. Numerical modeling of a vibrating source resting on a circular footing with/without confined cell is studied at different cell depth and diameter using Plaxis dynamic version 8.2. A series of axisymmetric model with geometry damping were run via Rayleigh damping after check and validate the program. The effect of confinement on the deformation behavior and dynamic response of supporting soil were investigated. The consequence of such technique on the peak particle velocity and excess pore water pressure in addition to sub-grade damping was analyzed. The results showed that, installing the cell with minimum diameter closer to the foundation. Based on numerical simulations, the peak particle velocity and the excess of pore water pressure for monitored point at the free field are reduced by 90%. This method can be considered an alternative technique to increase the sub-grade damping by 230%. Using barrier in the form of confined cell is an effective screening of vibration that can be achieved by proper interception, scattering and diffraction of surface waves.

Key words: Generator, foundation, dynamic response, sand, confinement.

# 1. INTRODUCTION

Vibration and dynamic responses produced from traffic, vibrating equipments, construction activities, earthquakes and machinery can cause extensive damage to both the structure and sub-structure in extreme circumstances. Dynamic motions initiated in the soil are instantaneously transmitted to foundations causing adverse effect on both the supporting soil and the superstructures. Damages may occur due to instability of soil which results in extensive ground movements including differential movement. The initiating cause can be often identified as the adverse response of the soil-foundation system under dynamic forces. Vibrating, rotating, reciprocating and impacting equipments create machine-induced vibration and/or shock, which are transmitted into their supporting system and the soil. Rotating machines and equipments that are not properly balanced produce centrifugal forces which in turn create steady state and random vibrations. Machines generating pulses or impacts, such as forging presses, injection molding, impact testers, hammers, centrifugal pumps and compressors are the most predominate sources of vibration and shock. Several theoretical and experimental studies have been carried out based on the experimental investigation of Woods (1968) on controlling of surface waves by open trenches. Beskos et al. (1986) and Dasgupta et al. (1990) conducted two-dimensional and three- dimensional boundary element studies on vibration isolation using open and in-filled trenches. Simplified models for designing open and in-filled trenches developed by Ahmad and Al-Hussaini (1991). Active isolation using open trenches to control the vibration dissipation to adjacent area is discussed by Ahmad et al. (1996) using a 3-D circumstances. Developing a finite/infinite element scheme to study isolation effectiveness of open/in-filled trenches and elastic foundations in reducing ground vibrations caused by the passage of trains are studied by (Yang and Hung 1997; Hung et al. 2004; Ju 2004). Di Mino et al. (2009) used open trenches for vibration control. Klein et al. (1997) adopted a 3-D boundary element code to study the screening effect of open trenches. Adam and Estorff (2005) employed coupled boundary element and finite element algorithm to study the attenuation of train-induced building vibrations using open trenches. Alzawi and El Naggar (2001) performed full-scale experimental study on open and geofoam filled trenches supported by 2-D finite element approach. The scopes of all the previous works are limited to the study of vibration isolation by single trenches (open or in-filled) except for the investigation of Hwang and Tu (2006) on the screening performance of several shallow open trenches. El-Naggar and Chehab (2005) studied the vibration barriers for shock-producing equipments. Jesmani et al. (2011) explored the efficiency of geometrical properties of an open trench in the ground vibration as an active isolation of deep foundations resting on a homogenous half-space clay soil using a three dimensional finite element method (FEM).

Based on the paper in literature, these researches are mainly focused on reducing the vibration by open and in-filled trenches in which the Rayleigh waves play a significant role in transmission of ground vibrations. This technique was needed to excavate a large quantity of earth to construct such trenches and required to transfer it far from the site so it considered uneconomic method. In addition, sometimes it can not be excavated in the site due

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to adjacent building and street. These trenches also cause a ground loss which tend to damage of the existence structures

Exceptionally, minor researchers investigated the effect of lateral containment/confinement by sheet pile placed below the embankment as a method of seismic mitigation, confirmed by Kimura *et al.* (1997) and Elgamal *et al.* (2002). Consequently, in the present paper, a new alternative technique is used for mitigating the machine disturbance using confined cell. The cell is used as a new lateral confinement tool compared with other technique of vibration isolation.

This paper provided an alternative technique for vibration isolation of machine foundation instead of in-filled sand trenches or force isolation. This suggested technique depends on using confined cell below the machine foundation. The cell is used as a new lateral confinement tool compared with other technique of vibration isolation. The technique is low cost and does not need to excavate the site. Therefore, the present paper was aimed to study the effects of increasing the sub-grade stiffness and control the dynamic disturbance results from vibrating sources using confined cell locally below the vibration source foundation. The adopted technique may be considered a new economic strategy of vibration isolation from any excitation sources. The vibrating source on the suggested confined foundation is investigated via finite element method using Plaxis dynamic version 8.2 (2002).

# 2. NUMERICAL MODELING AND SELECTION OF PARAMETERS

The problem under investigation simulated a vibrating source like generator with circular reinforced concrete foundation with/without confined cell resting on a layer of medium sand (Fig. 1). Due to the three dimensional nature of the problem, an axisymmetric model is adopted. The physical damping due to the viscous effects is taken into consideration via Rayleigh damping.

The axisymmetric model was used with the 15 node element. The mesh was generated by the program and refined in the area around the footing. The subsoil is consisted of a deposit of sand layer of 15 m thickness, assumed to be linear elastic in the analysis. The properties of the adopted sand ( $\gamma = 20 \text{ kN/m}^3$ ,  $\nu = 0.3$ ,  $\phi = 35^\circ$ , E = 50000 kPa).

The model boundaries should be sufficiently far from the region of interest, to avoid disturbances due to possible reflections. Although special measures are adopted in order to avoid spurious reflections (absorbent boundaries), there is always a small influence and it is still a good habit to put boundaries far away. In a dynamic analysis, model boundaries are generally taken further away than in static analysis. So, the Raleigh damping is considered at vertical boundaries and taken ( $\alpha$ ,  $\beta = 0.01$  as specified in the manual). The Raleigh damping can also be used to define the plastic properties of soil during analysis procedure. The ground water table is assumed at the ground surface to consider the excess pore water pressure, thus the soil material is assumed to be undrained.

The vibrating source is a generator founded on a 0.50 m thick of reinforced concrete footing of diameter equal to 1 m as stated in the manual model of Plaxis. Oscillations caused by the generator are transmitted through the footing into the soil. These oscillations with frequency of 10 Hz and amplitude of 10 kN/m<sup>2</sup> as shown in Fig. 2. In addition to the weight of the footing, the stress of the generator is assumed 10 kN/m<sup>2</sup> and it was modeled as a uniformly distributed load.

The footing has a weight of 125 kN, it is assumed to be elastic in the finite element analysis and simulated as an elastic beam element. Its plate properties are normal stiffness,  $EA = 19 \times 10^6$  kN/m, flexure rigidity, EI = 937500 kN/m<sup>2</sup>/m and the Poisson's ratio v = 0.20.

The confining cell was made of unplasticized polyvinyl chloride (UPVC) with different depth and diameters. The thickness of all the confining cells adopted in this study is 4 mm. Its actual strength depends on the wall thickness, uniformity, rate of loading and temperature of plastic materials. The mechanical properties of the tested cell are shown in Table1. The models were run at different cell depth (Hc/D) and diameter (Dc/D) see Fig. 1; where, Hc is the cell height, D is the footing diameter and Dc is the cell diameter. The investigated parameters of Hc/D are 0.25, 0.50, 1, 1.5 and 2, while the Dc/D are in range of 1.10, 1.30, 1.50 and 1.75. The cell is also simulated as an elastic beam element with input parameters of axial stiffness EA and bending stiffness EI which are calculated according to cell diameter. For soil structure interaction, the interface element is used between the soil and the cell by default value as a rigid case.

The selected monitored point #1 at the foundation level within the confined zone, whereas point #2 is located outside the confined zone in the free field at a distance of 0.6D from the face of footing. These points were used to identify their performance during the dynamic excitation.



Fig. 1 Schematic view of model and problem under investigation

## 3. ANALYSIS PROCEDURE

A series of dynamic numerical models was run for the system with/without confined cell at different studied parameters (cell depths and diameters). The calculation procedure involves three calculation phases. In the first phase, the footing is built and static load (weight of the generator) is applied. The second phase is the situation when the generator is running. In this phase, a vertical harmonic load with a frequency of 10 Hz and amplitude of 10 kN/m<sup>2</sup> is applied to simulate the vibrations transmitted by the generator. Five cycles with a time interval of 0.50 sec are considered. In the third phase the generator is turned off at estimated end time of 1 sec and the soil vibrates freely. All parameters in the dynamic analysis are set to zero. The last two stages involve dynamic calculations.

The screening effect of the confined cell which acted as a barrier is measured in the terms of amplitude reduction factor (Am) defined by Woods (1968). The amplitude reduction factor (Am) is not uniform over a range of investigation. It is, therefore, logical to express the degree of isolation in terms of average amplitude reduction factor (Am). Lesser the value of Am, better is the screening or mitigating effect and vice-versa.

For assessment of cell effectiveness, the amplitude reduction ratio, (Am), is the ratio between amplitude with confined cell and amplitude without confined cell.

The system effectiveness can be evaluated based on the observed displacement, velocity or acceleration with and without confined cell. Here, the results are presented in the form of Am in monitored displacement, which is calculated by normalizing the post-cell peak particle total displacement ratio (Am is the ratio between total displacement for footing on confined cell to displacement of without cell), for observed points #1 and #2.

#### Table 1 The properties of used UPVC cell (PVC Handbook 2005)

Specific gravity	1.4
Tensile modulus	28 10 <sup>3</sup> kPa
Tensile strength	55 10 <sup>3</sup> kPa
Maximum hydraulic pressure for 1 h at 23°C	23 Bar
Water absorption at 100°C for 24 h	$4 \text{ mg/cm}^3$



Fig. 2 Applied load-time curve in the analysis

#### 3.1 Verification of the Finite Element Model

The verification of the control FE models (vibration isolation by different methods) was previously made as stated by Jesmani et al. (2011) and other investigators in literature. On the other hand, neither experimental nor analytical results could be found in the literature for comparison in respect to the proposed confined cell below the foundation under dynamic load. However, in order to check the validity of the current confined cell below the foundation, the results of experimental tested of open trenches as a vibration isolation method as reported by Alzawi and El Naggar (2011) was used. The developed model was verified by comparing the results from full scale field experimental test presented in the case of open trench. All the parameters of the model were copied from the data presented in Alzawi and El Naggar (2011). The source of excitation was a Lazan type (MO2460) mechanical oscillator with eccentric masses. The oscillator was driven by a 7.5 HP 220 V three phase motor capable of generating sinusoidal force of 23.5 kN peak-to-peak.

The chosen ground motion during the exciting has frequency of 40 Hz. The present validate model aimed at determining the vertical velocity particles at different distance from the vibratory source in case of open barrier. The comparison of the field and numerical model under a given case is provided only for the case where the source of vibration is located at the first location from the barrier (2.5 m). The adopted dimension of wave barrier or the trench wall is 20 m length, 0.25 width and 3 m depth. The same soil condition is also adopted in the finite element analysis by Plaxis. Fig. 3 presents the relationship between the normalized vertical soil particle velocity at different distance from the source of disturbance for the proposed finite element analysis and experimental tests. The numerical results follow the trend of the field results and a good agreement is achieved. Accordingly, in the view of the fact that the adopted Plaxis dynamic version is capable of predicting the behavior of foundation on confined sub-grade under source of vibration. It was decided to use the computer package for the analyses proposed in this research.



Distance from the vibration source (m)

Fig. 3 Comparison of field and *FE* model attenuation results for exciting frequencies 40 Hz

# 4. RESULTS AND DISCUSSION

# 4.1 Effect of Cell on the Behavior of Foundation Sub-grade

It is well known that the body waves for any vibrating sources propagate radically outward from the source along hemispherical wave fronts and that Rayleigh wave propagates radically outward on a cylindrical wave front. All of the waves encounter an increasingly larger volume of material as they travel outward. While using confined cell as vertical barrier around the foundation can modify and alter the wave propagation from radical to downward displacement propagation due to barrier effect of such cell. The existence of confined cell below the foundation is considered an active screening or active isolation. Placing the confined cell or barrier can provide a screening effect due to significant increase in sub-grade stiffness during dynamic loading. The cell can provide a gradual sub-grade densification during loading stages; as a result, the deformation characteristics sub-grade are distinctly modified. A progressive densification is took place within the confined sub-grade during the dynamic loading. Hence, the cell can be controlled both the vertical and horizontal displacement induced from the dynamic effect as plotted in Fig. 4 for the total displacement vectors. This figure demonstrated that the effectiveness of using such cell in controlling the sub-grade displacement induced from vibration source compared with normal footing without cell. It has been found that the cell acted as a vertical barrier which confined the sub-grade and prevented the heave at the surface along each side of footing without cell (Fig. 4(a)).

It is observed that the installation of the cell produced significant lateral confinement. This confinement has an effective role to prevent the lateral spreading associated with dynamic excitation below the foundation (Fig. 4(b)). The cell prevented the enclosed foundation soil from moving outside to the free field. It also modified the direction of the total sub-grade displacement to be dissipated below the confined region and no clear deformation or heave presented adjacent to the cell. Figure 4 is again confirmed the effect of adopted cell in decreasing the foundation displacement during loading stages. It noticed that for footing without cell, the footing displacement (point #1) increased with the increase of time while the cell is significantly reduced the total foundation displacement by as much as 75% for cell geometry (Hc/D = 1.50 and Dc/D = 1.1). It can be concluded that the confined cell sharply increased the sub-grade densification; consequently, the foundation displacement is reduced. The cell acted as a vertical barrier and reinforced element that increased the sub-grade shear strength due to containment effect.

## 4.2 Effect of Cell Depth and Diameter

In order to study the effect of cell barrier geometry on the vibration control, Fig. 5 illustrates a significant isolation effect in the total displacement amplitudes for both footing with / without cell barrier at point #1. It can be seen that the cell has modified the time displacement behavior. The existence of such cell can sharply decreased the total foundation displacement because the cell provided the lateral constraint tool to the sub-grade particle. The presence of the cell prevents the particles from moving to the region adjacent and outside the confined zone. So the soil



Fig. 4 The total displacement vectors for footing with/without cell under dynamic effect



Fig. 5 The time displacement curve for footing with/without cell for monitored point #1

particle moves only downward under the footing due to the confinement effect. In addition, the dynamic loading from the vibrating sources can be assumed as a dynamic compaction/repeated load during the loading stages; consequently, the confined soil was gradually densified. The total displacement of foundation without cell was found to be 0.16 while this value dropped to 0.038 m when the cell geometry is Hc/D = 1.5 and Dc/D = 1.10. This again confirms the effectiveness of the confined cell to control the foundation settlement during the dynamic loading.

As a general trend, for all amplitudes, the reduction value increases with an increase of cell depth. It can be seen that as the cell depth increased the amplitude reduction in displacement distinctly reduced. The cell should be installed with sufficient anchorage depth with minimum cell diameter closed to footing in order to provide significant isolation effect. By regarding Fig. 6, it can be concluded that there is no benefit from increasing the penetration depth of the cell beyond a limit value of Hc/D = 1.5. Further than this ratio, there is inappreciable effect because of the amplitude reduction in displacement remained constant. A significant decrease in reduction factor (Am) is noticed when the cell is installed for a depth less than 1.5B. This suggests that when Hc/D < 1.5 the depth of cell provides only partial confinement to the soil. The maximum improvement in reduction factor is related to both cell depth and location of the monitored point. It is observed that as the cell diameter decreases and installed close to the footing with sufficient embedded depth the good screening effect is obtained. A reduction level up to about 22% at point #1 at cell geometry of Hc/D = 1.5 and Dc/D = 1.10 can be achieved. While this percent is reached to 18% for point #2 at the free field outside the confined cell. This again confirmed the effectiveness of cell barrier to mitigate the displacement induced from vibration source. The effect of cell diameter is investigated and shown in Fig. 7 for point #2. It has been found that increasing cell diameter leads to increase in the amplitude reduction factor. This factor was found to be 45% at the cell geometry of Hc/D = 1.5 and Dc/D = 1.75. While this reduction dropped to 18% for normalized cell depth of Dc/D = 1.10. This again justified the effectiveness of cell when it is installed with minimum diameter closed to footing to provide sand confinement situation which controlled the propagation of vibration to adjacent soil.

Generally, this confinement has an effective role in preventing the lateral spreading associated with vibration below the foundation. The screening devices by the adopted cells are provided near the source of vibration, and then it is termed as active screening or active isolation. Effective screening of vibration may be achieved by proper interception, scattering and diffraction of surface waves by using barriers like the adopted cell.

#### 4.3 Effect of Cell in the Peak Particle Velocity

On the other hand, the effect of confined cell on the particle velocity data was further analyzed to determine the peak particle velocity for point #2 located in the free field, outside the confined cell. The variation of peak particle velocity in case of footing sub-grade with/without cell is illustrated in Fig. 8. It is seen that there is a substantial reduction in peak particle velocity for point #2 due to confinement effect when the cell installed with minimum diameter and sufficient depth. Further, the reduction in peak particle velocity reached to 50% of its initial value of footing without cell for the cell geometry of Hc/D = 1.00 and Dc/D =1.10. While this percentage reduction is found to be 90% at Hc/D= 1.50 and Dc/D = 1.10. It is noticed that the gradual reduction in the peak velocity of soil particles in the free field confirmed the significant of cell which provided an obstruction for the surface waves. It can be concluded that the adopted cell decays the vibrations that transmitted to adjacent soil as clearly shown in the dropping off the velocity values of point #2 in the free field.

#### 4.5 Effect of Cell in the Excess of Pore Water Pressure

In order to study the effect of confined cell in the induced pore water pressure at adjacent soil in the free field point #2, Fig. 9 shows the relation between the reduction of the excess pore water with time at different cell depth. The induced excess pore



Fig. 6 The variation of amplitude displacement reduction factor with normalized cell depth for monitored point #1 and #2



Fig. 7 Variation of displacement reduction factor with normalized cell diameter for monitored point #2

water pressure from the vibration source can be controlled using the adopted cell. The cell prevented the dissipation of the pore water pressure to the region around the confined cell. At the same cell diameter (Dc/D = 1.10), the excess pore pressure is decreased as the cell depth increased, compared with normal footing without cell. The excess pore water pressure is reduced by as much as 60% to 90% of its initial value of foundation without cell for the cell depth Hc/D = 1.00 and 1.5, respectively. It is also noticed that, at small cell diameter with sufficient embedment depth, the pore water pressure migration has totally gone below the confined zone. The excess pore water pressure is developed under the confined zone; as a result, the induced excess pore water pressure form the vibration source is partially to totally eliminate. This is also confirmed the effectiveness of confined cell on absorbing the disturbance due to densification effect. Since the confined block were able to limit and relief lateral migration of pore water pressure outside the confined cell as distinctly observed in Fig. 9 for point #2.



Fig. 8 Time versus velocity curve for point #2 at Dc/D = 1.10





Fig. 9 Time versus excess pore water pressure curve for point #2 at Dc/D = 1.10

# 4.6 Effect of Cell in the Damping Properties of Sub-grade

The influence of placing cell below the foundation of vibrating source on the shear modulus and damping characteristics of sub-grade soil is well established. Monitored point at foundation level and within the confined zone, point #1 was selected to determine both the shear modulus and damping coefficient at different cell depth. These parameters were obtained by back analysis in which the shear wave velocity (Vs) of sub-grade can be extracted from the numerical analysis at different cell depth. Once shear wave velocity was obtained, the shear modulus (G) of sub-grade can be determined from the following equation:

$$Vs = \sqrt{\frac{G}{\rho}} \tag{1}$$

where  $\rho$  is the soil density,  $\rho = \gamma/g$ . ( $\gamma$  is the unit weight of sub-grade soil and g is the gravity). Therefore, the values of damping coefficient (*C*) of sub-grade were determined relative to the obtained values of the shear modulus from the following equation as stated by Hardin and Drnevich (1972),

$$C = \frac{3.4r_o^2}{1-v}\sqrt{G\rho}$$
(2)

where  $r_o$  is the equivalent radius of foundation and  $\nu$  is poison's ratio of soil. In order to study the effect of cell on the dynamic sub-grade properties, the relationship between damping ratio and cell depth ratio (Hc/D) is shown in Fig. 10. The ratio of damping cofficients ( $\zeta$ ) is defined as the ratio between the damping coefficients of cell foundation system to damping coefficient of normal foundation sub-grade without cell ( $\zeta = C_{\text{with cell}} / C_{\text{without cell}}$ ). has been found that, the confinement effect due to cell was modified and increased the shear modulus with upon increase in Hc/Dratio. As a result, the damping ratio increased with the increase of cell depth. The parameter of vertical cell below the footing considerably produced a progressive densification during the earthquake. Hence the sub-grade density is increased and the damping coefficient is also increased. The improvement in damping coefficient is attained to 230% for the cell depth of Dc/D = 1.10. It indicated that, the confined soil within the cell acted as a coherent compacted mass, which may be tended to damper and provided an effective screening for vibration. This damper can absorb the induced vibration and act as an active vibration isolation system, particularly decreases the soil disturbance.



Fig. 10 Ratio of damping coefficients versus normalized cell depth

## 5. CONCLUSIONS

In this research, a two-dimensional finite element analysis of a vibration isolation using confined cell below the foundation of a generator in active system has been conducted with the validated computer program Plaxis (8.2). Based on the obtained results, the following conclusions can be drawn:

- 1. The screening performance of a confined cell is an effective method in active isolation case to control the deformation induced from the vibration source. It can be achieved by proper interception, scattering and diffraction of surface waves.
- 2. The optimal cell depth and diameter for achieving an ideal reduction in ground displacement amplitude are found to be Hc/D = 1.5 and Dc/D = 1.10.

- 3. The cell barrier reduced the total foundation displacement by as much as 75% at optimal cell geometry.
- Using confined cell can partially to totally eliminate the displacement in the free field adjacent to cell; as a result, the geometrical damping is improved.
- 5. At optimal cell geometry, the peak particle velocity and the excess of pore water pressure for the monitored point at the free field are reduced by 90% of its initial value.
- 6. Installing the cell below the foundation is produced gradual sub-grade densification; as a result, the sub-grade soil damping is improved by 230%. Consequently, the confined soil behaves as if a damper with an effective screening for vibration.

Finally, it is recommended that additional research should be carried out in order to: (1) study the effect of soil type and soil relative density, (2) apply this technique in the full scale experimental test in the site using different dynamic sources.

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