Journal of GeoEngineering, Vol. 10, No. 1, pp. 11-16, April 2015
http://dx.doi.org/10.6310/jog.2015.10(1).2

FINITE DIFFERENCE SCHEME FOR FINITE STRAIN CONSOLIDATION CONSIDERING SECONDARY COMPRESSION

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ABSTRACT

This paper incorporates a simple secondary compression model into the finite strain one-dimensional consolidation equation to calculate the volumetric strain time curve. An explicit finite difference method is used to solve the differential consolidation equation. Results are compared with the laboratory test data with widely different load increment ratio and results calculated from another conventional consolidation equation expressed by the excess pore pressure. The validity for the assumption of the coefficient of volume compressibility defined by the primary compression and the adaptability of the explicit finite difference consolidation analysis are also discussed. It is emphasized that the analysis proposed in this paper is convenient to a rule of trial and error in order to check the coefficient of volume compressibility defined by the primary compression if it is reasonable or not.

Key words: One dimensional consolidation, secondary compression, clay, finite difference method.

1. INTRODUCTION

In a conventional one-dimensional consolidation test, the soil specimen is confined laterally by a rigid metal ring 6 cm inside diameter and 2 cm in depth and consolidated in the vertical direction. The maximum distance for pore water to flow for the drainage is about 1 cm as the specimen is located between two highly permeable metals. The rate of consolidation is obtained by observing the consolidation settlement at frequent time intervals and the specimen is left under a given constant load for about one day until the next load increment is added. Fig. 1 shows the relationship between the compression index and the typical one-dimensional consolidation settlement time curves. The settlement of some clays under sustained loading continues almost indefinitely but the primary consolidation component finally ought to reach the ultimate settlement at the end of the primary consolidation. All of the primary consolidation occurs in very short time, usually in less than about one hour. Additional secondary compression will occur if the specimen is left under a given load more than one day. Accordingly, it is difficult to observe the typical inverted "*S*" shape of the entire time-compression curve with pronounced primary consolidation effects when plotted to a semi-logarithmic graph. Secondary compression is notable after primary consolidation is completed but may occur also during the primary consolidation period (Taylor 1948). As only the total settlement which consists of primary and secondary compression can be read on dial gage in the conventional consolidation test, it is impossible to measure separately the secondary compression involved in the total compression during primary consolidation.

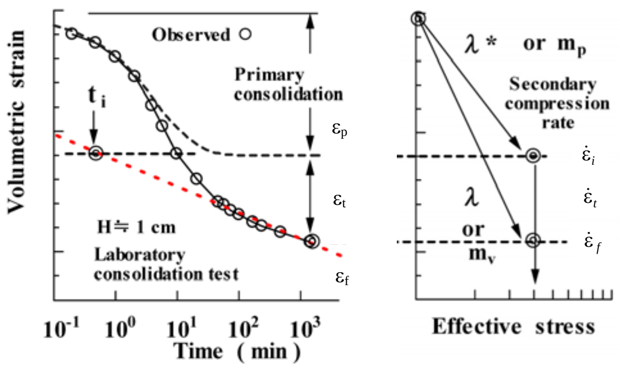

Fig. 1 Primary and secondary settlement-time curve (for illustration only)

Consequently, the secondary compression behaviors during primary consolidation is not fully understood by the experimental evidence. If the compression index λ defined by the total compression is adopted to evaluate the primary compression, the studies on primary and secondary compression are doubtful. The compression index or the coefficient of compressibility calculated by the total compression at the elapsed time of one day includes the effects of secondary compression. It should be determined by the amount of primary compression. It is the purpose of this paper to examine the assumption for the ratio of primary and secondary compression involved in the total compression and also to present a simple one-dimensional finite difference analysis in terms of finite strain consolidation taking account of the secondary compression.

Finite strain consolidation theories have been independently established by Mikasa (1963) and Gibson *et al*. (1967), although it is well known that the conventional consolidation theory based on the dissipation of excess pore pressure has been developed by Terzaghi (1943). This paper compared the two kinds of the consolidation analyses according to the different equation. It is shown that the calculation owing to the finite strain consolidation equation is simple and can be solved relatively easily.

Manuscript received October 14, 2014; revised December 15, 2014; accepted January 7, 2015.
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2. CONSOLIDATION EQUATIONS AND SECONDARY COMPRESSION

Assuming Darcy's law and the linear stress strain relation for saturated clays to be valid, the continuity condition that governs one-dimensional consolidation can be expressed as follows. where ε is the total volumetric strain (= vertical strain), t is the time, m_v is the coefficient of volume compressibility, σ_y is the applied stress component in the y-direction, u is the excess pore pressure, k is the permeability. Equation (1) is the basic differential equation of Terzaghi's consolidation theory (Terzaghi 1948) and the second term $\partial\sigma_y / \partial t$ of Eq. (1) means the rate of the time-dependent load.

$$\frac{\partial\varepsilon}{\partial t} = \frac{m_v(\partial\sigma_y - \partial u)}{\partial t} = \frac{\partial}{\partial y}\left(-\frac{k}{\gamma_\omega}\frac{\partial u}{\partial y}\right) \Rightarrow \frac{\partial u}{\partial t} = c_v\frac{\partial^2 u}{\partial y^2} + \frac{\partial\sigma_y}{\partial t} \tag{1}$$

$$\frac{\partial\varepsilon}{\partial t} = \frac{\partial}{\partial y}\left(\frac{k}{\gamma_\omega\, m_v}\frac{\partial\varepsilon}{\partial y}\right) = c_v\frac{\partial^2\varepsilon}{\partial y^2} \tag{2}$$

Equation (2) of the finite strain consolidation developed by Mikasa (1963) is well known to be equivalent to Eq. (1). Although the term of the time-dependent load cannot be found out, the derivation of Eq. (2) does not require that the applied load is kept constant during a process of consolidation. In the case of time-dependent load, the volumetric strain at the drainage boundary is a function of the applied load. The relation between the vertical effective stress increment and the vertical (= volumetric) strain is given by Eq. (3).

$$m_v(=\text{const.})\,\Delta\sigma'_y(\propto \text{time}) = \Delta\varepsilon = m_v(\propto \text{time}) * \Delta\sigma'_y(=\text{const.}) \tag{3}$$

The left side relation in Eq. (3) means the case of the time- dependent loading and the right side one may be equivalent to the elasto-visco-plastic behavior. When taking account of secondary compression, the volumetric strain behavior at the drainage boundary is analogous to the case of time-dependent loading. It is also well known that it is very easy for Eq. (2) to deal with the time-dependent loading. In one-dimensional consolidation the rate of the total volumetric strain $\dot{\varepsilon}$ has been traditionally divided in primary and secondary compression component.

$$\dot{\varepsilon}(= \partial\varepsilon / \partial t) = \dot{\varepsilon}_p(= -m_p\,\dot{u}) + \dot{\varepsilon}_s \tag{4}$$

where $\dot{\varepsilon}_p$ and $\dot{\varepsilon}_s$ are the rate of primary and secondary compression, respectively, m_p is the coefficient of volume compressibility defined by the primary volumetric strain and the superposed "·" implies time rate.

$$\frac{\partial u}{\partial t} = c_v^*\frac{\partial^2 u}{\partial y^2} + \frac{\partial\sigma_y}{\partial t} + \frac{\dot{\varepsilon}_s}{m_p} \tag{5}$$

where $c_v^*\left(= k / (\gamma_w \times m_p)\right)$ is the coefficient of consolidation defined by using the primary volumetric strain. The third term of Eq. (5) $\dot{\varepsilon}_s / m_p$ is equivalent to the pore pressure induced by secondary compression (Takeda *et al*. 2012). Consolidation Eqs. (2) and (5) can be solved by the explicit finite difference solution under the following boundary and initial conditions (6) and (7), respectively.

$$\begin{bmatrix} t=0\ \&\ H \ge y \ge 0\,;\ \varepsilon = 0\ \&\ \dot{\varepsilon} = \dot{\varepsilon}_s = 0 \\ t \ge 0\ \&\ y = H\,;\ \partial\varepsilon/\partial y = 0 \\ t > 0\ \&\ y = 0\,;\ \Delta\varepsilon = f(\Delta\sigma', \Delta\varepsilon_s) \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} t=0\ \&\ H \ge y \ge 0\,;\ u = u_0(=\Delta\sigma_y) \\ t \ge 0\ \&\ y = H\,;\ \partial\varepsilon/\partial y = 0 \\ t > 0\ \&\ y = 0\,;\ u = 0 \end{bmatrix} \tag{7}$$

where H is the length of the longest drainage path and u_0 is the initial excess pore water pressure. It is obvious that Eqs. (6) and (7) do not include the condition concerning one-dimensional deformation. Finite difference consolidation analysis assumes that the change in excess pore pressure during consolidation merely leads to the increase of the vertical strain.

If the finite element method is adopted for the consolidation analysis, the coupled matrix equation based on Biot's theory can take account of one-dimensional compression conditions and be written as follows (Smith 1982)：

$$\begin{bmatrix} \underline{K} & \underline{C} \\ \underline{C}^T & \Delta t * \underline{P} \end{bmatrix}\begin{bmatrix} \underline{d_{t+\Delta t}} \\ u_{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{C}^T & \underline{0} \end{bmatrix}\begin{bmatrix} d_t \\ u_t \end{bmatrix} + \begin{bmatrix} \underline{F_t} + \underline{F_s} \\ \underline{0} \end{bmatrix} \tag{8}$$

where $\underline{K}$ is the stiffness matrix, $\underline{P}$ is the permeability matrix, $\underline{C}$ is the coupling matrix, d_t is the nodal displacements and u_t is the nodal excess pore pressure. Subscript "t" means a time. $\underline{F_t}$ are the external nodal loads and $\underline{F_s}$ are the equivalent nodal force converted from secondary compression. The total strain components $\underline{\varepsilon}$ are calculated by using $d_{t+\Delta t}$ obtained from Eq. (8). and then, effective stresses $\underline{\sigma}$ and equivalent nodal forces $\underline{F_s}$ are obtained from Eqs. (9) and (10), respectively.

$$\underline{\sigma} = \underline{D}(\underline{\varepsilon} - \underline{\varepsilon_s}) \tag{9}$$

$$\underline{F}_s = \int \underline{B}^T \underline{D}\underline{\varepsilon_s}\, dv \tag{10}$$

where $\underline{D}$ is the stress-strain matrix, $\underline{B}$ is the strain-displacement matrix and $\underline{\varepsilon_s}$ is the strain components based on secondary compression. In this paper, the Bulk modulus K involved in $\underline{D}$ matrix is simply assumed to be equal to the inverse of m_p. K and the shear modulus G are expressed by Eqs. (11) and (12), respectively.

$$K = \frac{1}{m_p} \tag{11}$$

$$G = \frac{1.5(1-2\upsilon)}{(1+\upsilon)}K \tag{12}$$

where υ is Poisson's ratio.

In order to calculate secondary compression behaviors, Eq. (13) is used in this paper although the studies on secondary compression model are extensive and an analogous one is presented by the Authors in a previous paper (Takeda *et al*. 2012).

$$\Delta\varepsilon_s = \alpha \ln(t / t_i) \tag{13}$$

where t is the elapsed time at depth y of the consolidation layer and the time t_i means the beginning of secondary compression and is not equal to time for the end of primary consolidation.

If the magnitude of primary compression $\Delta\varepsilon_p$ is assumed and then, at a time t_f the total compression $\Delta\varepsilon_f$ is obtained from the consolidation test, secondary compression is given by

$$\Delta\varepsilon_s = \Delta\varepsilon_f - \Delta\varepsilon_p \ ; \quad m_p = \Delta\varepsilon_p / \Delta\sigma_y \tag{14}$$

Substituting the above relation into Eq. (13), the time t_i can be calculated from the following Eq. (15) as

$$t_i = t_f \ \exp(-\Delta\varepsilon_s / \alpha) \tag{15}$$

The numerical procedure and secondary compression model described above are used to predict the settlement-time curve in one-dimensional consolidation tests.

3. EXAMPLE OF CALCULATION

3.1 Soil Parameters

Figure 2 presents the volumetric strain-time curves calculated by Eq. (2) together with the observed values reported by Aboshi (1973).

The procedure of determining the parameters is as follows:

1. The coefficient of volume compressibility m_v is calculated from the volumetric strain $\Delta\varepsilon_f$ of one day after. The coefficient of volume compressibility m_p defined by the amount of primary compression is arbitrary assumed to be less than the value of m_v. The ratio of m_p to m_v is named primary consolidation ratio in Japan.
2. The coefficient of consolidation c_v is estimated by the square root of time method.
   $T_v \times H^2 / t_{90} = 0.848 \times 1^2/14.5 = 0.06$ (cm^2/min)
   This value is affected by secondary compression. c_v value should be determined by the primary consolidation time curve although there is little possibility.
3. The coefficient of secondary compression α is easily determined from the final slope of the semi-logarithmic plot of the consolidation-time curve after the primary consolidation has ended.
   $\alpha = 0.43 \times \Delta\varepsilon / \log(t_2 / t_1)$
   $= 0.43 \times 0.0155/\log(1440/180) = 3.3 \times 10^{-3}$
4. If the value of m_p is assumed, the value of $\Delta\varepsilon_s$ obtained from Eq. (14) corresponds to $t_f = 1440$ min and t_i is calculated by Eq. (15). As shown in Fig. 2, four kinds of the value of m_p are used in the trial and error calculation.

As can be seen in Fig. 2 secondary compression begins after the end of primary consolidation in the case of $m_p / m_v = 0.9$. The predicted values using $m_p / m_v = 0.5$ underestimates the strain.

Good agreement between the predicted observed volumetric strain-time curves may be seen to be close when the value of m_p is assumed to be equal to 0.7 or 0.6. Figure 3 shows the calculated volumetric strain-time curves in the pervious boundary. These curves are equivalent to the time dependent behaviors under the consolidated drained creep test for clays with an infinite permeability. The magnitude of assumed primary compression can be confirmed by the calculated values before the time t_i Almost time t_i may be less than that of the end of primary consolidation. t_i corresponds to the drainage distance H = 1 cm and depends on $\Delta\varepsilon_f$ at t_f.

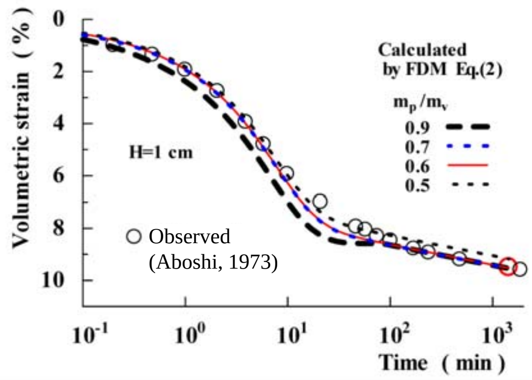

Fig. 2 Volumetric strain-time curves

3.2 Maximum Drainage Distance H and t_i

There is little experimental evidence of the effect of sample thickness on secondary compression. Ladd *et al*. (1977) present two cases, Hypothesis *A* and *B* for time effects on one-dimensional consolidation time curves. Aboshi (1973) has shown one dimensional consolidation time curves on specimens with several initial thickness and the observed results have supported Hypothesis *A* in which the curve of thin sample simply displaced in proportional to the squared ratio of the drainage distance as shown in Fig. 4.

The basic soil parameter already given are used to predict the behavior of thick sample (H = 10 cm) set up with the same boundary conditions as the thin sample (H = 1 cm).

If t_i is assumed to be independent to the change of the drainage distance, the predicted volumetric strain-time curves are denoted by the broken line in Fig. 4. However, if t_i depends on H^2 rule, the predicted curves are shown by solid lines. The observed curve tends to substantiate the latter assumption. It can be considered that an important point is how to postulate the time t_i for field clay layers with different drainage distance.

3.3 Another Consolidation Equations

Figure 5 presents the comparison of results denoted in Fig. 2 with calculated values due to Eqs. (5) and (8) in terms of excess pore pressure. Please note that the results by Eq. (8) were obtained with FEM, while the results by Eq. (2)-finite strain consolidation and Eq. (5)-conventional consolidation theory were obtained by FDM.

In finite difference analysis, the maximum drainage distance is divided into 45 equal layers ($\Delta y = H/45$). However, in finite element calculations, it is divided into 10 equal elements as shown Fig. 6. The same parameters described the above section are used again to the calculation based on Eqs. (5) and (8).

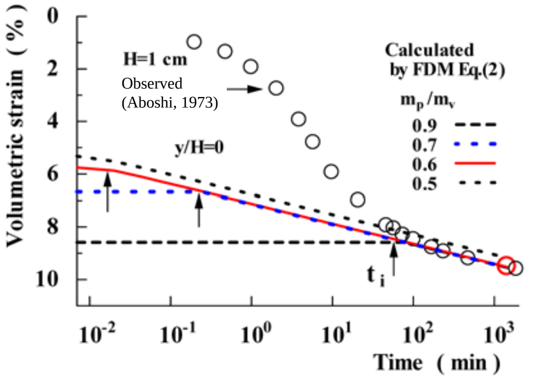

Fig. 3 Calculated volumetric strain-time curves at the drainage boundary

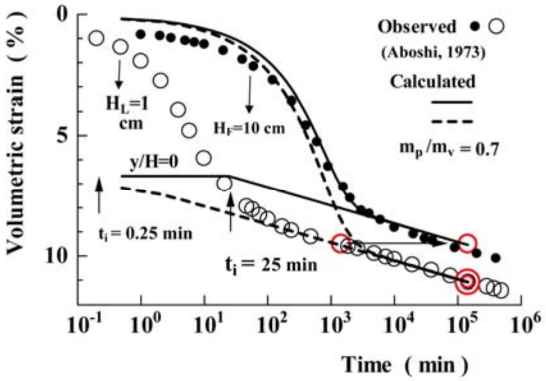

Fig. 4 Volumetric strain-time curves with different drainage distance

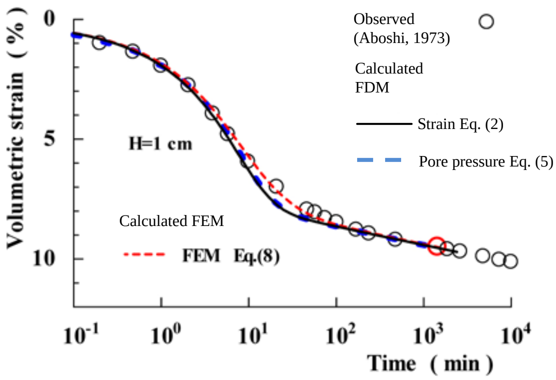

Fig. 5 Volumetric strain-time curves

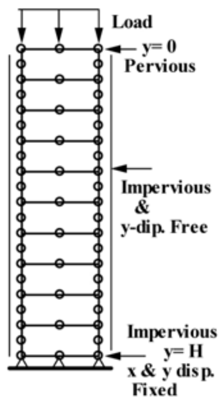

Fig. 6 FE mesh

The best obtained fits are shown in Fig. 5. As can be seen, there is reasonable agreement among the predicted volumetric strain-time curves.

4. LABORATORY INVESTIGATION

Average physical properties for clays are specific gravity = 2.64, liquid limit = 67% and plastic limit = 36%. Portions of different particle sizes are sand = 11%, silt = 55% and clay = 34%. The undisturbed clay samples are normally pre-consolidated under the vertical pressure 39.2 kPa.

One-dimensional consolidation tests are conducted for the various prescribed load increment ratio $\Delta\sigma_y / \Delta\sigma_{y0}$, which ranged widely from 0.25 to 1.5.

The prediction is again confirmed by finite difference analysis based on Eq. (2). Load increment and soil constants are given in Table 1. The coefficient of secondary compression α increases with the load increment ratio $\Delta\sigma_y / \Delta\sigma_{y0}$. It is difficult to determine 90% primary compression according to the square root of time method. Therefore the value of c_v with a under line is assumed by the curve fitting method.

As can be observed in Figs. 7 and 8, the proposed analysis due to Eq. (2) predicts also the remarkable accuracy the volumetric strain-time curves for the different loading increment ratios.

Figure 7 presents the results by means of the trial and error calculation in which the value is assumed to be equal to 0.9, 0.65 or 0.55. The numerical results by means of Eq. (2) for all experimental results showed that a ratio of 0.65 is a best fit, as shown in Fig. 8. From these curves it may be shown that for the assumed value of m_p / m_v, t_i tends to increase with the reduction of the load increment ratio. The agreement between the observed and predicted curves is again seen to be reasonable.

At present it is impossible to confirm whether the assumption used in above calculations is good or not.

Table 1 Soil constants related to consolidation test

Load (kPa)	9.8	14.7	19.6	29.4	39.2	58.9
m_v (1/kPa)	2.07	3.11	4.00	4.87	4.97	4.64
$\alpha \times 10^{-3}$	0.342	0.636	0.845	1.03	1.05	1.22
t_{90} (min)	×	×	×	7.5	4.2	3.6
c_v (cm^2/min)	<u>0.01</u>	<u>0.02</u>	<u>0.05</u>	0.1	0.18	0.20
t_i (min)	174	112	54	11	2.1	0.6

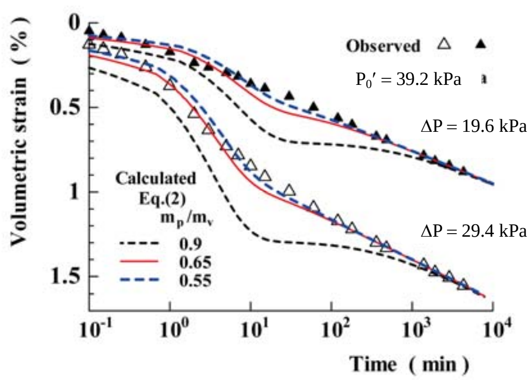

Fig. 7 Volumetric strain-time curves in trial and error calculations

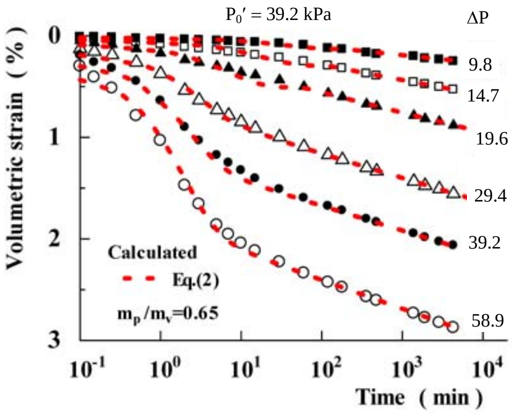

Fig. 8 Volumetric strain-time curves for various load increment ratio

5. CONCLUSIONS

Finite difference method of analyzing one dimensional consolidation taking account of secondary compression developed from the finite strain consolidation equation, appears to give reliable predictions of the volumetric strain-time curves for laboratory consolidation tests with widely different load increment ratio. However, the proposed analysis needs to assume the ratio of the primary and total volumetric strain. It is currently impossible to measure separately the primary compression included in the total compression. There is no reliable method other than a rule of trial and error. However, by the use of the proposed analysis, the effects of secondary compression taking place during the primary consolidation can be successfully interpreted.

In applying the laboratory consolidation test to practical field cases, it must be recognized that the problem of similitude consolidation of clay layers depend on not only the constitutive model of clays but also the assumption of soil constants used in the numerical solution. Finally, finite difference program used in this paper is listed in the appendix and is also available to the readers in this journal.

APPENDIX

One-dimensional consolidation analysis based on the explicit finite strain consolidation equation by Excel VBA.

Option Explicit

```
Sub FDM()

Range(Cells(1, 5), Cells(35, 7)).Select
Const n As Integer = 11            '  n = number of nodal point
Dim A(n) As Double, B(n) As Double
Dim H As Single, CV As Single, DP As Single, EF As Single, MV As Single
Dim AL As Single, VFD As Single, MP As Single, EP As Single, ES As
Single
Dim VID As Single
Dim U As Single, TV As Single, DT As Single, CT As Single, SUM As Single
Dim i As Integer, k As Integer, m As Integer

H = Range("B3")      '  H = Maximum drainage distance cm
CV = Range("B5")     '  CV =  Coefficient of consolidation cm2/min
MV = Range("B7")
' MV =  Coefficient of volume compressibility cm2/kgf
DP = Range("B9")     '  DP =  Load increment kgf/cm2
AL = Range("B11")    '  AL=α = Coefficient of secondary compression
VFD = AL / 1440
' VFD = Rate of secondary compression at time 1440 min
MP = Range("B14")
' MP =  Coefficient of volume compressibility cm2/kgf  defined by
' primary compression
EP = MP * DP
' EP =  MP*DP Primary compression at drainage boundary
A(1) = EP
' Primary consolidation copletes immediately after loading
EF = MV * DP         '  EF =  MV*DP Total compression at 1440 min
VID = VFD * Exp((EF - EP) / AL)
' VID= Initial rate of secondary compression
 ES = 0                ' ES = Secondary compression at time = 0

m = n - 1                          ' m = Number of element
DT = 0.5 * (H / m) ^ 2 / CV        ' DT =  Time increment min

For  k = 1  To  19
U = k * 5                          ' U= Degree of consolidation
If k < 10 Then                     ' TV= Time factor at every dU=5%
TV = 0.785 * (U / 100) ^ 2         ' i=19 ～  U= 95%
Else
TV = 1.781 - 0.405 * Log(100 - U)
End If
Cells(k, 5) = TV * H ^ 2 / CV      ' Cells(k,5) = Elapsed time min
Next k

' Set up  time interval  in secondary compression period
For k = 20  To  30                 ' Cells(30,5) = 1440 min
Cells(k, 5) = Cells(k - 1, 5) * 1.55   ' Constant 1.55 = variable
Next k

For i = 2  To  n      '  Zero clear strain in consolidation layer
A(i) = 0             ' Strain=0 at Nodal point i
Next  i              ' Nodal point of i=1 at drainage boundary
                       ' K=1～19   U = 5～95 %

For k = 1 To 30        ' k > 20 ～   Cal. of secondary compression
AGAIN:
For  i = 2  To  m      ' Consolidation equation in terms of strain
B(i) = 0.5 * (A(i - 1) + A(i + 1))
Next i
B(n) = A(m)            ' i = n = 11 at impermeable point

For i = 2 To n         ' B(i)= Strain after time DT
A(i) = B(i)
Next i
ES = ES + VID * Exp(-ES / AL) * DT
A(1) = EP + ES

CT = CT + DT           ' IF Elapsed time  CT  <  Cells(K, 5)
If  CT  <  Cells(k, 5)  Then  GoTo  AGAIN
' Go back  Label  AGAIN

SUM = 0.5 * (A(1) + A(n))
For  i = 2  To  m
SUM = SUM + A(i)       ' SUM = Sum of strain at all points
Next  i

Cells(k, 6) = SUM / m      ' Cells(i,6) = Average strain
Cells(k, 7) = ES
' Cells(i,7) = Secondary compression at drainage boundary

Next  k

End  Sub
```

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